

Haptic Display of Mathematical Functions for Teaching Mathematics to Students with Vision Disabilities: Design and Proof of Concept

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Abstract

We describe the design and initial implementation of a system for constructing a haptic model of a mathematical function for exploration using a PHANToM. The model is that of a block of balsa wood with the trace of the function carved into its surface.

Problem Statement

It is difficult to teach some kinds of mathematics without using graphical representations of functions. In this paper we describe the design of a proposed haptic application and the implementation of a prototype using a PHANToM which will allow a blind student to feel the shape of a function being studied.

Related Work

Some of our previous work has been in constructing PHANToM-based haptic maps for mobility training. [1] Other VE Lab members are constructing haptic models of atoms. [2]

Approach

Our software package is designed to allow the input of a function, expressed as a Fortran statement function definition such as

$$F(X) = 10.0 * X ** 2 - 2.0 * X + 1.0$$

Our package parses the textual representation of the function and then evaluates the function over an appropriate domain of values. The software then builds a 3-d model of a block of material on whose face the trace of the function is carved. The user can then use the PHANToM to follow the trace of the function. Optional commands will allow the user to specify the domain and range to be displayed, the presence or absence of the axes, and major and minor tick marks on the axes and major and minor grid lines.

Design

We address the design of the software in two sections: the user interface and the implementation of the haptic display.

User Interface

There are two phases to using the software: (1) defining the function to be displayed and (2) exploring the function using the PHANToM. The user enters the function using the computer keyboard and explores the function with the right hand controlling the PHANToM and the left hand optionally entering keyboard commands.

Defining the Function

The user inputs the function to be studied, expressed as a Fortran statement function definition. A beep or a voice audio cue indicates when the model of the function has been "carved."

Exploring the Function

The user then explores the function using the PHANToM. Optional audio cues can guide the user to move the PHANToM tip to the function trace. The shape of function can then be followed by moving the PHANToM tip along the "carved" groove.

We assume that the user is right handed and will use the left hand on the computer keyboard to enter commands that modify preferences or trigger various audio cues.

Using the Keyboard for Optional Commands

Some of the functions to be accessed by the left hand are the following:

- general application functions
 - save function
 - save function as
 - close function
 - quit application
- preferences (toggle on/off)
- provide audio feedback
- specify domain
- specify range

- display ("carve") grid
- display axes
- show major axis ticks
- show minor axis ticks
- show major grid lines
- show minor grid lines
- audio feedback
 - speak quadrant
 - speak location of function trace w.r.t. PHANToM tip (above or below, left or right)
 - speak (x,y) coordinates of current location of PHANToM tip
 - speak (x,y) coordinates of nearest point on function trace

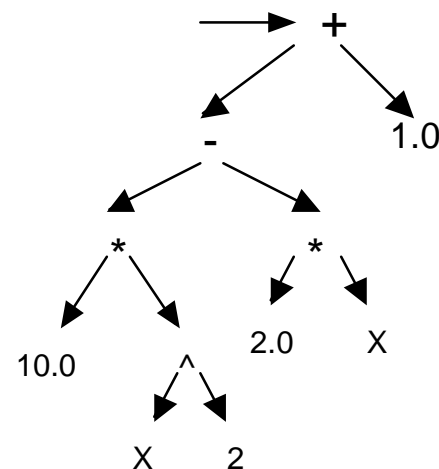
Architecture of the Application

The major functional components of the application are the function compiler and the haptic modeler.

Function Compiler

We have used flex and bison to construct a compiler which translates the function definition provided as a character string by the user into a tree which can then be used to evaluate the function for various values in the domain. In the tree, the internal nodes are the operators and the leaves are the operands. For example, the function

$F(X) = 10.0 * X ** 2 - 2.0 * X + 1.0$
is translated into:



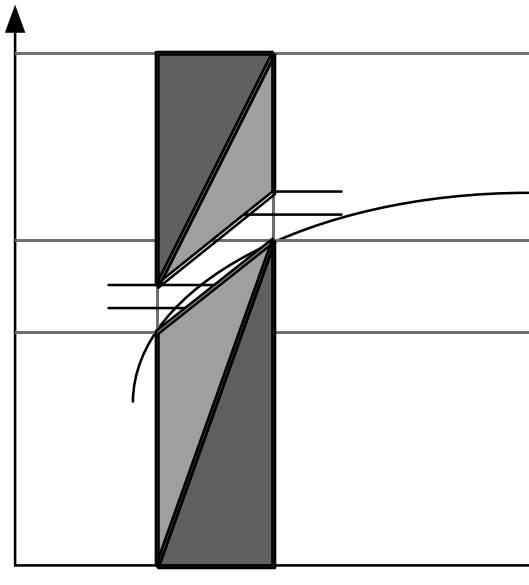
We support the sine, cosine, tangent, natural logarithm, and exponential functions.

Haptic Modeler

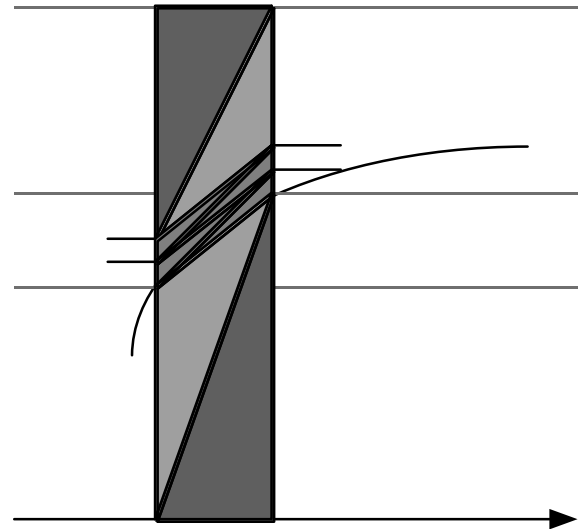
Let f be the function the user wishes to explore haptically. For each pair $(valueX, valueY)$, where $valueY=f(valueX)$, we construct five points in (x,y,z) space as follows:

```
(valueX+offsetX, offsetY, 0.0)
(valueX + offsetX, valueY +
  offsetY - gapY, 0.0)
(valueX + offsetX, valueY +
  offsetY, depthZ)
(valueX + offsetX, valueY + gapY
  + offsetY, 0.0)
(valueX + offsetX, sizeY +
  offsetY, 0.0)
```

We then build eight triangles for each x -value. Four of them are shown in gray in the following figure. These triangles are all coplanar.



Four other triangles are constructed in the gap between the two pairs of triangles shown above. They form a v-shaped trench below the plane of the previous triangles which is the groove traced by the PHANToM.



Implementation

We have a working prototype which allows the user to input a function and then explore the carving of the function using the PHANToM.

The current system includes some simple voice prompts. Initially the system speaks, "Welcome to Haptic Math! Place the PHANToM in its reset position and press <ENTER>."

When the user presses the "R" key, the system speaks "Expression" followed by a character by character reading of the current function, as entered so far.

When the user presses the <enter> key after entering a function, the system responds with "Displaying the following expression," followed by a reading of the function definition, followed by "Now you can feel the function."

Future Work

We are currently refining this system by:

- (1) adding the coordinate grid
- (2) implementing additional keyboard commands as indicated in the design

(3) adding error checking (for the syntax of the user's function and for domain and range values)

We have encountered a problem in displaying grid lines along with the function: it is difficult for the user to recognize which direction to move the PHANToM at a point where the function and a grid line intersect. We are trying several approaches, including using different shapes for the carving of the grid and the function, using vibration rather than a carved line to indicate when the PHANToM tip is near a grid line.

We continue to refine the user interface design based on consultation with teachers and others with limited vision.

There are three possible further steps for the project. We could develop a full haptic graphing calculator. Alternatively, we could add functionality for assisting in teaching specific mathematics courses. For example, we could add a front end that would allow the user to build parameterized conic sections, or we could display not only the trace of a function but also its derivative. We are also exploring working with K-12 teachers about potential related applications to curricula in geography, statistics, and astronomy

Bibliography

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http://www.sensable.com/community/PUG99/07_vanscoy.pdf
- [2] Harvey, Erica and Chaim Gingold, "Haptic Representation of the Atom," Proceedings of Information Visualisation 2000, London, July 19-21, 2000.

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PHANToM is a registered trademark of SensAble.