Probabilistic multiparty session types

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Introduction

- In general, an important feature of a probabilistic model is that it distinguishes between nondeterminism and probabilistic choice
  - a nondeterminism choice refers to the one made by an external process,
  - a probabilistic choice is a choice made internally by the process, and not controlled by an external process.
- Intuitively, a probabilistic choice is given by sets of alternative transitions, each transition having a certain probability of being selected, where the sum of all probabilities of one alternative set is 1.

Introduction

- To distinguish the differences between nondeterminism and probabilistic choices, consider the following simple example: *Alice* wrote a manuscript and intends to submit it to a journal. There, for some journals, she has to select from several editors (say three: *Bob*, *Carol* and *Diana*).
- It is the author's choice to which editor to send his work. This is a probabilistic choice (as it is under his control and the preference for which to select may depend on some previous interactions).
- Then the author waits for an answer. This is a nondeterministic choice (as the choice of what kind of answer he receives is out of hi's control).

Introduction

- We consider that the probabilistic choice is a choice made internally by the process, and not controlled by an external process.
- There are two possibilities for extending a model using probabilities:
  - to replace nondeterministic choices by probabilistic choices
  - to allow both probabilistic and nondeterministic choices.
- We take the second approach since when considering concurrent processes the concept of nondeterminism is necessary to describe the asynchronous character of the interleaving parallel composition.

Processes

$$
\begin{array}{rcll}
P & ::= & \vdots & \\
 & \mid & \sum_{p_i} p_i : s!\langle \tilde{e}_i \rangle; P_i & \text{(value sending)} \\
 & \mid & \sum_{j \in J} s?(\tilde{x}_j); P_j & \text{(value reception)} \\
 & \mid & \sum_{p_i} p_i : s \lhd l_i; P_i & \text{(label selection)} \\
 & \mid & s \rhd \{l_j : P_j\}_{j \in J} & \text{(label branching)} \\
 & \mid & \vdots &
\end{array}
$$

- Assume *Alice* knows *Bob* with whom it already had some scientific interactions, while about *Carol* she heard from her articles.
- Therefore, the probability that *Alice* chooses *Bob* to handle the review of her manuscript is higher than choosing *Carol*.
- Even if there exists a probability to choose *Diana*, this is very small as *Alice* does not know anything about her.

Example (Probabilistic Choice)

$$\begin{array}{lcl} Alice & = & 0.6 : submitB!\langle article\rangle; AliceB \\ & + & 0.3 : submitC!\langle article\rangle; AliceC \\ & + & 0.1 : submitD!\langle article\rangle; AliceD \end{array}$$

- After receiving a manuscript, an editor can perform various actions:
  - to accept the paper; usually the probability to accept a paper is small (e.g., 0.1);
  - to reject the paper;
  - to propose another editor, possible from another journal, as the paper does not fit the journal aims; the probability for this to happen is very small (e.g., 0.05);

Example (Probabilistic Choice)

$$\begin{aligned} Bob &= 0.10 : s \triangleleft accept; BobA \\ &+ 0.85 : s \triangleleft reject; BobR \\ &+ 0.05 : s \triangleleft propose; BobP \end{aligned}$$

- After sending her manuscript *Alice* knows she can expect that:
  - her paper is accepted;
  - her paper is rejected;
  - her paper is proposed to another editor;

Example (Nondeterministic Choice)

$$
\begin{aligned}
Alice \quad = \quad s \triangleright \{ & accept; AliceA \\
& reject; AliceR \\
& propose; AliceP \}
\end{aligned}
$$

$$\textstyle\sum_{p_i} p_i : s!\langle\tilde{e}_i\rangle; P_i \mid s : \tilde{h} \rightarrow_{p_i} P_i \mid s : \tilde{h} \cdot \tilde{v}_i \quad (\tilde{e}_i \downarrow \tilde{v}_i) \qquad \text{(ProbSend)}$$

$$\textstyle\sum_{p_i} p_i : s \lhd l_i; P_i \mid s : \tilde{h} \rightarrow_{p_i} P_i \mid s : \tilde{h} \cdot l_i \qquad \text{(ProbLabel)}$$

$$\textstyle\sum_{i \in I} s?(\tilde{x}_i); P_i \mid s : \tilde{v} \cdot \tilde{h} \rightarrow_1 P_i\{\tilde{v}/\tilde{x}_i\} \mid s : \tilde{h} \qquad \text{(NondetReceive)}$$

$$s \rhd \{l_j : P_j\}_{j \in J} \mid s : l_i \cdot \tilde{h} \rightarrow_1 P_i \mid s : \tilde{h} \quad (i \in J) \qquad \text{(NondetBranch)}$$

$$P \rightarrow_p P' \Rightarrow P \mid Q \rightarrow_p P' \mid Q \qquad \text{(Par1)}$$

$$P \rightarrow_p P' \text{ and } Q \rightarrow_q Q' \Rightarrow P \mid Q \rightarrow_{p \cdot q} P' \mid Q' \qquad \text{(Par2)}$$

$$P \equiv P' \text{ and } P' \rightarrow_p Q' \text{ and } Q' \equiv Q' \Rightarrow P \rightarrow_p Q \qquad \text{(Struct)}$$

Example

- After receiving a manuscript *Bob* sends it to two reviewers:
  - $Elliot = 0.15 : review!\langle accept\rangle; ElliotA + 0.85 : review!\langle reject\rangle; ElliotR;$
  - $Felix = 0.05 : review!\langle accept\rangle; FelixA + 0.95 : review!\langle reject\rangle; FelixR;$
- using rule (PAR2) it can be noticed that all possible evolutions are:
  - $Elliot \mid Felix \rightarrow_{0.0075} ElliotA \mid FelixA$, where $0.0075 = 0.15 * 0.05$
  - $Elliot \mid Felix \rightarrow_{0.0425} ElliotR \mid FelixA$, where $0.0425 = 0.85 * 0.05$
  - $Elliot \mid Felix \rightarrow_{0.1425} ElliotA \mid FelixR$, where $0.1425 = 0.15 * 0.95$
  - $Elliot \mid Felix \rightarrow_{0.8075} ElliotR \mid FelixR$, where $0.8075 = 0.85 * 0.95$
- it should be noticed that the sum of the probabilities of all evolutions equals 1, where $1 = 0.0075 + 0.0425 + 0.1425 + 0.8075$.

Global Types

$$
\begin{array}{rcll}
G & ::= & \sum_{p_i} q \rightarrow_{p_i} q' : k\langle S_i\rangle.G_i & \text{(probValues)} \\
 & \mid & \sum_{p_j} q \rightarrow_{p_j} q' : k\{l_j : G_j\} & \text{(probBranching)}
\end{array}
$$

Local Types

$$
\begin{array}{rcll}
T & ::= & \sum_{p_i} p_i : k!\langle S_i\rangle.T_i & \text{(send)} \\
 & \mid & \sum_{i\in I} k?(S_i).T_i & \text{(receive)} \\
 & \mid & k \oplus \{p_j : (l_j : T_j)\}_{j\in J} & \text{(selection)} \\
 & \mid & k\&\{l_j : T_j\}_{j\in J} & \text{(branching)}
\end{array}
$$

- We use the judgement $\Gamma \vdash P \triangleright \Delta$ which says that "under the environment Γ, process P is well-typed having typing Δ".
- We use notation $T@q$ (called located type) representing a local type T assigned to a participant q.

$$\frac{\forall i.\Gamma \vdash P_i \triangleright \Delta, \tilde{s} : T_i@q \qquad i \in J \qquad \sum_i p_i = 1}{\Gamma \vdash \sum_{p_i} p_i : s_k \triangleleft l_i; P_i \triangleright \Delta, \tilde{s} : k \oplus \{p_i : (l_i : T_i)\}_{i \in I}@q} \quad \text{(TSelect)}$$

$$\frac{\forall j.\Gamma \vdash P_j \triangleright \Delta, \tilde{s} : T_j@q}{\Gamma \vdash s_k \triangleright \{l_j; P_j\}_{j \in J} \triangleright \Delta, \tilde{s} : k\&\{l_j : T_j\}_{j \in J}@q} \quad \text{(TBranch)}$$

Definition (Evolution probability)

If $P \rightarrow_{p_1} P_1 \rightarrow_{p_2} P_2 \ldots \rightarrow_{p_k} Q$ then the probability to reach from P to Q equals $p = p_1 * p_2 * \ldots * p_k$. We denote this by $prob(P, Q) = p$.

Proposition

If we denote by $reach(P)$ all processes reachable from a well-typed process P that cannot further evolve, then

$$\sum_{Q \in reach(P)} prob(P, Q) = 1.$$

As processes interact, their dynamics is formalised by a type reduction relation $\Rightarrow$ on typing Δ:

- $\tilde{s} : \{\sum_{p_i} p_i : k!\langle\tilde{S}_i\rangle; T_i@q_1, \sum_{i'\in I'} k?(\tilde{S}_{i'}); T_{i'}@q_2, \ldots\}$
  $\Rightarrow_{p_j}\tilde{s} : \{T_j@q_1, T_{j'}@q_2, \ldots\}$, for $j \in I$, $j' \in I'$, $S_j = S_{j'}$
- $\tilde{s} : \{k \oplus \{p_i : (l_i : T_i)\}_{i\in I}@q_1, k\&\{l_{i'} : T_{i'}\}_{i'\in I'}@q_2, \ldots\}$
  $\Rightarrow_{p_j}\tilde{s} : \{T_j@q_1, T_j@q_2, \ldots\}$, for $j \in I \cap I'$.
- $\Delta\Rightarrow_p\Delta'$ and $\Delta'\Rightarrow_q\Delta''$ implies $\Delta\Rightarrow_{p\cdot q}\Delta''$

Theorem (subject congruence and reduction)

1. $\Gamma \vdash P \triangleright \Delta$ *and* $P \equiv P'$ *imply* $\Gamma \vdash P' \triangleright \Delta$.
2. $\Gamma \vdash P \triangleright \Delta$ *and* $P \rightarrow_{p_i} P'$ *imply* $\Gamma \vdash P' \triangleright \Delta'$, *where* $\Delta = \Delta'$ *or* $\Delta \Rightarrow_{p_i} \Delta'$.

- By using probabilities we are able to describe complex processes in which some behaviours are more likely to happen than others.
- An illustrative example is presented.
- Our approach is sound.
- Discussions: ....

Thank you!