Hashing

What is it?

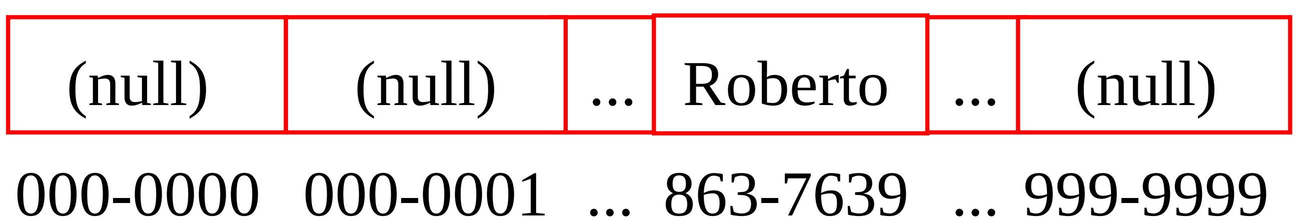
A form of narcotic intake?

A side order for your eggs?

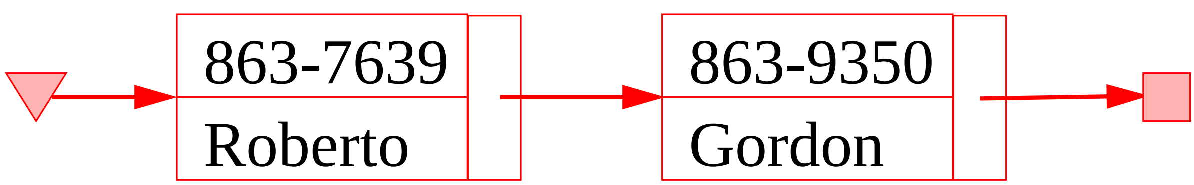
A combination of the two?

Problem

- RT&T is a large phone company, and they want to provide caller ID capability:
  - given a phone number, return the caller’s name
  - phone numbers are in the range R=0 to 10^7-1
  - want to do this as efficiently as possible ($$$)
- A few suboptimal ways to design this dictionary:
  - an array indexed by key: takes O(1) time, O(N+R) space -- huge amount of wasted space



  - a linked list: takes O(N) time, O(N) space



  - a balanced binary tree: O(lg N) time, O(N) space (you want fancy pictures here too? so read the slides from the RedBlack help session).

Another Solution

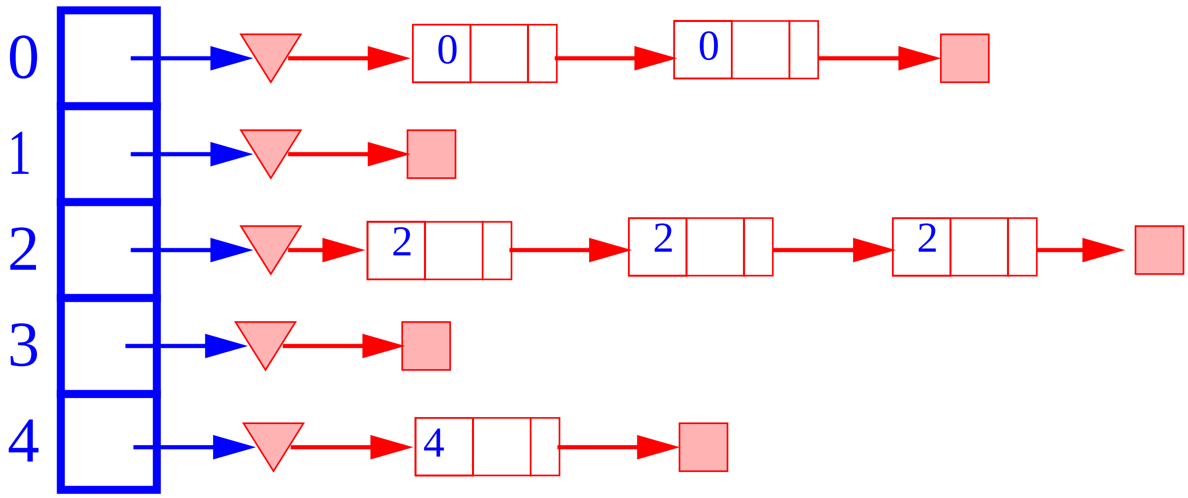
- We can do better, with a *Hashtable* -- O(1) expected time, O(N+M) space, where M is table size
- Like an array, but come up with a function to map the large range into one which we can manage
  - e.g., take the original key, modulo the (relatively small) size of the array, and use that as an index
- Insert (863-7639, Roberto) into a hashed array with, say, five slots
  - 8637639 mod 5 = 4, so (863-7639, Roberto) goes in slot 4 of the hash table

(null)	(null)	(null)	(null)	Roberto
0	1	2	3	4

- A lookup uses the same process: hash the query key, then check the array at that slot
- Insert (863-9350, Gordon)
- And insert (863-2234, Gordon). Don't skip this example!

Collision Resolution

- How to deal with two keys which hash to the same spot in the array?
- Use ***chaining***
  - Set up an array of links (a **table**), indexed by the keys, to **lists** of items with the same key



- Most efficient (time-wise) collision resolution
  - we'll talk about others later which use less space

Pseudo-code

- Any dictionary has 3 basic methods, and the constructor:
  - init
  - insert
  - find
  - remove

- Init
  - create table of M lists

- Insert(K)
  - index = h(K)
  - insert into table[index]

- Find(K)
  - index = h(K)
  - walk down list at table[index], looking for a match
  - return what was found (or error)

- Remove(K)
  - index = h(K)
  - walk down list at table[index], lookiing for a match
  - remove what was found (or error)

Hash Functions

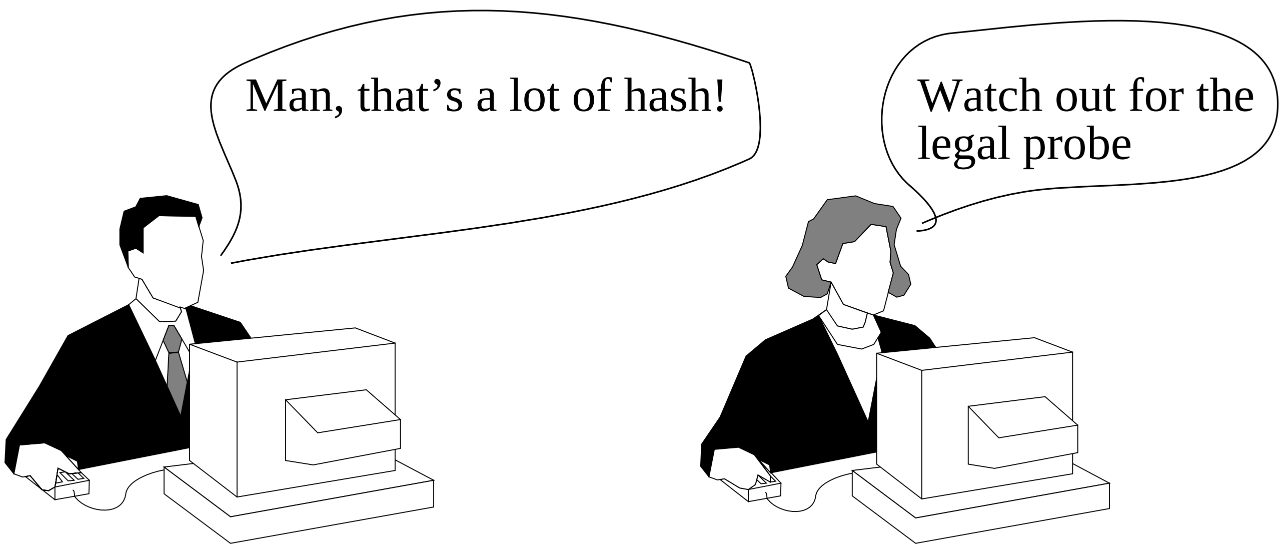
- Need to choose a good hash function
  - quick to compute
  - distributes keys uniformly throughout the table
- How to deal with hashing non-integer keys:
  - find some way of turning the keys into integers
    - in our example, remove the hyphen in 863-7639 to get 8637639!
    - for a string, add up the ASCII values of the characters of your string
  - then use a standard hash function on the integers
- Use the remainder
  - $h(K) = K \bmod M$
  - K is the key, M the size of the table
- Need to choose M
- $M = b^e$ (**bad**)
  - if M is a power of two, h(K) gives the e least significant bits of K
  - all keys with the same ending go to the same place
- M prime (**good**)
  - helps ensure uniform distribution
  - take a number theory class to understand why

Hash Functions (cont.)

- Mid-Square
  - $h(K)$ = middle digits of K^2
- I.E. Table size power of 10
  - h(4150130) = 21526 **4436** 17100
  - h(415013034) = 526447 **3522** 151420
  - h(1150130) = 13454 **2361** 7100
- I.E. Table power is power of 2
  - h(1001) = 10 **100** 01
  - h(1011) = 11 **110** 01
  - h(1101) = 101 **010** 01

More on Collisions

- A key is mapped to an already occupied table location
  - what to do?!?
- Use a collision handling technique
- We've seen *Chaining*
- Can also use *Open Addressing*
  - Double Hashing
  - Linear Probing



Linear Probing

- If the current location is used, try the next table location

```
linear_probing_insert(K)
   if (table is full) error

   probe = h(K)

   while (table[probe] occupied)
          probe = (probe + 1) mod M

   table[probe] = K
```

- Lookups walk along table until the key or an empty slot is found
- Uses less memory than chaining
  - don't have to store all those links
- Slower than chaining
  - may have to walk along table for a long way
- A real pain to delete from
  - either mark the deleted slot
  - or fill in the slot by shifting some elements down

Linear Probing Example

- h(K) = K mod 13
- Insert keys:

18 41 22 44 59 32 31 73

0	1	2	3	4	5	6	7	8	9	10	11	12

Double Hashing

- Use two hash functions
- If M is prime, eventually will examine every position in the table

```
double_hash_insert(K)
  if(table is full) error

  probe = h1(K)
  offset = h2(K)

  while (table[probe] occupied)
     probe = (probe + offset) mod M

  table[probe] = K
```

- Many of same (dis)advantages as linear probing
- Distributes keys more uniformly than linear probing does

Double Hashing Example

- h1(K) = K mod 13
  h2(K) = 8 - K mod 8
  - we want h2 to be an offset to add

18 41 22 44 59 32 31 73

0	1	2	3	4	5	6	7	8	9	10	11	12

Theoretical Results

- Let $\alpha = N/M$
  - the load factor: average number of keys per array index
- Analysis is probabilistic, rather than worst-case

Expected Number of Probes

	not found	*found*
Chaining	$1 + \alpha$	$1 + \frac{\alpha}{2}$
Linear Probing	$\frac{1}{2} + \frac{1}{2(1-\alpha)^2}$	$\frac{1}{2} + \frac{1}{2(1-\alpha)}$
Double Hashing	$\frac{1}{(1-\alpha)}$	$\frac{1}{\alpha} \ln \frac{1}{1-\alpha}$

Pretty Graph

Expected Number of Probes vs. Load Factor