Optimal Stopping of the Context Collection Process in Mobile Sensor Networks

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The considered setting

Consider a **Mobile Sensors Network** (MSN) with

- **sources**, *i.e.*, sensors that produce **context** (*e.g.*, humidity)
- **collectors**, *i.e.*, mobile nodes that receive, store and forward context to their neighbors.

**Context** is **quality-stamped**, *e.g.*, freshness.

The context **quality indicator decreases** with time.

The **aim** of a collector is to **gather** as many high-quality pieces of context as possible from sources and/or collectors.
The considered setting

communication range

source

collector
The problem

The collectors in a MSN:

- *forage* for high quality context and, then, *deliver* it to mobile context-aware applications;

- *undergo* a context collection process by exchanging data with neighboring collectors and/or sources *in light of* receiving context of *better* quality and/or *new* context;

- *cannot prolong* this process forever, since context quality decreases with time, thus, delivered context might be unusable for the application.
Some definitions

Context $c$ is represented as:

$$c = \langle p, u, x_u \rangle$$

where:

$p$ is contextual parameter/ type (e.g., temperature),
$u$ is contextual value (e.g., 30 °C),
$x_u$ is quality indicator of value $u$
Context quality indicator

Indicator $x_u \in [0, 1]$ indicates freshness of $u$.

- $x_u = 1$ indicates that $u$ is of maximum quality.
- $x_u = 0$ indicates that $u$ is unusable.

Context at time $t$ is called fresh if $x_u(t) > 0$; otherwise it is called obsolete.
Context quality indicator

Indicator $x_u(t)$ at time $t > 0$ is updated as follows:

$$x_u(t) = x_u(t-1) - 1/z , \quad x_u(0) = 1, \quad z \neq 0$$

- $z$ is the validity horizon for parameter $p$ in which value $u$ is considered usable.

- $z$ is application specific, e.g., $z = 10$min if $p$ is temperature, $z = 1$min if $p$ is wind-speed.

Notice: Alternative quality indicator functions can be, for instance, the inverse exponential function
Consider a **collector** which has collected a set of $N$ **fresh** pieces of context, $C = \{c_1, c_2, \ldots, c_N\}$, referred to as **local context**.

Let collector receive context $q$ from a neighbor collector. Collector increases its **local context** in **type** and/or **quality** as follows:

- If $q$ is **obsolete** then collector **discards** $q$;
- If $q$ is **fresh** and there is some local context $c$ with the same type of $q$ but less fresh than $q$ then collector **replaces** $c$ with $q$;
- If $q$ is **fresh** and there is no other local context of the same type, collector **inserts** $q$ into $C$;
Degree of completeness

Local context $C$ is quantified through degree of completeness (DoC), $Y$, defined as the random variable [1]:

$$Y = N \cdot \sum_{k=1}^{N} X_k$$

- $N$ is the current number (quantity) of collected pieces of context; $N \in \{0, 1, 2, \ldots, m\}$, $m > 0$.
- $X_k$ is the current quality indicator of the $k$th contextual parameter in $C$.

Degree of completeness

When the collector decides to stop the collection process at some time, it wants to achieve the highest expected value of \( Y \).

Hence, the collector has to find an optimal stopping time \( t \) of the collection process which maximizes:

\[
E[Y_t] = E[N_t \cdot \sum_{k=1}^{N_t} X_k^t]
\]
Optimal Stopping Theory (OST)

• Choose the **best** time to **take** a decision of performing a certain action.

• **Observe** the current state of a system and decide whether to:
  - **continue** the process or
  - **stop** the process, and incur a certain cost.

...the **discounted sum** problem, the **odds** algorithm, the **secretary** problem, the **parking** problem, the **asset-selling** problem, etc.
Application to context collection problem

➤ Decision

☐ *When* to **stop** collecting pieces of context from neighboring collectors/sources and deliver them to the application.

➤ Cost

☐ *Quality* of local pieces of context decreases with time.

☐ *Serving* obsolete context to the application.

➤ Approach

☐ *Adoption* of the OST **discounted sum** problem
Discounted sum problem in context collection

The decision of the collector at time $t$ is:

- stop and deliver local context to the application, or
- continue the process and update local context

Let us define a tolerance threshold $\theta \in (0, m^2)$ such that:

If $Y > \theta$ Then local context is significantly adequate for the collector’s requirements in terms of quantity and quality.
Discounted sum problem in context collection

Consider the indicator function:

\[ I_t = \begin{cases} 
1, & \text{if } Y_t > \theta \\
0, & \text{otherwise} 
\end{cases} \]

and the cumulative sum up to time \( t \):

\[ S_t = \sum_{n=1}^{t} I_n \]

The problem is to decide how large \( S_t \) should get before the collector stops, i.e., we have to determine a time \( t \) such that the supremum

\[ \sup_t E[\beta^t S_t] \]

is attained, \( 0 < \beta < 1 \).
Discounted sum problem in context collection

**Optimal Stopping Rule:** Observe $I_t$ value at time $t$ and stop at the first time for which it holds true that:

$$S_t \geq \frac{\beta}{1 - \beta} (1 - F_Y(\theta))$$

- $F_Y(y)$ is the cumulative distribution function of $Y$.
- $\beta$ is *discount factor* indicating the necessity of collector to take a decision;
  - collector requires a rather extended time horizon for deciding on deliver context when $\beta$ is high.
Performance & Comparative Assessment

- **Simulation setup**
  - MSN of 100 nodes; number of sources \( \omega = \{5, 10, 20\} \)
  - Mobility model: Random Waypoint
  - Validity horizon \( \zeta \sim U(2,10)\)min.
  - Tolerance threshold \( \theta \in [0.2, 0.7] \)
  - Maximum quantity of contextual parameters \( m = \{10, 20\} \)

- **Comparison Schemes**
  - **Scheme C**: Randomized policy: collectors stop the process at a random time instance
  - **Scheme B**: Finite-Horizon policy [1]: collectors stop the process based on a pre-defined deadline \( T \); adoption of OST

- **Metric**
  - Normalized average value of DoC delivered to the application;
    - the higher DoC is, the higher context **quality** and **quantity** is delivered to the application
The Probability Density Function (PDF) of the decision delay $\Delta t^*$, i.e., interval between following collection processes, for diverse number of contextual parameters $m$. 
Performance & Comparative Assessment

The DoC for schemes A, B, & C

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Performance & Comparative Assessment

The PDF of normalized DoC for all schemes.
Performance & Comparative Assessment

The normalized DoC for all schemes against tolerance threshold $\theta$. 
Conclusions

• A solution to the *context collection problem* based on Optimal Stopping Theory;

• Collectors *autonomously* take time-optimized context delivery decisions *without* a deadline;

• Collectors deliver context of high quality and quantity within *short* delays;

• Our scheme performs *well* when dealing with applications which require context of high quality and quantity (*i.e.*, high tolerance threshold)
Thank you!