Online Learning: Introduction to Multi-Armed Bandits

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Introduction

Offline vs Online Learning

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- Online learning algorithms learn from incoming sequences of data.

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- In online learning there is no distinction between training and test set
- Statistical assumption regarding the generative process

Online Learning

As a Game

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- The game proceeds as follows *(i)* predictor chooses an instance *(ii)* environment chooses an instance *(iii)* predictor incurs a reward (or loss) and *(iv)* predictor uses the loss/reward to improve the next round selection. Consider spam detection as an example.

Online Learning

Performance Measure

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- Regret is defined as $\mathrm{R}(T) = \sum_{t=1}^{T} (\hat{y}_t - y_t)$
- If regret is linear, the algorithm is not learning anything.

Online Learning

General Online Learning Problems

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General Online Learning Problems

- Online classification, Online regression
- For online classification and regression problems we can use algorithms like FTL, FTRL etc.
- Our main focus is on online learning problems involving exploration-exploitation trade-off. One instantiation of such a problem is multi-armed bandits (MAB)

Online Learning

Multi-Armed Bandit Problem

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- Two reward distribution settings are studied within the MAB framework *(i)* stochastic and *(ii)* adversarial.

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- The objective of the two problem settings are different.
- Classical algorithms fails in pure-exploration settings.

Stochastic Bandits

Upper Confidence Bound Algorithm (UCB)

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Upper Confidence Bound Algorithm (UCB)

- UCB is one of the most popular stochastic MAB algorithms
- Try all the actions once
- At round t, pick the action a that maximizes $r_a(t) + \sqrt{\frac{2 \ln t}{t_a}}$
- One of the simplest algorithm that attains logarithmic regret

Non-Stochastic Bandits

EXPonential-weight for EXPloration and EXPloitation

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- Regret of the EXP3 is of the order $O(\sqrt{kT \ln k})$

Query Recommendations with Bandits

Contextual Bandits For Query Recommendations

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- We propose to select the queries based on max-utility.

Contextual Bandits For Query Recommendations

MAB algorithms work by randomly picking two arms and estimating statistical properties by running on-the-fly hypothesis testing. We propose to estimate the sampling probabilities using the similarity between context vectors.

Let $s_{j,i} = \text{sim}(a_j, a_i)$. $S^i_{\geq}(\varepsilon) = \{a_j : \text{sim}(a_j, a_i) \geq \varepsilon\}$ and $S^i_{<}(\varepsilon) = \{a_j : \text{sim}(a_j, a_i) < \varepsilon\}$ are the partition of the arms for a given similarity threshold value $\varepsilon > 0$.

We define:

$$s_{j,i}(\varepsilon) = p(a_j | a_j \in S^i_{\geq}(\varepsilon)) = \frac{s_{j,i}}{\sum_k s_{k,i} [\![a_k \in S^i_{\geq}(\varepsilon)]\!]}$$

$$\bar{s}_{j,i}(\varepsilon) = p(a_j | a_j \in S^i_{<}(\varepsilon)) = \frac{s_{j,i}}{\sum_k s_{k,i} [\![a_k \in S^i_{<}(\varepsilon)]\!]},$$

Contextual Bandits For Query Recommendations

Given the currently playing arm a_i, our hypothesis is that the arms are sampled according to the joint distribution $p(a_j, a_k | a_i, \varepsilon) = p(a_j, a_k | (s_{j,i}, s_{k,i}) \geq \varepsilon \vee (s_{j,i}, s_{k,i}) < \varepsilon)$.

The marginal conditional probability can be seen as the normalized preference score for a specific arm to be played next.

Our next assumption is that there exists a utility function which maps these preference scores to utilities.

This assumption is similar to the Lipschitz assumption.

A theorem due to [**Ortega2010**] states that only the ln function satisfies the above requirements.

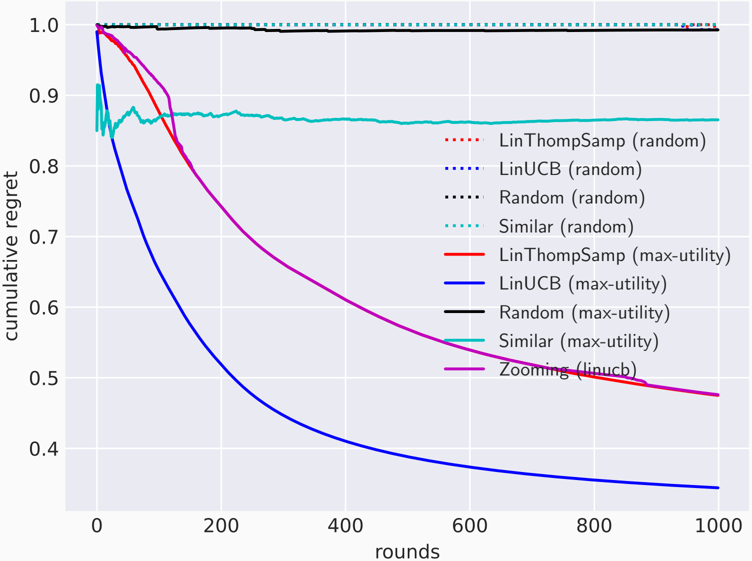
Contextual Bandits For Query Recommendations

This leads us to the following optimization problem for finding the candidate set.

$$\max_{\substack{\mathcal{C} \subseteq \mathcal{A}' \\ |\mathcal{C}| \leq k}} \log \left(g \left(\mathcal{C}, \{a_i\} \right) \right)$$

This optimization problem is submodular and it can be solved in lear time very efficiently using simple greedy algorithm

Contextual Bandits For Query Recommendations



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Adversarial Bandits with Self Supervised Experts

- We study the query recommendation problem in the adversarial setting.

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Adversarial Bandits with Self Supervised Experts

- We study the query recommendation problem in the adversarial setting.
- In the stochastic setting, we assume that there is a fixed query that the user will be interested in issuing with optimal utility.

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Adversarial Bandits with Self Supervised Experts

- We make use of the auto-regressive models as experts.

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Adversarial Bandits with Self Supervised Experts

- We make use of the auto-regressive models as experts.
- We train N auto-regressive models on the historical query logs.

Query Recommendations with Bandits

Parameters : parameter η

Initialization: $W_0 = 1, r_0 = 1, \mathcal{C}_0 = \emptyset, \hat{w}_i = 0$

for $t = 1, 2, \ldots T$ **do**

$q_t = get\_currently\_executing\_query()$

$\mathcal{C}^t = \bigcup_{e \in \mathcal{E}} query\_by\_score\_threshold(\mathcal{Q}, q_t, \epsilon)$

$\mathcal{C}_t = \mathcal{C}^t \cup \mathcal{C}_{t-1}$

$w_{i,t} = \frac{\eta \hat{r}_{t-1}[i \in \mathcal{C}_t \setminus \mathcal{C}_{t-1}]}{(1-\eta)|\mathcal{C}_{t-1}||\mathcal{C}_t \setminus \mathcal{C}_{t-1}|} + \hat{w}_{i,t} 1[i \in \mathcal{C}_{t-1}]$

$p_{i,t} = \frac{(1-\eta) w_{i,t} 1[i \in \mathcal{C}_t \setminus \mathcal{C}_{t-1}] + (1-\eta) w_{i,t} 1[i \in \mathcal{C}_{t-1}]}{\sum_i w_{i,t} 1[i \in \mathcal{C}_t]} + \frac{\eta}{|C_t|}$

Draw I_t from $\mathcal{C}_t$ according to p_t

Play I_t and observe gain $r_{I_t,t}$

$r_t = r_{t-1} + r_{I_t,t}$

Calculate the pseudo-reward values $\hat{r}_{i,t} = \frac{r_{i,t}[[I_{i,t}=1]]}{p_{i,t}}$

$\hat{w}_{i,t+1} = w_{i,t} \exp\left(\eta \hat{r}_{i,t}\right)$ for $j \in \mathcal{C}_t$

end

Query Recommendations with Bandits

Regret Analysis
Taking $\eta = \frac{1}{\sqrt{T|\mathcal{C}_T|}}$, and considering only the leading terms, per-round regret is

$$L_T - L^\star \leq O\left(\sqrt{\frac{|\mathcal{C}_T|}{T}}\right) \tag{1}$$

This regret matches with the regret that can be achieved if the size of $\mathcal{C}$ is fixed and known in advance.

Thank You

Questions..........?