Stability of Switched Linear Systems

A frequency-domain approach

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Overview

We seek to investigate stability properties of switched linear systems of the form $\dot{x} = A(t)x,\ A(t) \in \{A_1, A_2\}$. In contrast to time-invariant systems, instability can occur due to switching between stable constituent systems. This raises a number of questions related to stability for such systems.
It has been shown that in practice, periodic stability implies absolute stability (except for the marginal case). For a given *periodic* switching signal stability of the system can be determined by computing the spectral radius of the state-transition matrix for one period. We make suggestions how to approach stability questions for single-input single-output systems in the frequency domain.

Problem

Class of systems:

- two linear SISO systems
- periodic switching
- two switches per period

Objectives:

- sufficient condition for stability
- for arbitrary switching signal
- identify classes of stable switching signals

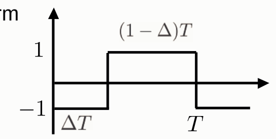
Model description

Switching system

$$\dot{x} = A(t)x, \qquad A(t) \in \{A_1, A_2\}$$

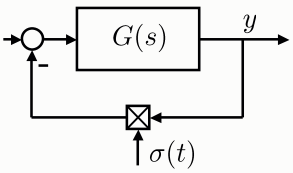
A_1, A_2 Hurwitz, in companion form

$$\dot{x} = Ax + \sigma(t)bc^T x$$



Switching signal

$$\sigma(t) \in \{-1, 1\}$$



Transfer function

$$G(s) = c^T (sI - A)^{-1} b$$

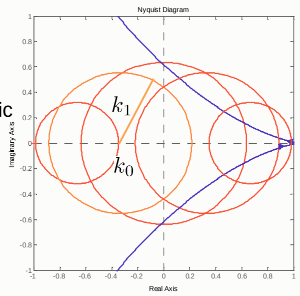
Harmonic Balance [*Harry Power*]

Assumption:
G(s) has low-pass characteristic

Approximate $\sigma(t)$ by its first harmonic

Harmonic balance gives

$$\frac{-1}{G(j\omega)} = k_0(\Delta) + k_1(\Delta)e^{-j2\omega t_0}$$



* *We gratefully acknowledge helpful discussions with Paul Curran*

Convolution matrix

- For periodic solutions:

$$Y(j\omega) = G(j\omega) \cdot \Sigma(j\omega) * Y(j\omega)$$

- switching signal and output are periodic therefore have discrete spectra
- Let $\nu = [y_{-N} \dots y_0 \dots y_N]^{\mathsf{T}}$ the Fourier coefficients of $Y(j\omega)$

  then for periodic solutions: $\nu = \Gamma(j\omega, N)K(\Delta, N)\nu$

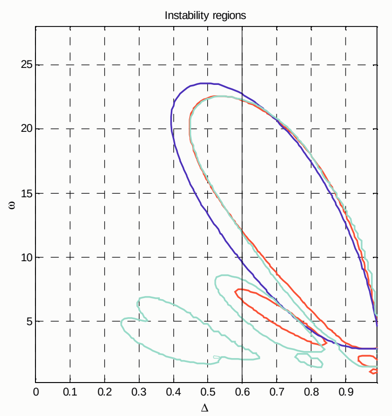
  with $\Gamma(jw, N) = \mathrm{diag}\{G(jN\omega) \dots G(0) \dots G(-jN\omega)\}$

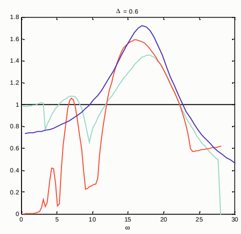
$$K(\Delta, N) = \begin{bmatrix} k_0 & 0 & k_{-1} & \dots & k_{-N} & 0 & \dots & 0 \\ 0 & k_0 & 0 & k_{-1} & \dots & k_{-N} & 0 & 0 \\ \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\ 0 & 0 & k_N & \dots & k_1 & 0 & k_0 & 0 \\ 0 & \dots & 0 & k_N & \dots & k_1 & 0 & k_0 \end{bmatrix}$$

- Then the spectrum can be reproduced if $\Gamma(j\omega, N)K(\Delta, N)$ has an eigenvalue equal to 1.

> Proposition
> The maximum real eigenvalue of $\Gamma(j\omega, N)K(\Delta, N)$ is an approximation for stability of the periodic switched system.

Example





Conclusion & further work

Results

- approximation for absolute stability (sufficient condition)
- design tool for switched systems
- good approximation of true stability boundaries
- (un)stable switching signals can be identified

Further work

- does method have computational advantages?
- can dwell-time problem be investigated?
- approach might be powerful for analysis of smooth functions

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