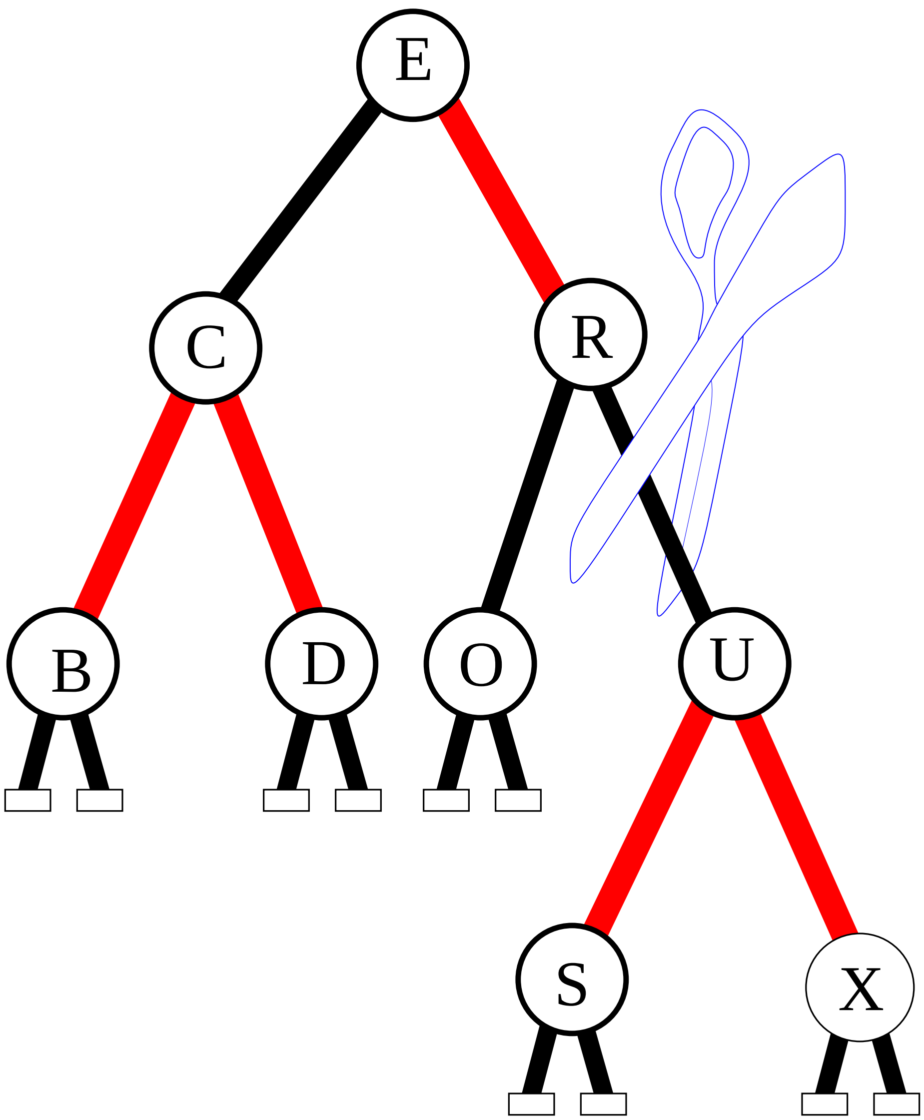
Deletion from Red-Black Trees

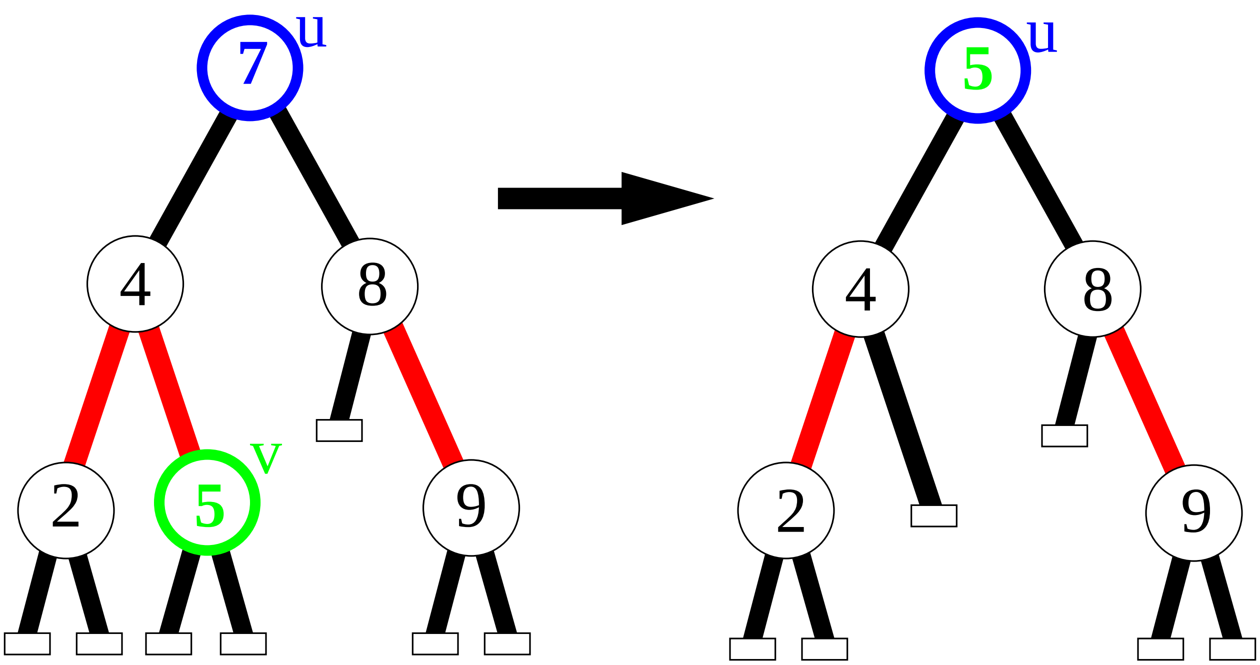


Setting Up Deletion

As with binary search trees, we can always delete a node that has at least one external child

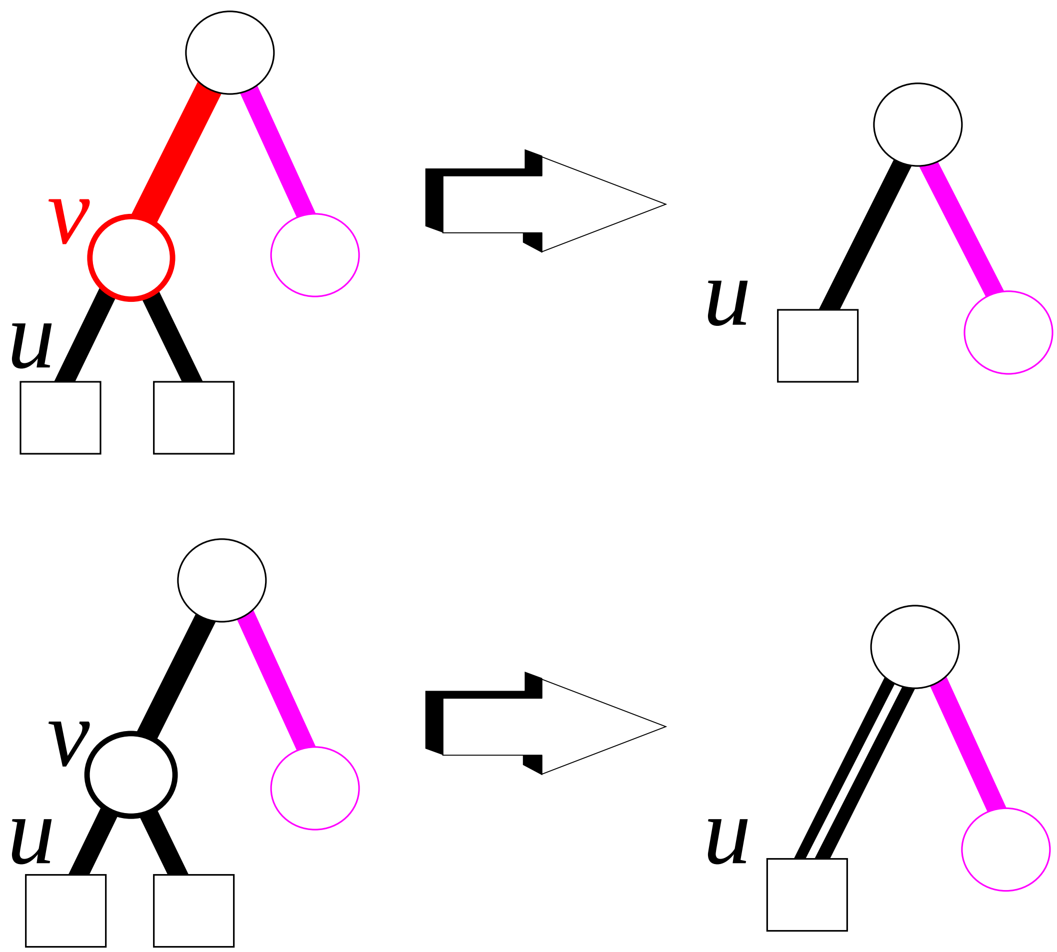
If the key to be deleted is stored at a node that has no external children, we move there the key of its inorder predecessor (or successor), and delete that node instead

Example: to delete key 7, we move key 5 to node u, and delete node v



Deletion Algorithm

1. Remove ***v*** with a removeAboveExternal operation
2. If ***v*** was red, color ***u*** black. Else, color ***u*** ***double black***.



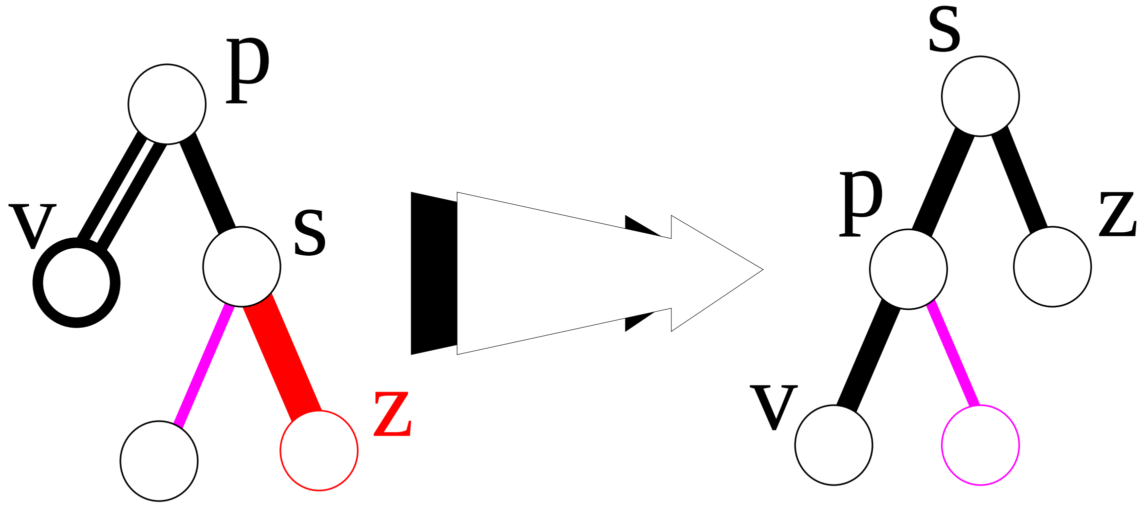
3. While a ***double black*** edge exists, perform one of the following actions ...

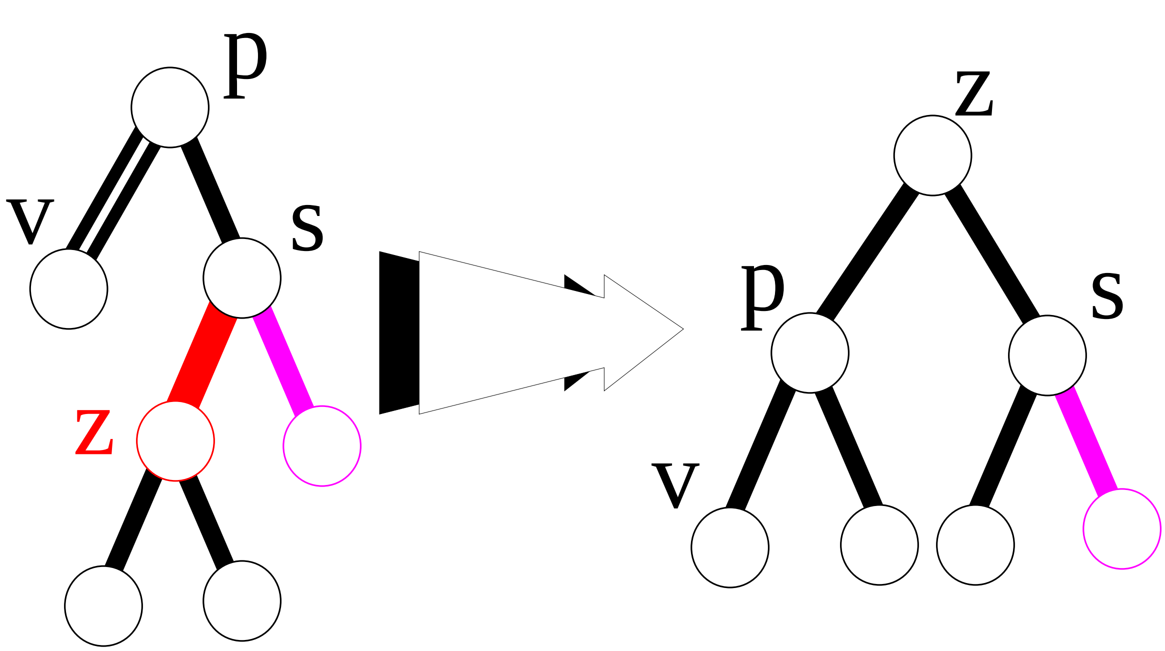
How to Eliminate the Double Black Edge

- The intuitive idea is to perform a "***color compensation***"
- Find a red edge nearby, and change the pair (***red*** , ***double black***) into
  (***black*** , ***black***)
- As for insertion, we have two cases:
  - ***restructuring***, and
  - ***recoloring (demotion***, inverse of promotion)
- Restructuring resolves the problem locally, while recoloring may propagate it two levels up
- Slightly more complicated than insertion, since two restructurings may occur (instead of just one)

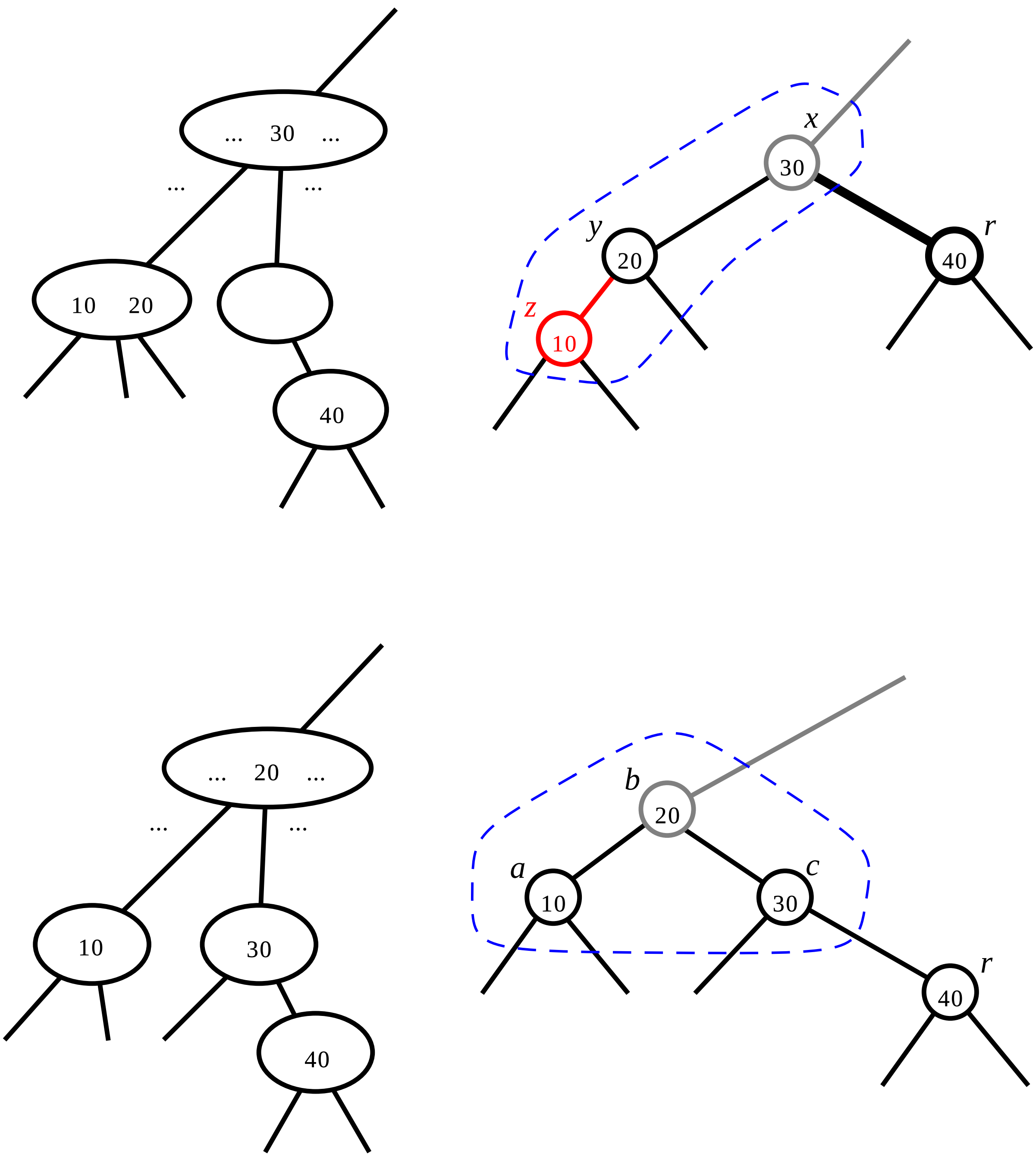
Case 1: black sibling with a red child

- If sibling is **black** and one of its children is **red**, perform a ***restructuring***



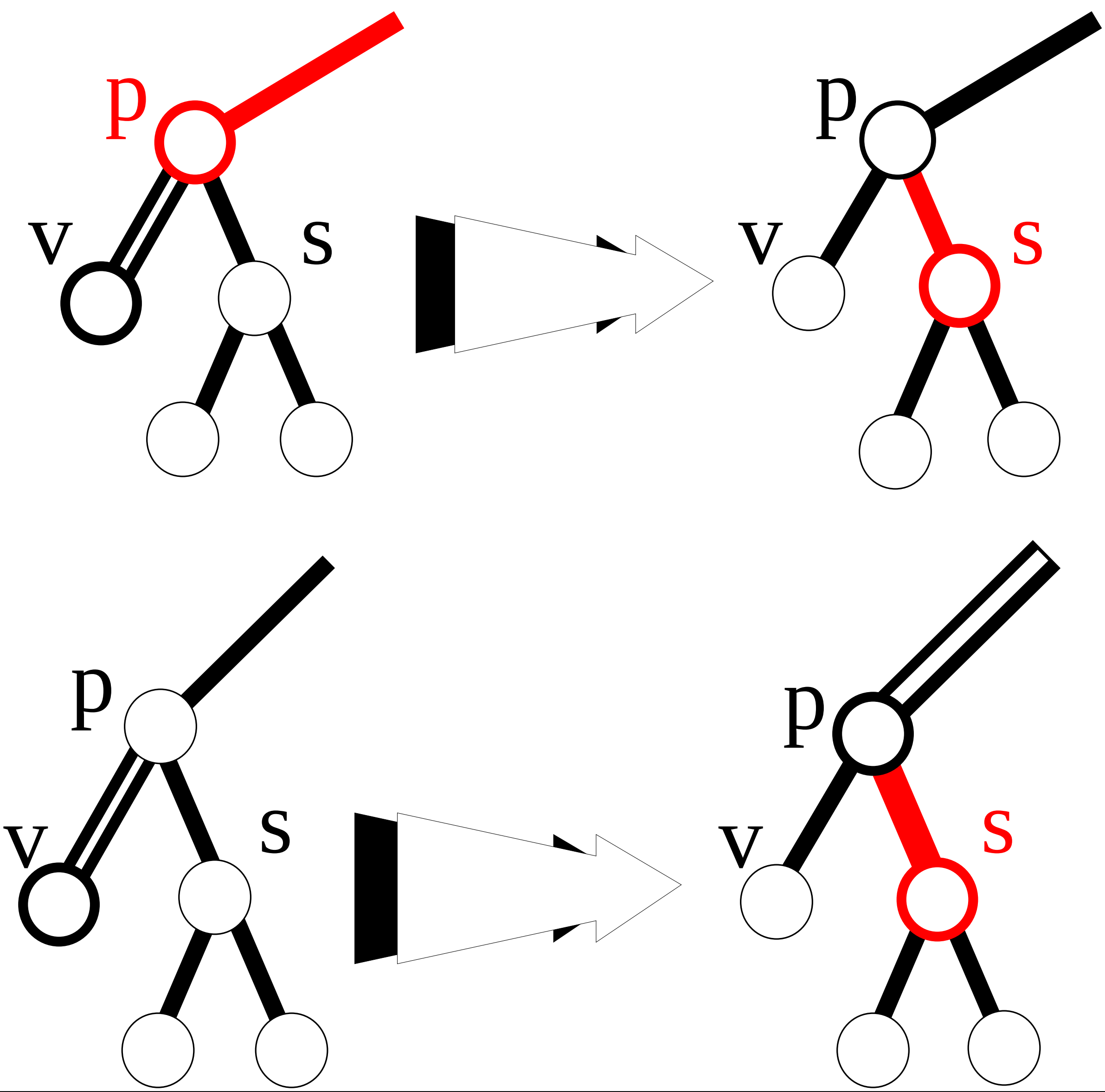


(2,4) Tree Interpretation

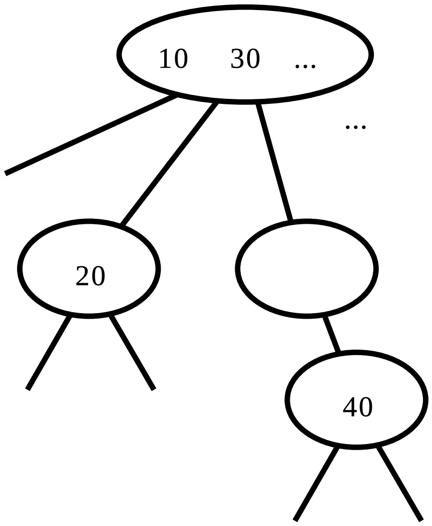


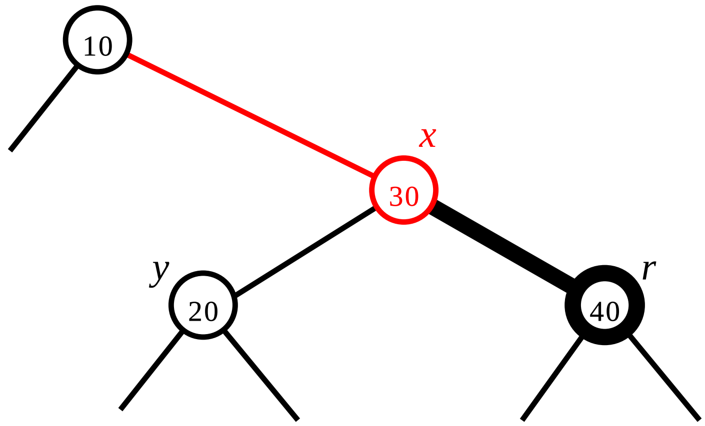
Case 2: black sibling with black childern

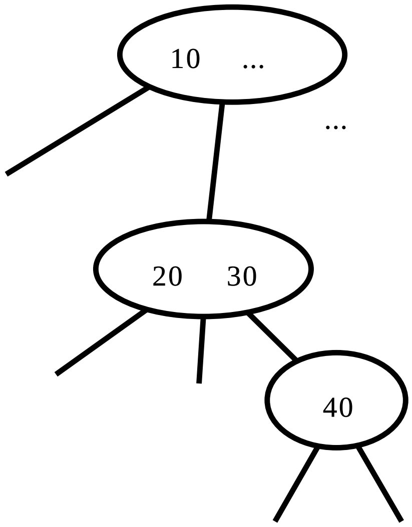
- If sibling and its children are **black**, perform a ***recoloring***
- If parent becomes **double black**, ***continue*** upward

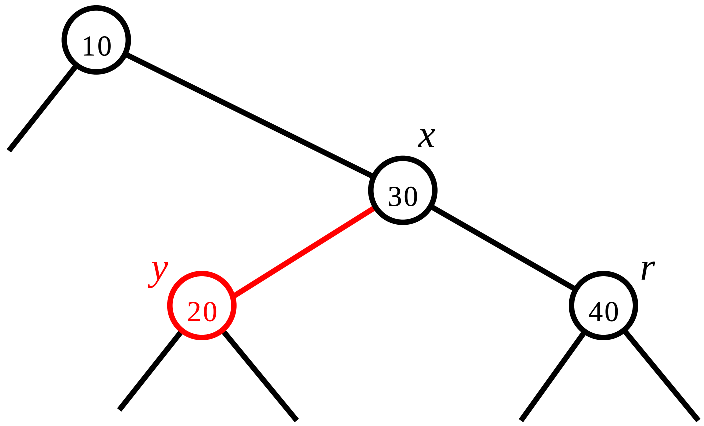


(2,4) Tree Interpretation



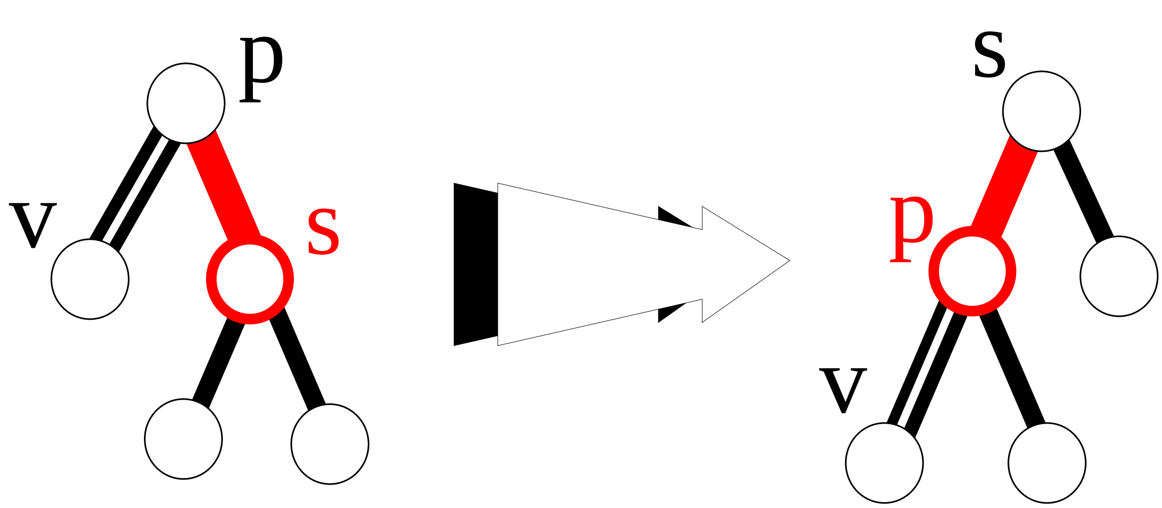






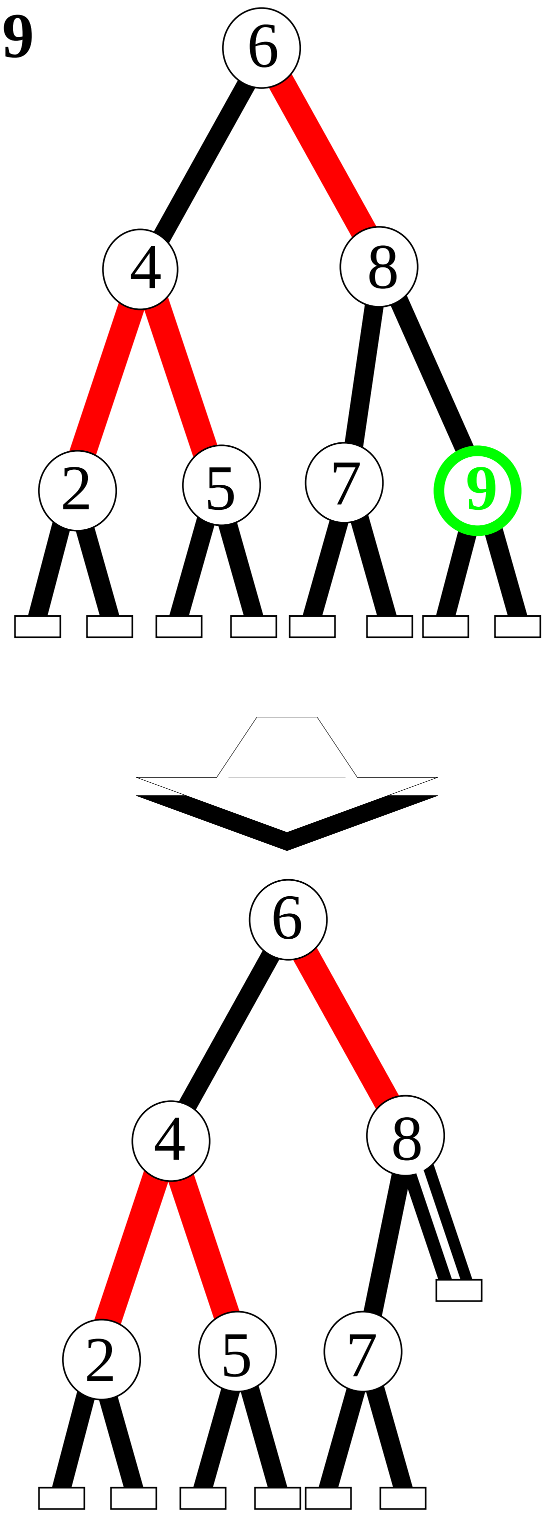
Case 3: red sibling

- If sibling is red, perform an ***adjustment***
- Now the sibling is **black** and one the of previous cases applies
- If the next case is recoloring, there is no propagation upward (parent is now red)



How About an Example?

Remove 9



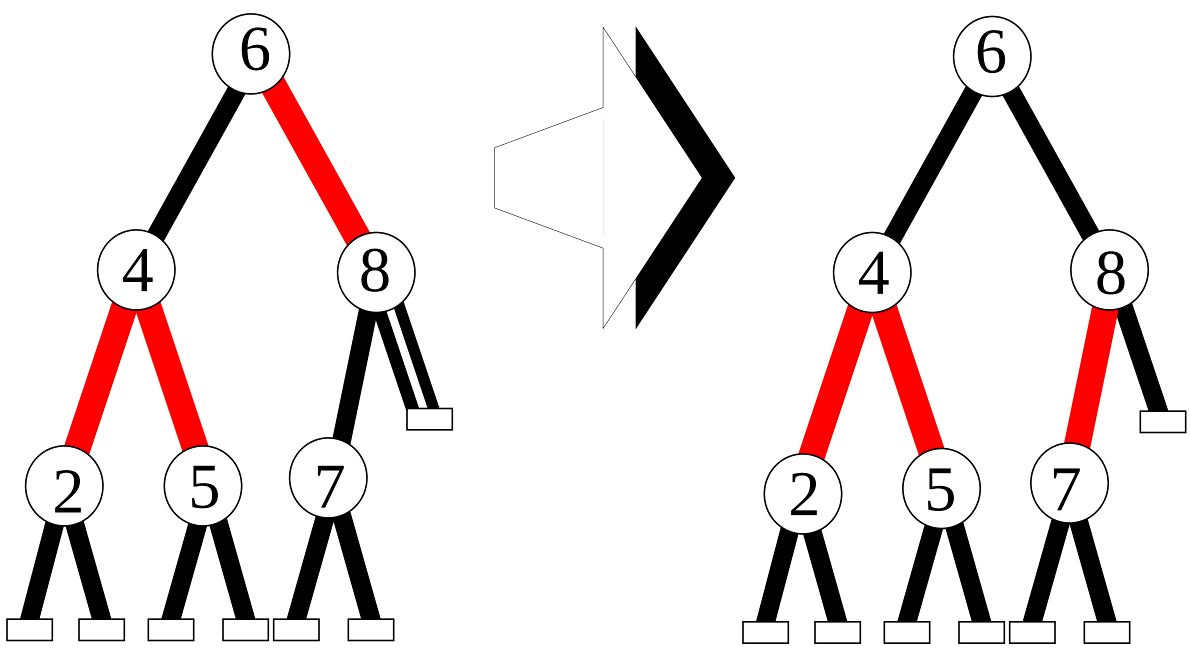
Example

What do we know?

- Sibling is black with black children

What do we do?

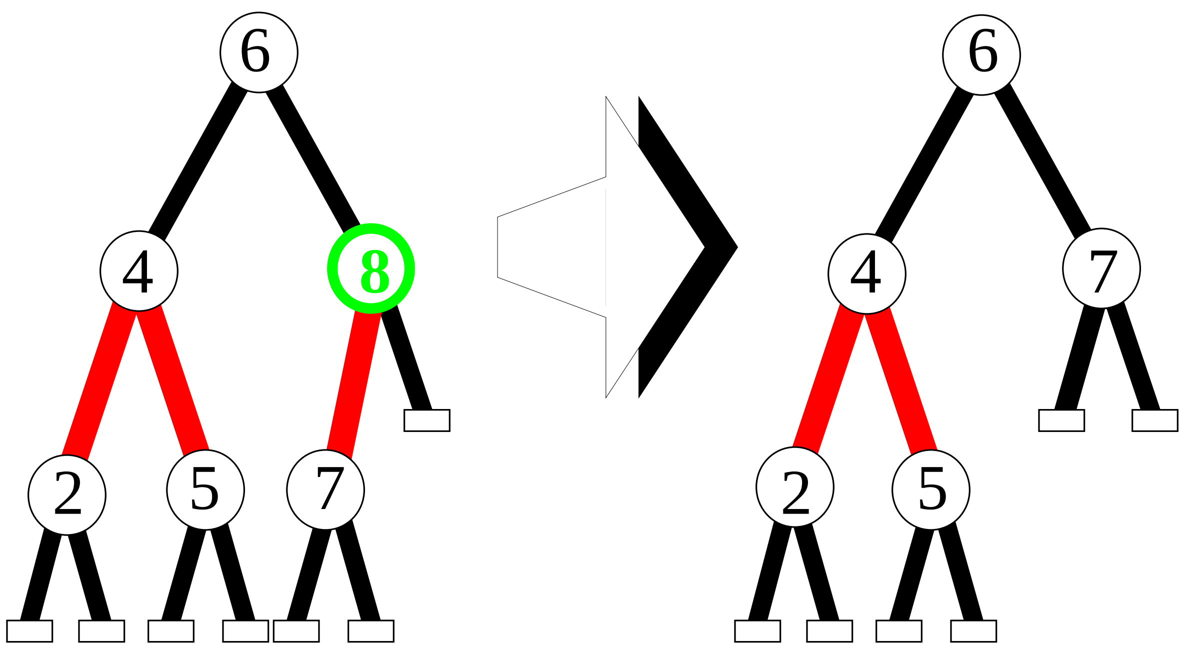
- Recoloring



Example

Delete 8

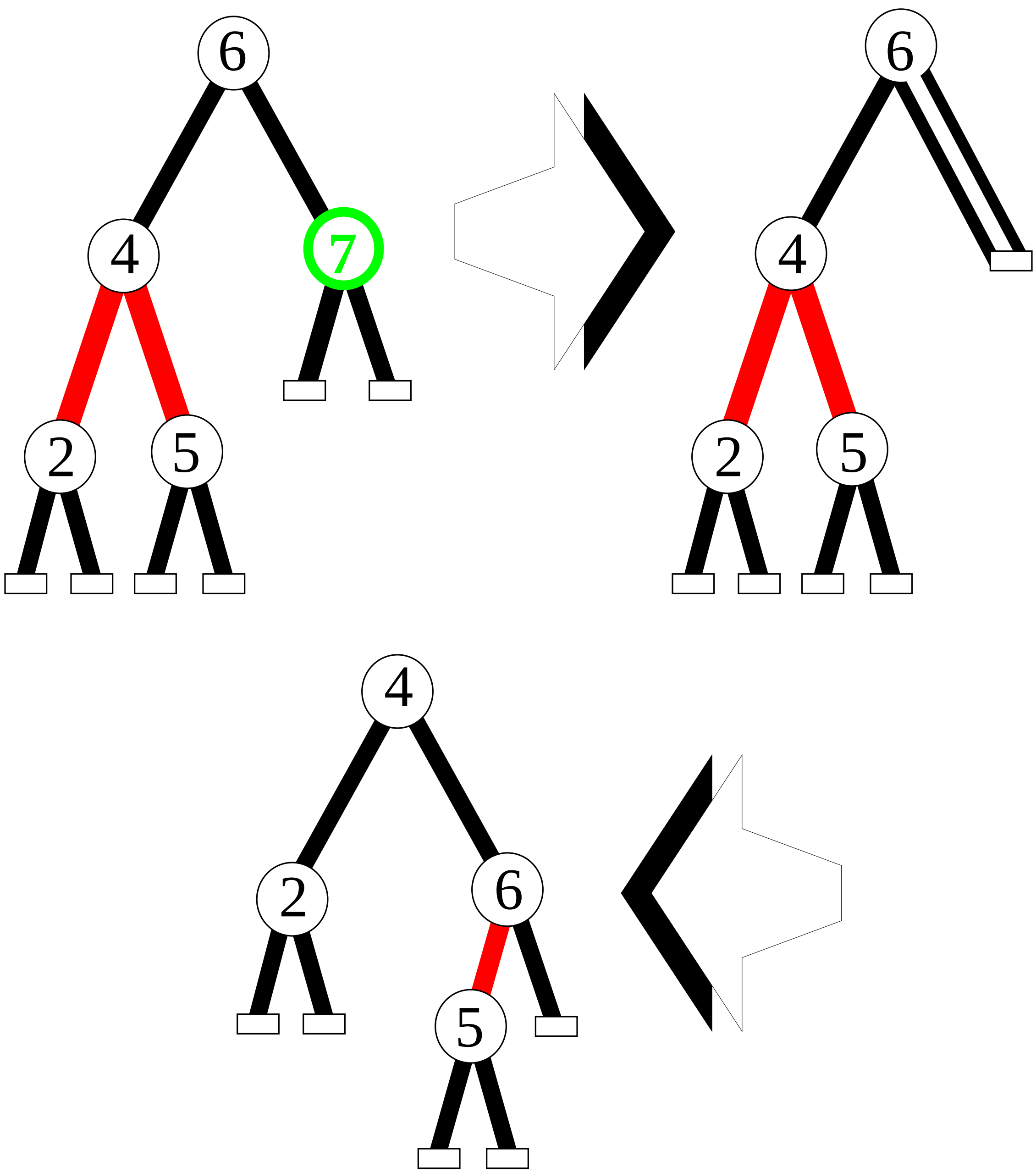
- no double black



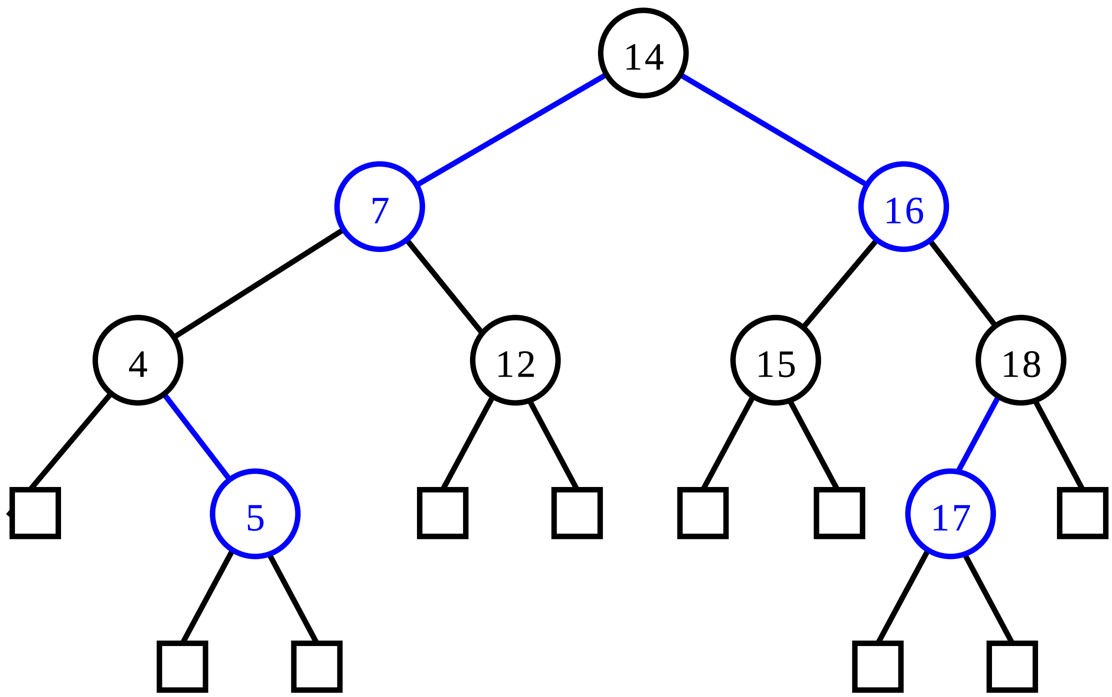
Example

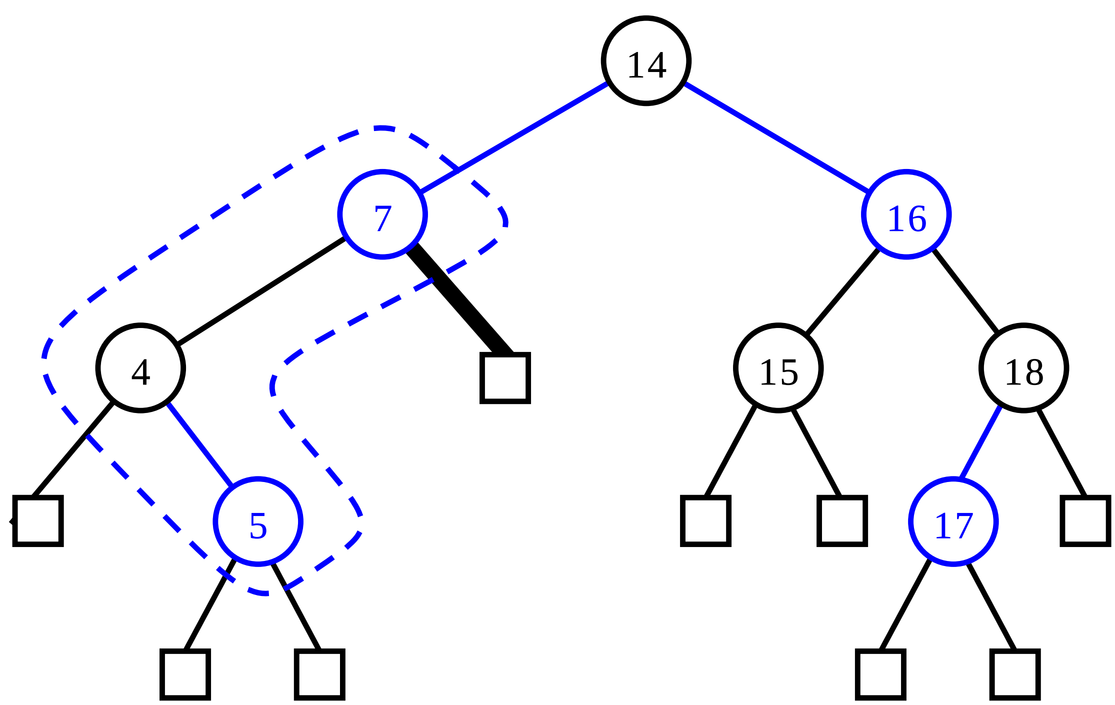
Delete 7

- Restructuring

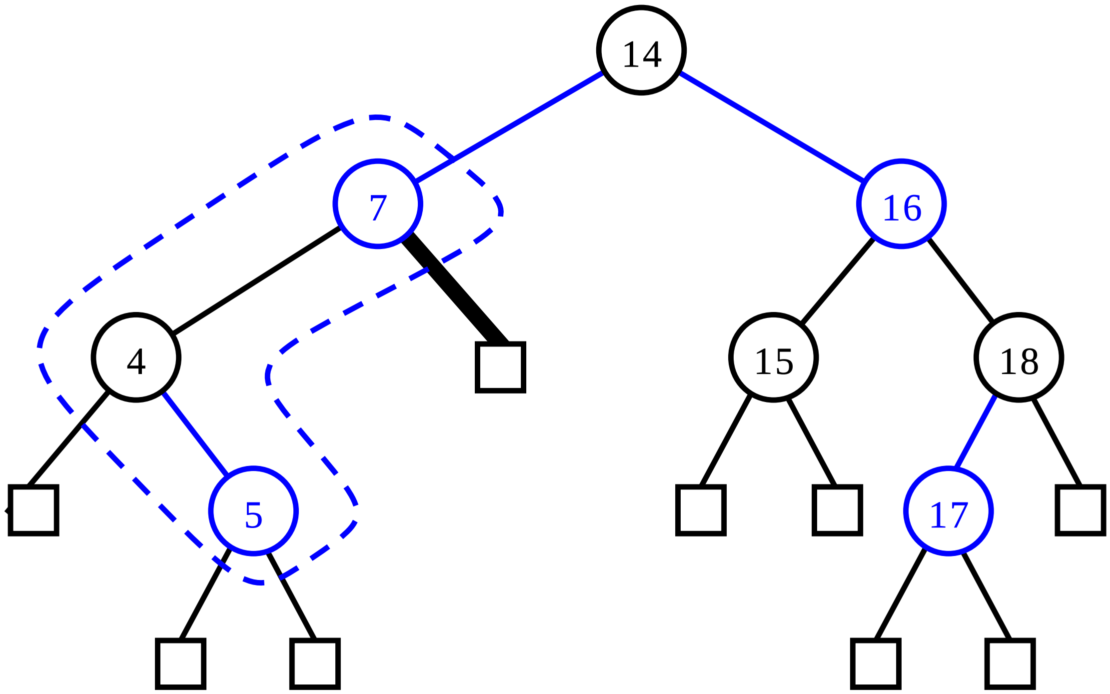


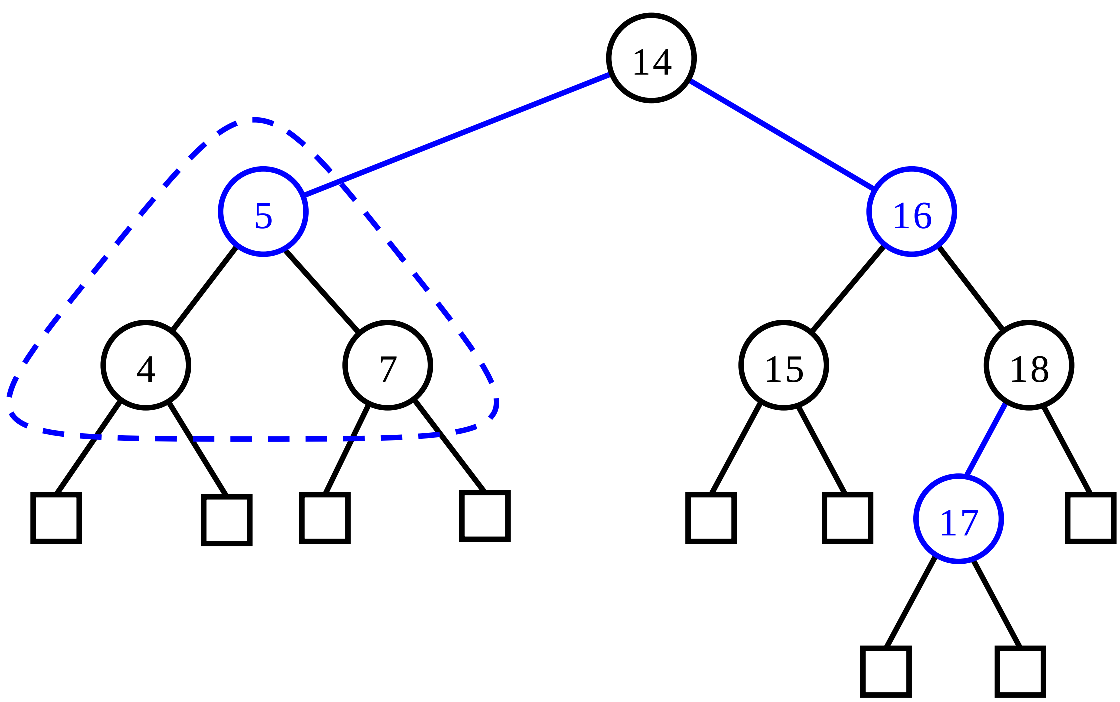
Example





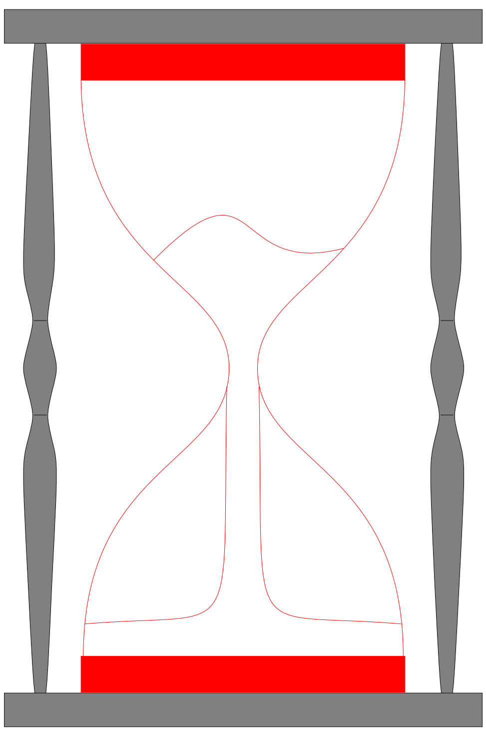
Example





Time Complexity of Deletion

Take a guess at the time complexity of deletion in a red-black tree . . .

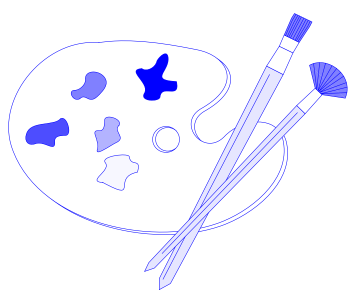


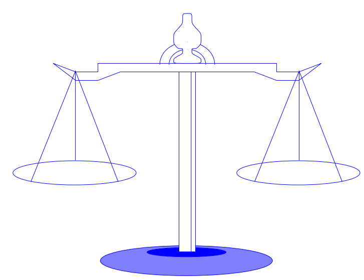
$O(\log N)$

What else could it be?!

Colors and Weights

Color	Weight
red	0
black	1
double black	**2**





Every root-to-leaf path has the same weight

There are no two consecutive red edges

- Therefore, the length of any root-to-leaf path is at most twice the weight

Bottom-Up Rebalancing of Red-Black Trees

- An insertion or deletion may cause a local ***perturbation*** (two consecutive red edges, or a **double-black** edge)

- The perturbation is either
  - ***resolved locally*** (restructuring), or
  - ***propagated*** to a higher level in the tree by recoloring (promotion or demotion)

- O(1) time for a restructuring or recoloring

- At most one restructuring per insertion, and at most two restructurings per deletion

- O(log N) recolorings

- Total time: O(log N)

Red-Black Trees