Control of Sideslip and Yaw Rate in 4-Wheel Steering Cars

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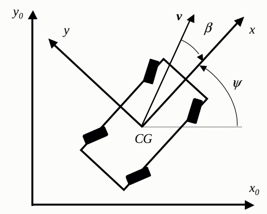
Abstract

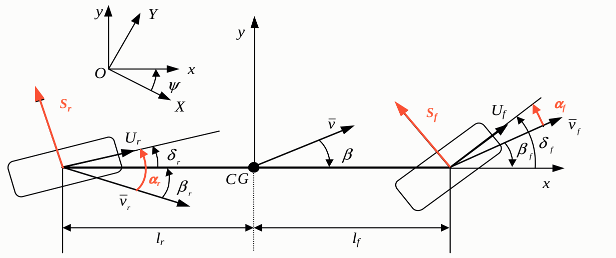
A new steering control structure for cars equipped with 4-wheel steering is presented. This control structure is based on a simplified linear model that captures the main features of the lateral dynamics of 4-wheel steering cars at constant speed. The proposed control structure allows for the decomposition of an originally 2-by-2 MIMO control design problem into two SISO control design problems by using individual channel decomposition. The control design can be carried out using classical Bode-plot based techniques and results in very simple sideslip and yaw rate controllers valid for the entire speed operating envelope.

1. The control problem



- Track reference yaw rate ($\dot{\psi}$) and sideslip signals (β) with the highest possible closed-loop bandwidth (Desirable: 3 Hz).
- Reject any disturbances in sideslip and yaw rate (i.e. those caused by wind gusts or μ–split braking) with highest possible bandwidth.
- Robustness to uncertainties and parameter changes (e.g. tyre stiffness).
- Speed operating envelope: 10–60 m/s.

2 Single-track linear model of 4-wheel steering dynamics

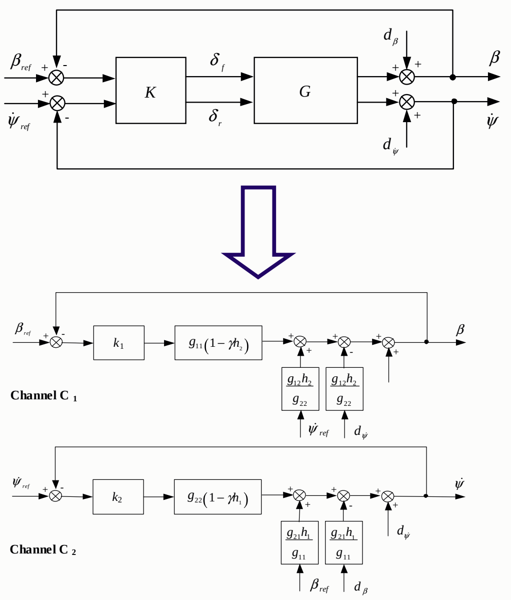


$$\dot{x} = Ax + Bu \qquad y = Cx + Du$$

$$u = \begin{bmatrix}\delta_f \\ \delta_r\end{bmatrix} \quad y = x = \begin{bmatrix}\beta \\ \dot{\psi}\end{bmatrix} \quad A = \begin{bmatrix} -\frac{C_f + C_r}{m v_x} & \frac{C_f l_f - C_r l_r}{m v_x^2} + 1 \\ \frac{C_f l_f - C_r l_r}{I_{zz}} & -\frac{C_f l_f^2 + C_r l_r^2}{I_{zz} v_x} \end{bmatrix} \quad B = \begin{bmatrix} -\frac{C_f}{m v_x} & -\frac{C_r}{m v_x} \\ \frac{C_f l_f}{I_{zz}} & -\frac{C_r l_r}{I_{zz}} \end{bmatrix} \quad C = \begin{bmatrix}1 & 0 \\ 0 & 1\end{bmatrix} \quad D = \begin{bmatrix}0 & 0 \\ 0 & 0\end{bmatrix}$$

$$G(s) = C(sI - A)^{-1}B + D = \begin{bmatrix} g_{11}(s) & g_{12}(s) \\ g_{21}(s) & g_{22}(s) \end{bmatrix}$$

3. Individual channel decomposition (Diagonal controller)



4. Control structure

Original yaw rate equation

$$\ddot{\psi} + \frac{C_f l_f^2 + C_r l_r^2}{I_{zz} v_x}\dot{\psi} = \frac{C_f l_f - C_r l_r}{I_{zz}}\beta + \frac{C_f l_f}{I_{zz}}\delta_f - \frac{C_r l_r}{I_{zz}}\delta_r$$

Input transformation

$$\Delta_1 = \delta_f + \frac{C_r}{C_f}\delta_r$$

$$\Delta_1 = \delta_f - \frac{C_r l_r}{C_f l_f}\delta_r$$

Cross-feedback

$$\Delta_2 - K\beta \quad \text{with } K = 1 - \frac{C_r l_r}{C_f l_f}$$

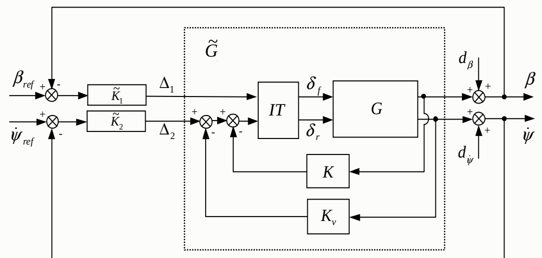
Speed-dependent feedback

$$\Delta_2 - K_v\dot{\psi}$$

$$\text{with } K_v = K_0 - \frac{C_f l_f^2 + C_r l_r^2}{C_f l_f v_x},\ K_0 \text{ constant}$$

Resulting yaw rate equation

$$\frac{I_{zz}}{C_f l_f}\ddot{\psi} + K_0\dot{\psi} = \Delta_2$$



Design based on $\tilde{G}$: Upper-triangular with speed-invariant yaw rate dynamics

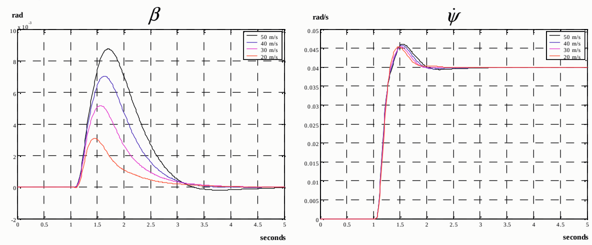
4. Control design

- Design is carried out in the frequency domain using an improved linear model of the car that includes time delay and actuator dynamics.
- The controller structure allows for the design of two controllers $\tilde{K}_1$ and $\tilde{K}_2$ valid for the entire operating envelope.
- The design is based on $\tilde{g}_{11}$ and $\tilde{g}_{22}$, respectively.
- Bandwidth separation is imposed (BW of Channel 1<<BW of Channel 2) in order to improve cross-channel disturbance rejection.
- Good phase and gain margins are obtained with integral control in Channel 1 and PID control in Channel 2.

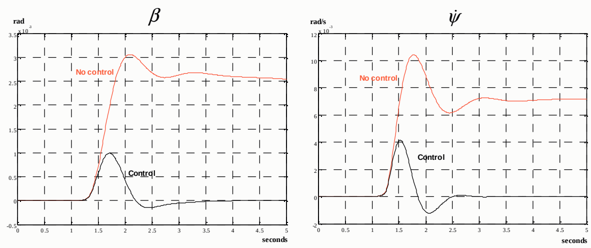
5. Simulation results

Using a non-linear two-track model of a Mercedes S Class

- Step reference of 0.04 rad/s in yaw rate. Maintain sideslip at 0 rad



- μ–split braking (1-0.2). Initial speed: 40 m/s. Braking: 9 m/s in 4 seconds



Acknowledgment: Thanks to Jens Kalkkuhl, DaimlerChrysler Research, Stuttgart

European Union RTN Summer School on Multi-Agent Control, NUI Maynooth, Sept. 8th – 10th 2003