Student-Project Allocation Problem with Ties

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In the **Student-Project Allocation problem (SPA)**, we seek to find a **matching** – which is an assignment of students to projects offered by lecturers, based on student preferences over projects and the maximum number of students that each project and lecturer can accommodate. Variants of SPA in the literature involve (a) lecturer preferences over (i) students, (ii) projects, (iii) (student, project) pairs, and (b) no lecturer preferences at all. See [1] for a detailed recent survey. All of these variants assume that preferences are strictly ordered. However, this might not be achievable in practice. In this work, we study **SPA with lecturer preferences over Students with Ties (SPA-ST)**.

1. An instance of SPA-ST

Students	Lecturers	
s_1: (p_2 p_1)	$l_1\{2\}$: s_3 (s_1 s_2)	l_1 offers $p_1\{1\}, p_2\{1\}$
s_2: p_2 p_3	$l_2\{1\}$: (s_3 s_2)	l_2 offers $p_3\{1\}$
s_3: p_3 p_1		

Figure 1: Ties in the preference lists are indicated by round brackets, i.e., s_1 is indifferent between p_2 and p_1, while s_2 prefers p_2 to p_3. The capacity for each project and lecturer is enclosed in curly braces.

In this context, we seek a **stable matching**. Given that ties are involved, three types of stability arise [2], namely **weak stability, strong stability and super-stability**. Informally, we say a matching is (i) *weakly stable*, (ii) *strongly stable*, or (iii) *super-stable*, if there is no student and lecturer such that if they decide to form an arrangement outside the solution, respectively,
(i) both of them would be better off,
(ii) one of them would be better off and the other no worse off, or
(iii) neither of them would be worse off.

2. Weakly stable matchings

Students	Lecturers	
s_1: ($\boxed{p_2}$ p_1)	$l_1\{2\}$: s_3 ($\boxed{s_1}$ s_2)	l_1 offers $p_1\{1\}, p_2\{1\}$
s_2: p_2 p_3	$l_2\{1\}$: ($\boxed{s_3}$ s_2)	l_2 offers $p_3\{1\}$
s_3: $\boxed{p_3}$ p_1		

Figure 2: Each of lecturer l_1 and l_2 would not be better off rejecting her assigned student for s_2. Hence the matching in red square is weakly stable – it matches 2 students.

Students	Lecturers	
s_1: ($\boxed{p_2}$ p_1)	$l_1\{2\}$: $\boxed{s_3}$ ($\boxed{s_1}$ s_2)	l_1 offers $p_1\{1\}, p_2\{1\}$
s_2: p_2 $\boxed{p_3}$	$l_2\{1\}$: (s_3 $\boxed{s_2}$)	l_2 offers $p_3\{1\}$
s_3: p_3 $\boxed{p_1}$		

Figure 3: Another weakly stable matching – it matches 3 students.

- The weakly stable matchings in Fig. 2 and Fig. 3 have different sizes.
- The problem of finding a maximum size weakly stable matching is known to be NP-hard; further, a $\frac{3}{2}$-approximation algorithm is described in [3].

3. Strongly stable matchings

Students	Lecturers	
s_1: (p_2 $\boxed{p_1}$)	$l_1\{2\}$: s_3 ($\boxed{s_1}$ $\boxed{s_2}$)	l_1 offers $p_1\{1\}, p_2\{1\}$
s_2: $\boxed{p_2}$ p_3	$l_2\{1\}$: ($\boxed{s_3}$ s_2)	l_2 offers $p_3\{1\}$
s_3: $\boxed{p_3}$ p_1		

Figure 4: l_1 would be no worse off rejecting s_2, in order to take on s_1 for p_2, but s_1 would not be better off getting assigned to p_2. Hence the matching is strongly stable.

- Not every SPA-ST instance admits a strongly stable matching.
- However, in cases where it can be achieved, it should be preferred over a matching that is merely weakly stable. Why? Clearly, strong stability will prevent s_2 and s_3 from undermining the matching in Fig. 3 by persuading or bribing l_1 and l_2 respectively, in order to take them on for p_2 and p_3 respectively. **Problem 1: An algorithm for strong stability in SPA-ST.**

4. Super-stable matchings

With respect to Fig. 4, s_1 and l_1 would be no worse off forming an arrangement such that s_1 becomes assigned to p_2, hence the matching is not super-stable. In fact, this instance does not admit a super-stable matching.

So why should we care about super-stability?

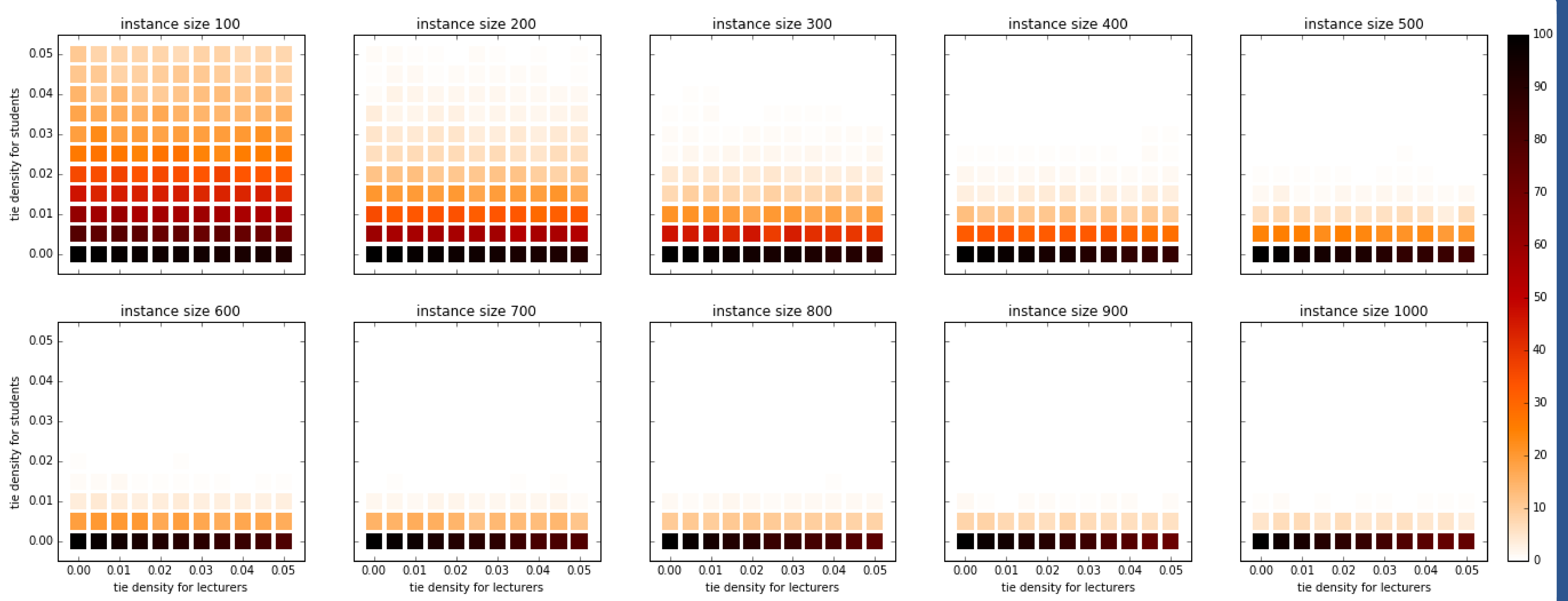
- If some students have incomplete information regarding projects and rank them equally in ties, and if a super-stable matching M exists, then no matter how the ties are resolved (representing the true preferences), M would be stable.
- All weakly stable matchings are of the same size, equal to the size of M.
- Moreover, super-stability $\Longrightarrow$ strong stability $\Longrightarrow$ weak stability.

Hence the motivation for the following. **Problem 2: An algorithm for super-stability in SPA-ST.** See [4] for our solution to this problem.

An Empirical Analysis from our Solution to Problem 2

- Each of the coloured square boxes represent the proportion of 1000 randomly-generated instances that admits a super-stable matching.
- As we increase the tie density in the students' and lecturers' preference lists, this proportion reduces (see the colour bar transition from 100% to 0%). The proportion reduces further as the size of the instance increases.

Interesting observation: when ties are only allowed in the lecturers' preference lists, a significant proportion of the randomly-generated instances admitted a super-stable matching.



Future work: (i) Investigate how other parameters (e.g., position of ties in the preference lists) affect the existence of a super-stable matching, (ii) Derive theoretical bounds on the probability of a super-stable matching existing, given an arbitrary SPA-ST instance, (iii) Attempt Problem 1.

References

[1] M. Chiarandini, R. Fagerberg, and S. Gualandi. Handling preferences in Student-Project Allocation. *Annals of Operations Research*, 2018.
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[3] F. Cooper and D.F. Manlove. A 3/2-approximation algorithm for the Student-Project Allocation problem. In *Proceedings of SEA 2018*, vol. 103 of *LIPIcs*, pg 8:1–8:13.
[4] S. Olaosebikan and D.F. Manlove. Super-stability in the Student-Project Allocation problem with Ties. CoRR (2018). Available from `http://arxiv.org/abs/1805.09887`.

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