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# **Context Compression: using Principal Component Analysis for Efficient Wireless Communications**

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# Objective

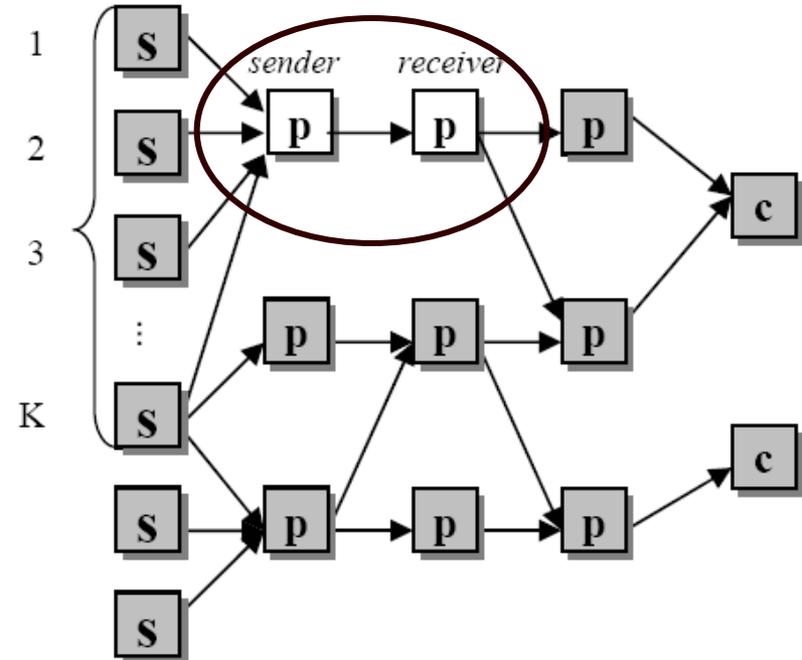
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- **Improving** energy efficiency in Wireless Sensor Networks
- **Compressing** contextual information **prior** to transmission based on the *current* **Principal Components** of the sampled data.
- **Exploitation of the natural characteristics** of the pieces of context



# Network model

- A set of **sensor** nodes (sources), **processing** nodes (relays), and **sink** nodes (consumers).
- Paths leading from the sources to sinks through processing nodes.
- A node is battery-powered (energy-constrained).



**s:** source node

**p:** processing node

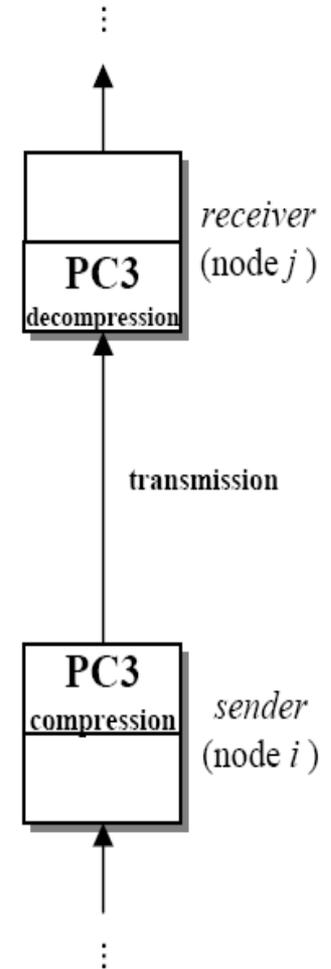
**c:** sink node

→ : context stream

# Sender and Receiver node

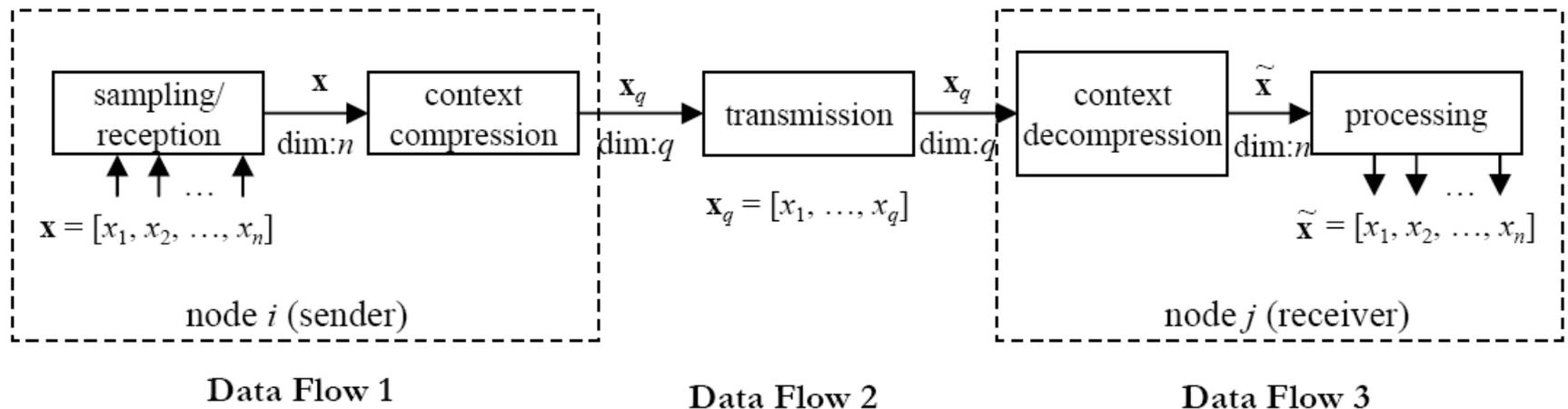
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- Consider a **distributed compression algorithm** between sender  $i$  and receiver  $j$ :
  - Node  $i$  performs context compression and forwards the compressed contextual data (stream) to node  $j$ ,
  - Node  $j$  performs context decompression in order to reproduce the original contextual data.
- Subsequently, node  $j$  can further act as a sender node for the upstream nodes and so on...



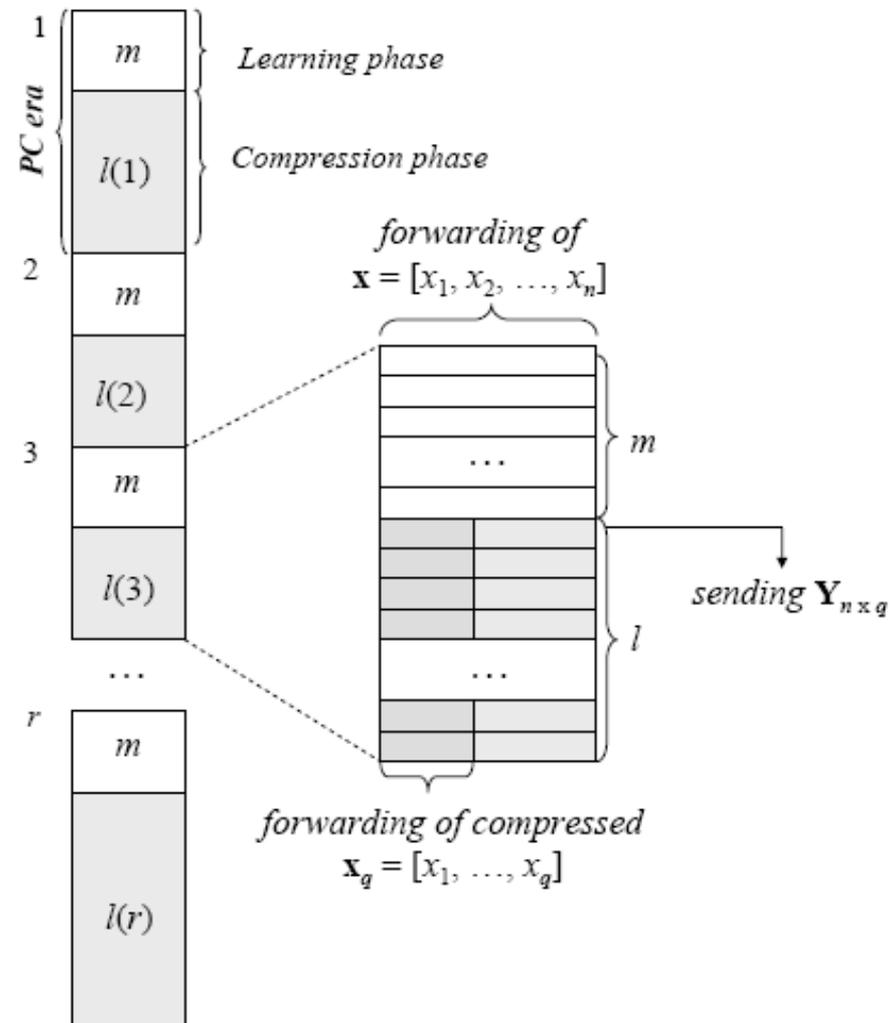
# Sender and Receiver node (focus)

- Node  $i$  captures the  $n$ -dimensional context vector (CV)  $\mathbf{x}$  ;
- Node  $i$  compresses  $\mathbf{x}$  to a  $q$ -dimensional CV ( $q < n$ ),  $\mathbf{x}_q$  and forwards it to upstream node  $j$ .
- Node  $j$  reproduces the  $n$ -dimensional context vector,  $\tilde{\mathbf{x}}(t)$  for further processing (or forwarding to upstream nodes).



# Sender node $i$ (focus)

- $t$ : discretized time domain
- $\mathbf{x}_i(t) = [x_{ik}(t)]$ ,  $k = 1, \dots, n$  be the CV of  $n$  measurements collected by node  $i$  at time  $t$
- $x_{ik}(t)$ : the  $k$ th contextual component (e.g., temperature, humidity, wind speed).
- Node  $i$  gathers the last  $m > 0$  received CVs  $\mathbf{x}(t-m)$ ,  $\mathbf{x}(t-m+1)$ ,  $\dots$ ,  $\mathbf{x}(t)$ , thus, forming a  $m \times n$  matrix  $\mathbf{X}$  consisting of  $m$  CVs.
- Based on  $\mathbf{X}$ , node  $i$  obtains the corresponding **principal components** (PCs) and, thus, reduces (compresses) a  $\mathbf{x}(t)$  to  $\mathbf{x}_q(t)$ .



# Principal Component Analysis (PCA)

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- A mechanism that performs *lossy* data compression
- PCA discovers a **linear** relationship among the contextual components  $x_k$ .
- PCA keeps the components that **better** describe the variance of the sample **X**.
  - The trade-off here is between compression (**count of principal contextual components retained**) and compression fidelity (**the variance preserved**).

# Principal Component Analysis (PCA)

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- Given a set of  $m$  multivariate CVs of dimension  $n$ ,  $\mathbf{x}(t)$ ,  $1 \leq t \leq m$ , the PC basis is obtained by minimizing the optimization function:

$$J_q(\mathbf{x}(t), \mathbf{y}_k) = \frac{1}{m} \sum_{t=1}^m \left\| \mathbf{x}(t) - \sum_{k=1}^q \mathbf{y}_k \mathbf{y}_k^\top \mathbf{x}(t) \right\|^2$$

# Compression through PCA

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- Dimensionality reduction from  $n$  to  $q$  with accuracy  $a\%$ , i.e., the minimum number of PCs,  $q^*$ , that describe at least the  $a\%$  of the variance of the projection of CVs on the PC basis expressed by the eigenvalues  $\lambda_k$ .
- Then:
  - we obtain a **compression** of  $\mathbf{x}(t)$  of dimension  $q < n$ :

$$\mathbf{x}_q(t) = \mathbf{Y}^\top \mathbf{x}(t)$$

- we obtain an **approximation** of  $\mathbf{x}(t)$ :

$$\tilde{\mathbf{x}}(t) = \mathbf{Y} \mathbf{x}_q^\top(t)$$

# Context compression scheme

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- **Learning phase (lasts for  $m$ ):**
  - Node  $i$  learns the PCs of the last  $m$  measurements. Node  $i$  gathers the most recent  $m$  CVs. During this history window, i.e., for  $1 \leq t \leq m$ , node  $i$  forwards **each** received  $\mathbf{x}(t)$  to the peer node  $j$ .
  - Once  $m$  CVs are received at node  $i$  then node  $i$  can determine the  $q$  principal components forming the  $\mathbf{Y}$  matrix w.r.t. the recent  $m$  measurements and  $a = 90\%$ .

# Context compression scheme

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## ➤ Compression phase (lasts for $l$ ):

- Node  $i$  forwards to node  $j$  the  $\mathbf{Y}$  ( $n \times q$ ) matrix only **once**.
- The *trained* node  $i$  forwards to node  $j$  only the values of the  $q$  principal contextual parameters for a finite time period  $l > 0$

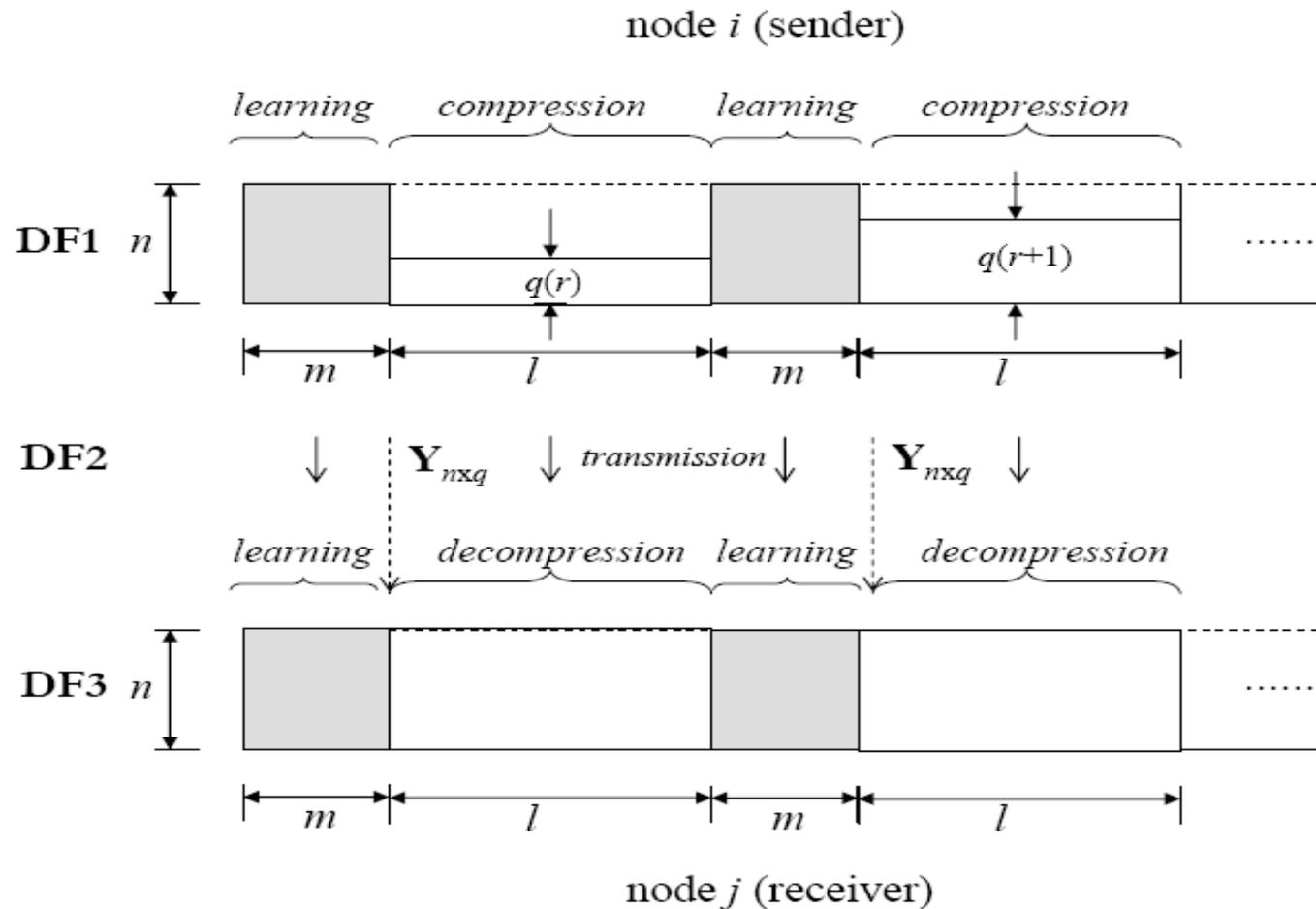
$$\mathbf{x}_q(t) = \mathbf{Y}^\top \mathbf{x}(t)$$

- Node  $i$  forwards the compressed CV to node  $j$ .
- On the other side, in the (de)-compression phase, node  $j$  has the required information (i.e., the  $\mathbf{Y}$  ( $n \times q$ ) matrix) to reproduce / approximate

$$\tilde{\mathbf{x}}(t) = \mathbf{Y} \mathbf{x}_q^\top(t)$$



# The Data Flow



# Error...

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- For the finite period  $l$  we assume that
  - (i) the **number** of the PCs remain unchanged and
  - (ii) the **order** of the corresponding eigenvalues does not change.
- The node  $j$  re-produces the CVs during the  $l$  period inducing the re-production / reconstruction error:

$$e^R(t) = \| \tilde{\mathbf{x}}(t) - \mathbf{x}(t) \|$$

# Adaptive mechanism

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- The value of  $l(r)$  for the  $r$ -th compression phase **is not a-priori known** and, additionally, there is no knowledge about the underlying data distribution w.r.t. the PCs.
- A controller  $A(l)$  adjusts the period  $l(r+1)$  of the  $(r+1)$ -th compression phase based on the error  $e(l)$  and the  $l(r)$  value of the  $r$ -th compression phase.
- Adaptation rule

$$l(r+1) = l(r) + a(r) : a(r) \in \{-1, 0, 1\}$$

# Periodic error

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- Periodic error  $e(\ell)$ : the average value of the relative error  $e^*(t)$  within a period  $\ell$

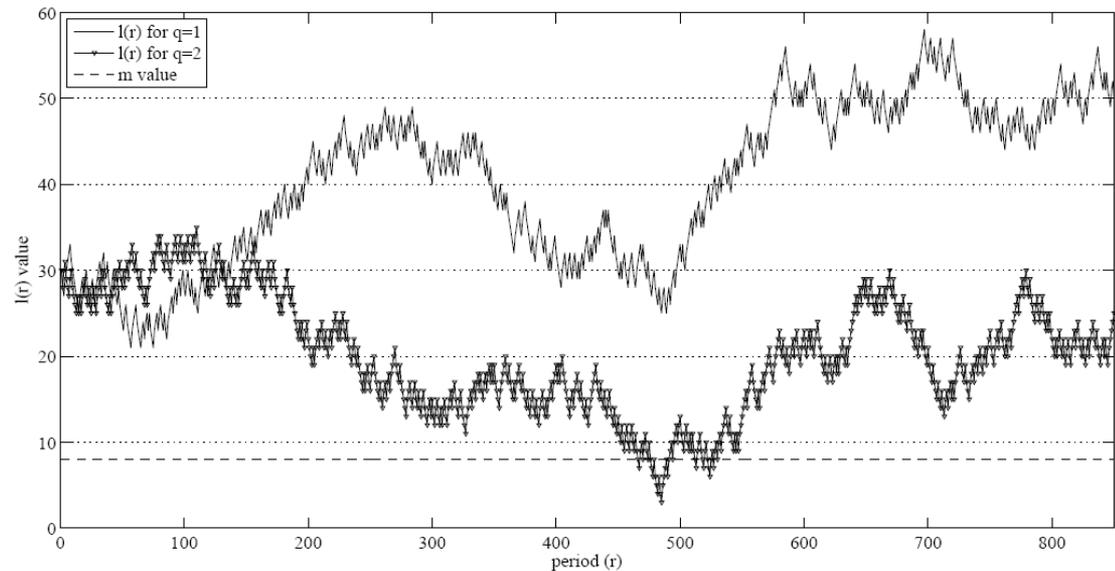
$$e(\ell) = \frac{1}{\ell} \sum_{t=m}^{m+\ell} e^*(t)$$

# Adaptive mechanism

- The control parameters for the adaptation rule are:
  - $\Delta e(l)$  : change in periodic error
  - $\Delta l$  : change in compression phase length

THE ADAPTATION RULES FOR  $a(r)$

	$\Delta e(l)$		
$\Delta l$	$< 0$	$0$	$> 0$
$- a(r) $	-1	0	+1
$0$	0	+1	-1
$+ a(r) $	+1	0	-1



# Performance assessment

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- Real sensor readings of **temperature** (T), **humidity** (H), and **wind speed** (W).
- Information collected from experiments of the Sensor and Computing Infrastructure for Environmental Risks (SCIER) system, capable of delivering valuable real time information regarding a natural hazard (e.g., fire) and both monitoring and predicting its evolution.
- The corresponding contextual vectors are of  $n = 7$  dimensions.
- Experiment with **32.25** hours of sensing.

# Performance assessment

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- Mica2 energy consumption model;
  - Mica2 operates with a pair of AA batteries that approximately supply 2200 mAh with effective average voltage 3V. It consumes 20mA if running a sensing application continuously which leads to a lifetime of 100 hours.
  - The packet header is 9 bytes (MAC header and CRC) and the maximum payload is 29 bytes. Therefore, the per-packet overhead equals to 23.7\% (lowest value).
  - For each contextual value the assumed payload is **4 bytes** (float).
  - A Mica2 message contains up to 7 contextual values (message payload: **28 bytes per message**).

# Performance assessment

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Table 1: Energy costs

Node operation mode	Energy cost
Instruction execution	4 nJ/instruction
Idle – Stand by	9.6 mJ/s - 0.33 mJ/s
Transmitting - Receiving	720 nJ/bit - 110 nJ/bit

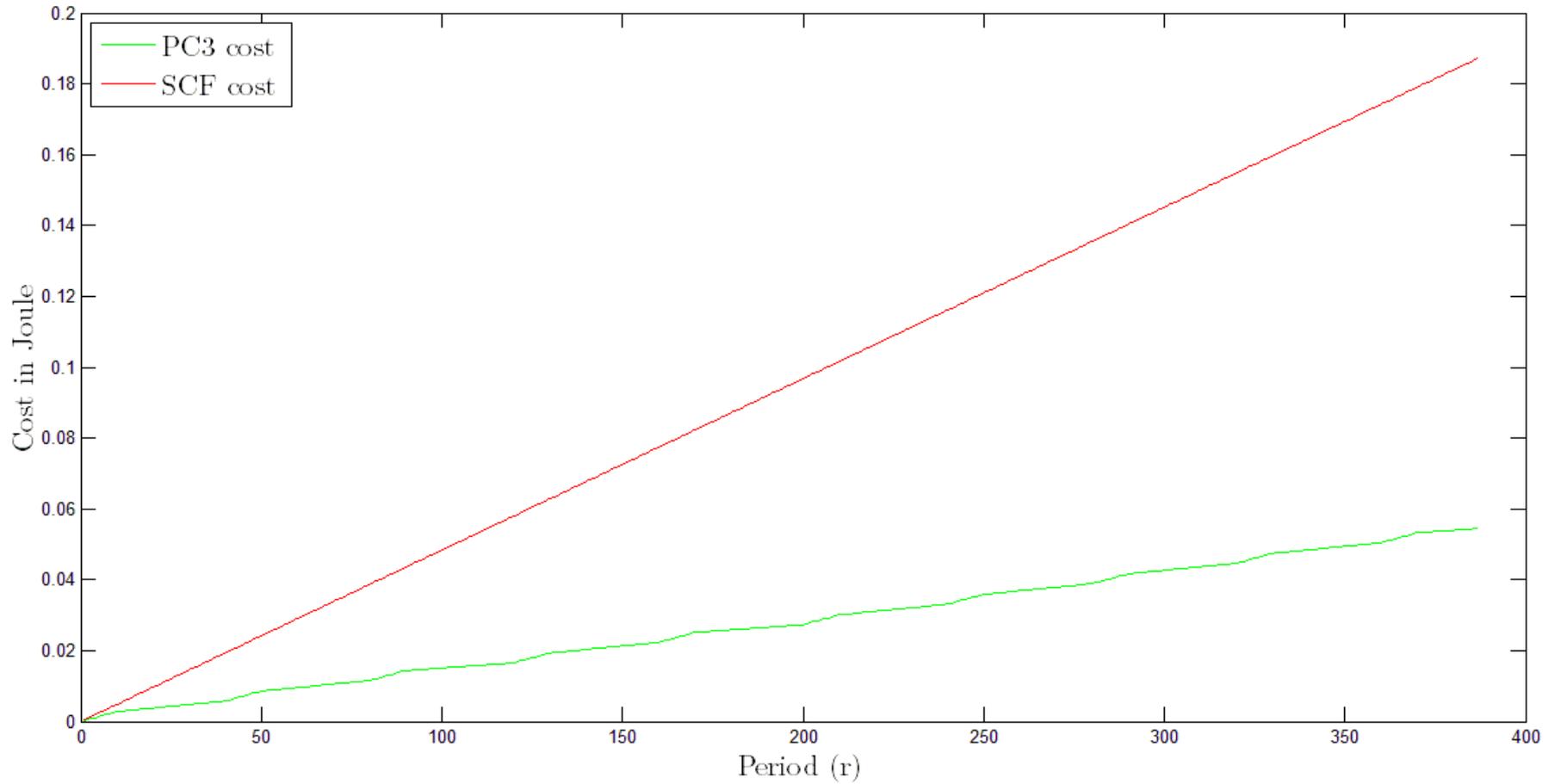
# Total energy cost

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$$c(t) = c(t - 1) + c_R(t) + c_T(t) + c_I(t) + c_0(t)$$

- $c_R$ ,  $c_T$  are receive (rx) and transmit (tx) costs for CVs and the periodic transmission/reception of the  $\mathbf{Y}$  matrix, respectively,
- $c_I$  is the energy cost for the CPU instructions of the PCA (compression and decompression)
- $c_0$  is the state transition cost.

# Cost



# Gain & Efficiency

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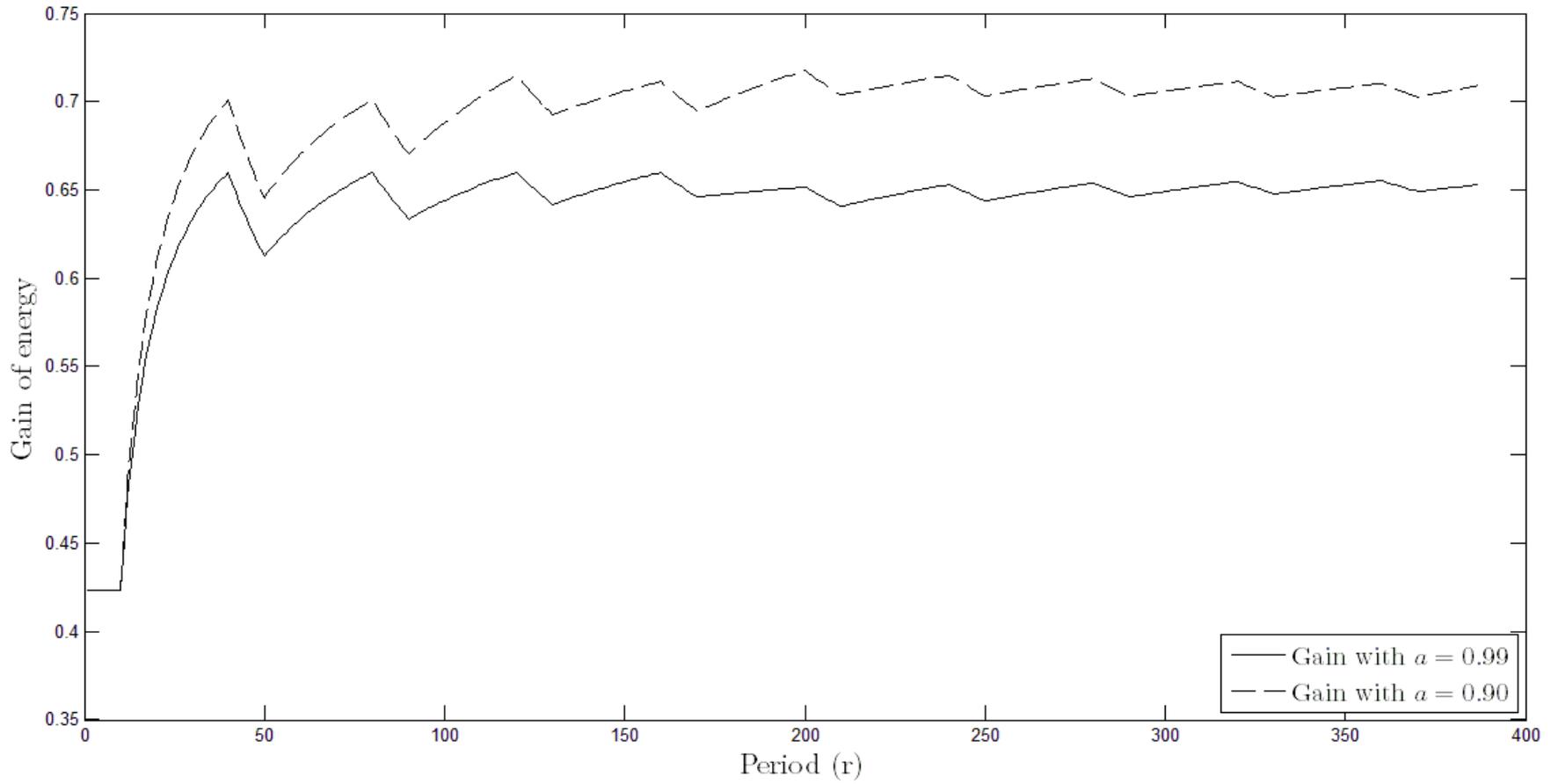
➤ **Gain:** The percentage cost gain  $g(t) \in [0, 1]$  when applying PC3 w.r.t. SCF by using the energy costs  $c_{PC3}(t)$  and  $c_{SCF}(t)$

$$g(t) = \frac{c_{SCF}(t) - c_{PC3}(t)}{c_{SCF}(t)}$$

➤ **Efficiency:**  $w(t) \in [0, \infty)$  for a finite time horizon up to  $t$  is the portion of energy cost  $c(t)$  out of the data accuracy  $1-e(t)$

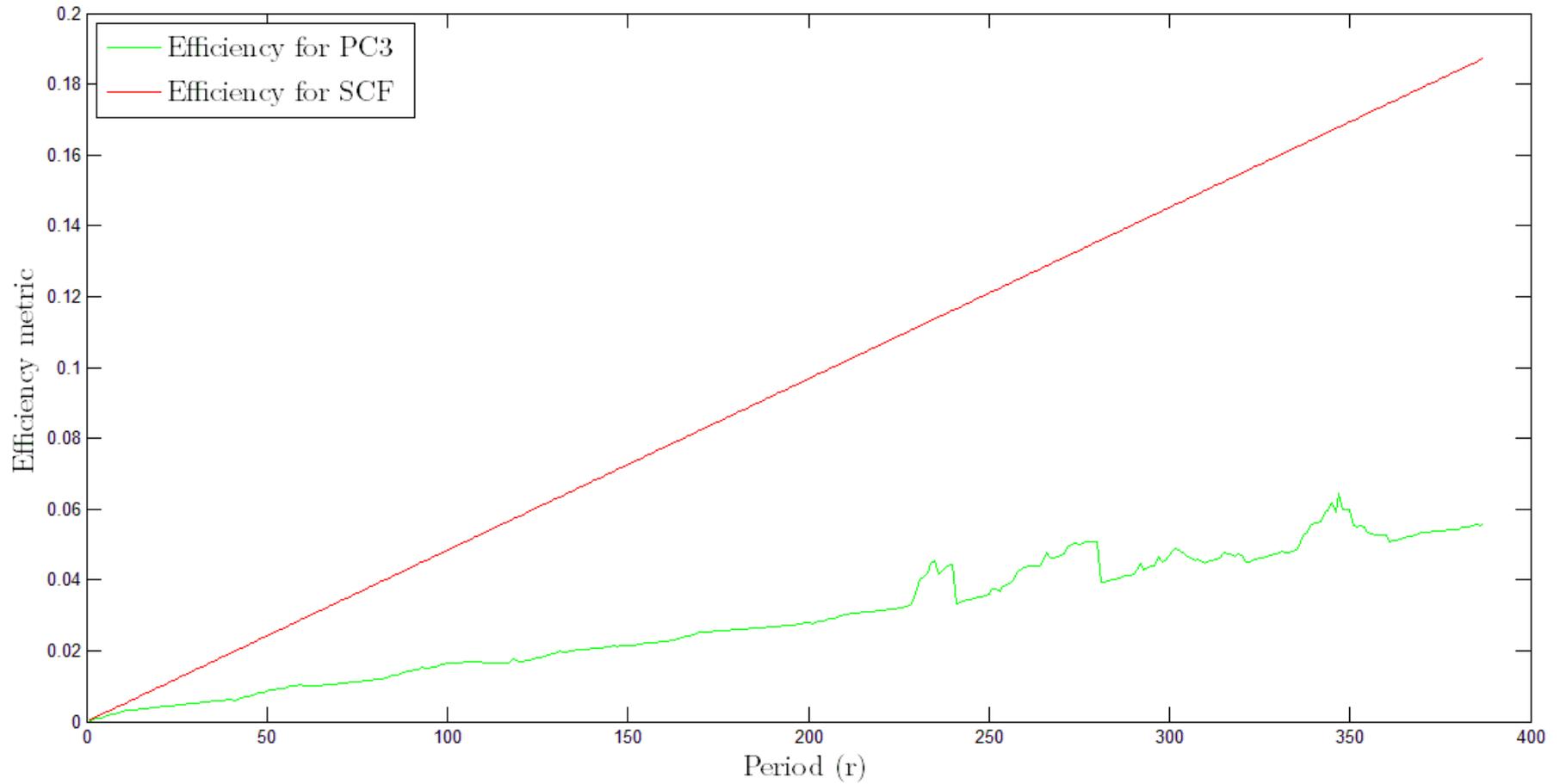
$$w(t) = \frac{c(t)}{1-e(t)}$$

# Performance



# Performance

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# Thank you!

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