Week 20 Tutorial Solutions

Background: Binary Coded Decimal

1. The BCD representation of 74_{10} is 01110100 because $7_{10} = 0111_2$ and $4_{10} = 0100_2$.
2. 10000101 is the BCD representation of 85_{10} because $1000_2 = 8_{10}$ and $0101_2 = 5_{10}$.
3. The BCD representation of 19_{10} is 00011001 and the BCD representation of 1_{10} is 0001. Adding the BCD representations gives 00011010 which is not a valid BCD representation because $1010_2 = 10_{10}$ and 10 is not a single digit. Ordinary binary addition generates a carry when the sum in the rightmost 4 bits exceeds 15, but for BCD addition we should generate a carry when this sum exceeds 9.

Ternary: Base 3 Numbers

4. Calculate the decimal equivalent of each ternary number: 3 times the first digit, plus the second digit.

BCT: Binary Coded Ternary

5. The BCT representation of 20_3 is 1000 because $2_3 = 10_2$ and $0_3 = 00_2$.
6. 1001 is the BCT representation of 21_3 because $10_2 = 2_3$ and $01_2 = 1_3$. In decimal it's 7 because $2 \times 3 + 1 = 7$.

Converting Binary to BCT

7. The solution follows the suggested steps.

   (a)

3-bit binary	000	001	010	011	100	101	110	111
Decimal	0	1	2	3	4	5	6	7
2-digit ternary	00	01	02	10	11	12	20	21
BCT	0000	0001	0010	0100	0101	0110	1000	1001

(b) The truth table is copied from the top and bottom rows of the table in (a).

x	y	z	a	b	c	d
0	0	0	0	0	0	0
0	0	1	0	0	0	1
0	1	0	0	0	1	0
0	1	1	0	1	0	0
1	0	0	0	1	0	1
1	0	1	0	1	1	0
1	1	0	1	0	0	0
1	1	1	1	0	0	1

(c) Karnaugh maps:

a

	$\overline{y}$	y	y	$\overline{y}$
$\overline{x}$	0	0	0	0
x	0	1	1	0
	$\overline{z}$	$\overline{z}$	z	z

b

	$\overline{y}$	y	y	$\overline{y}$
$\overline{x}$	0	0	1	0
x	1	0	0	1
	$\overline{z}$	$\overline{z}$	z	z

c

	$\overline{y}$	y	y	$\overline{y}$
$\overline{x}$	0	1	0	0
x	0	0	0	1
	$\overline{z}$	$\overline{z}$	z	z

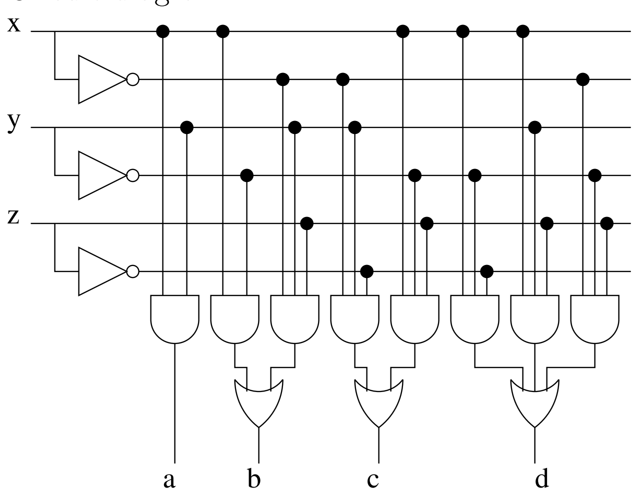
d

	$\overline{y}$	y	y	$\overline{y}$
$\overline{x}$	0	0	0	1
x	1	0	1	0
	$\overline{z}$	$\overline{z}$	z	z

(d) The Karnaugh maps give us the following definitions of a, b, c, d. Note that c and d just remain as sums of minterms.

$$
\begin{aligned}
a &= xy \\
b &= x\overline{y} + \overline{x}yz \\
c &= \overline{x}y\overline{z} + x\overline{y}z \\
d &= x\overline{y}\,\overline{z} + xyz + \overline{x}\,\overline{y}z
\end{aligned}
$$

(e) Circuit diagram:



BCT Addition

8. The solution follows the suggested steps.

(a) Here is the addition table for single-digit ternary numbers.

+	0	1	2
0	00	01	02
1	01	02	10
2	02	10	11

(b) Converting the addition table into BCT we have:

+	00	01	10
00	0000	0001	0010
01	0001	0010	0100
10	0010	0100	0101

(c) This is the truth table:

x	y	z	w	a	b	c	d
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1
0	0	1	0	0	0	1	0
0	0	1	1				
0	1	0	0	0	0	0	1
0	1	0	1	0	0	1	0
0	1	1	0	0	1	0	0
0	1	1	1				
1	0	0	0	0	0	1	0
1	0	0	1	0	1	0	0
1	0	1	0	0	1	0	1
1	0	1	1				
1	1	0	0				
1	1	0	1				
1	1	1	0				
1	1	1	1				

(d) Karnaugh maps:

a

	$\overline{y}$	y	y	$\overline{y}$	
$\overline{x}$	0	0	0	0	$\overline{w}$
$\overline{x}$	0	0			w
x	0				w
x	0			0	$\overline{w}$
	$\overline{z}$	$\overline{z}$	z	z	

b

	$\overline{y}$	y	y	$\overline{y}$	
$\overline{x}$	0	0	1	0	$\overline{w}$
$\overline{x}$	0	0			w
x	1				w
x	0			1	$\overline{w}$
	$\overline{z}$	$\overline{z}$	z	z	

c

	$\overline{y}$	y	y	$\overline{y}$	
$\overline{x}$	0	0	0	1	$\overline{w}$
$\overline{x}$	0	1			w
x	0				w
x	1			0	$\overline{w}$
	$\overline{z}$	$\overline{z}$	z	z	

d

	$\overline{y}$	y	y	$\overline{y}$	
$\overline{x}$	0	1	0	0	$\overline{w}$
$\overline{x}$	1	0			w
x	0				w
x	0			1	$\overline{w}$
	$\overline{z}$	$\overline{z}$	z	z	

(e) By finding the largest possible squares and rectangles which cover the 1s (including some blanks), we get the following formulae. The formula for a is a special case—it turns out that $a = 0$ in all 9 non-blank positions, so we can just take $a = 0$ as the definition.

$$
\begin{aligned}
a &= 0 \\
b &= xw + yz + xz \\
c &= x\overline{z}\,\overline{w} + \overline{x}\,\overline{y}z + yw \\
d &= \overline{x}\,\overline{y}w + y\overline{z}\,\overline{w} + xz
\end{aligned}
$$

(f) Circuit diagram: