A Parallel, Backjumping Subgraph Isomorphism Algorithm using Supplemental Graphs

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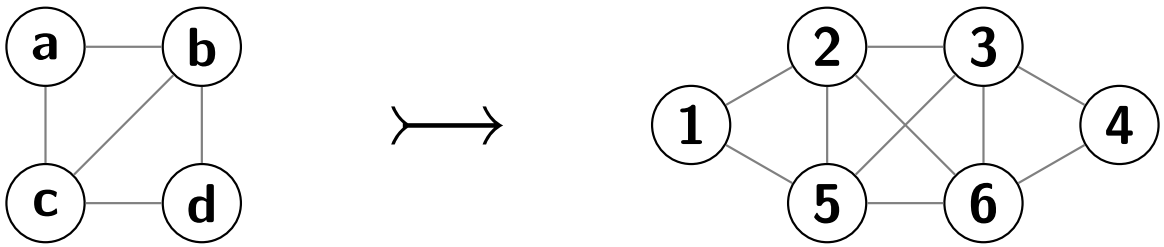
The Subgraph Isomorphism Problem

- Given a little *pattern* graph and a large *target* graph, find "a copy of" the pattern inside the target.



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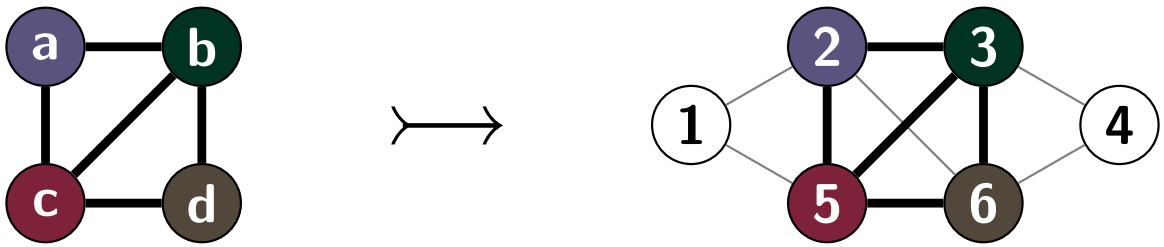


Applications

- Bioinformatics and chemistry.
- Computer vision and pattern recognition.
- Fraud detection and law enforcement.
- Model checking.
- Social network analysis.
- Code generation in compilers.
- Diseased cows.

The Basic CP Model

- One variable for each vertex in the pattern graph, with the domains being the vertices of the target graph.
- If two vertices are adjacent in the pattern, their values must be adjacent in the target.
  - In the *induced* variant, non-adjacent vertices must be mapped to non-adjacent vertices. We only discuss the non-induced variant in this talk.
- Each pattern vertex must be given a different value.
- Often enhanced: for example, we can filter based upon degree.
- Another perspective: find an injective mapping from the pattern to the target, which preserves adjacency.

Selected Previous Work

- VF2: backtracking search, interesting heuristics. Widely used.

  A (Sub)Graph Isomorphism Algorithm for Matching Large Graphs. *Luigi P. Cordella, Pasquale Foggia, Carlo Sansone and Mario Vento*. IEEE Trans. Pattern Anal. Mach. Intell., 2004.

- LAD: locally all-different, and neighbourhood degree sequences.

  AllDifferent-based filtering for subgraph isomorphism. *Christine Solnon*. Artif. Intell., 2010.

- SND: distance-based filtering.

  Scoring-Based Neighborhood Dominance for the Subgraph Isomorphism Problem. *Gilles Audemard, Christophe Lecoutre, Mouny Samy Modeliar, Gilles Goncalves and Daniel Porumbel*. CP 2014.

Being Clever is Expensive

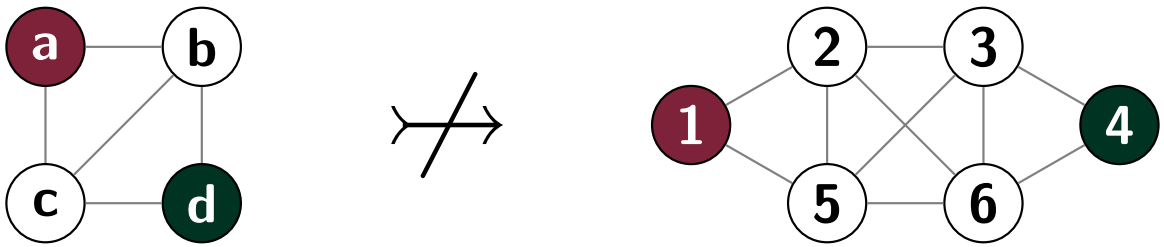
- We want to work with up to 1,000 pattern vertices, and 10,000 target vertices.
- When LAD and SND fail, they often manage less than one recursive call per second, particularly with larger target graphs.

Our Approach

- Cheaper inference: 10^4 to 10^6 recursive calls per second.
- Expensive preprocessing once at the top of search, rather than computing distances and degree sequences during search.
- FC-CBJ instead of MAC.
- A counting-based all-different propagator, which does more than pairwise-$\neq$ AC but less than GAC.
- Bit-parallelism for all major operations.
- Thread-parallel search.

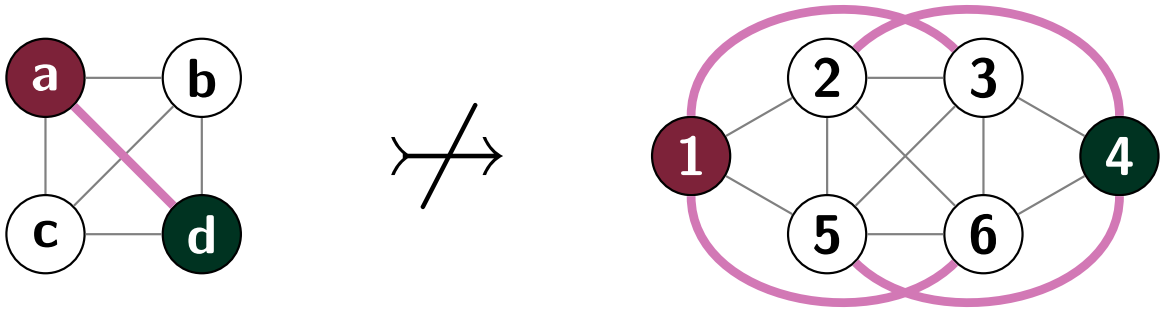
Supplemental Graphs

- Adjacent vertices must be mapped to adjacent vertices.
- Used by SND:
  - Vertices that are distance 2 apart must be mapped to vertices that are within distance 2.
  - Vertices that are distance k apart must be mapped to vertices that are within distance k.



Supplemental Graphs

- G^d is the graph with the same vertex set as G, and an edge between v and w if the distance between v and w in G is at most d.
- For any d, a subgraph isomorphism $i : P \rightarrowtail T$ is also a subgraph isomorphism $i^d : P^d \rightarrowtail T^d$.



Supplemental Graphs

- We can do something stronger: rather than looking at distances, we can look at (simple) paths, and we can count how many there are.
- This is NP-hard in general, but only lengths 2 and 3 and counts of 2 and 3 are useful in practice.
- We construct these graph pairs once, at the top of search, and use them for degree-based filtering at the top of search, and "adjacency" filtering during search.

Counting-Based All-Different

- One pass through the variables, from smallest domain to largest.
- Track unions of domains as we go along.
- Eliminate any detected Hall sets from future variables.
- This can miss some deletions, but has a very fast bitset implementation for large domains.

Backjumping

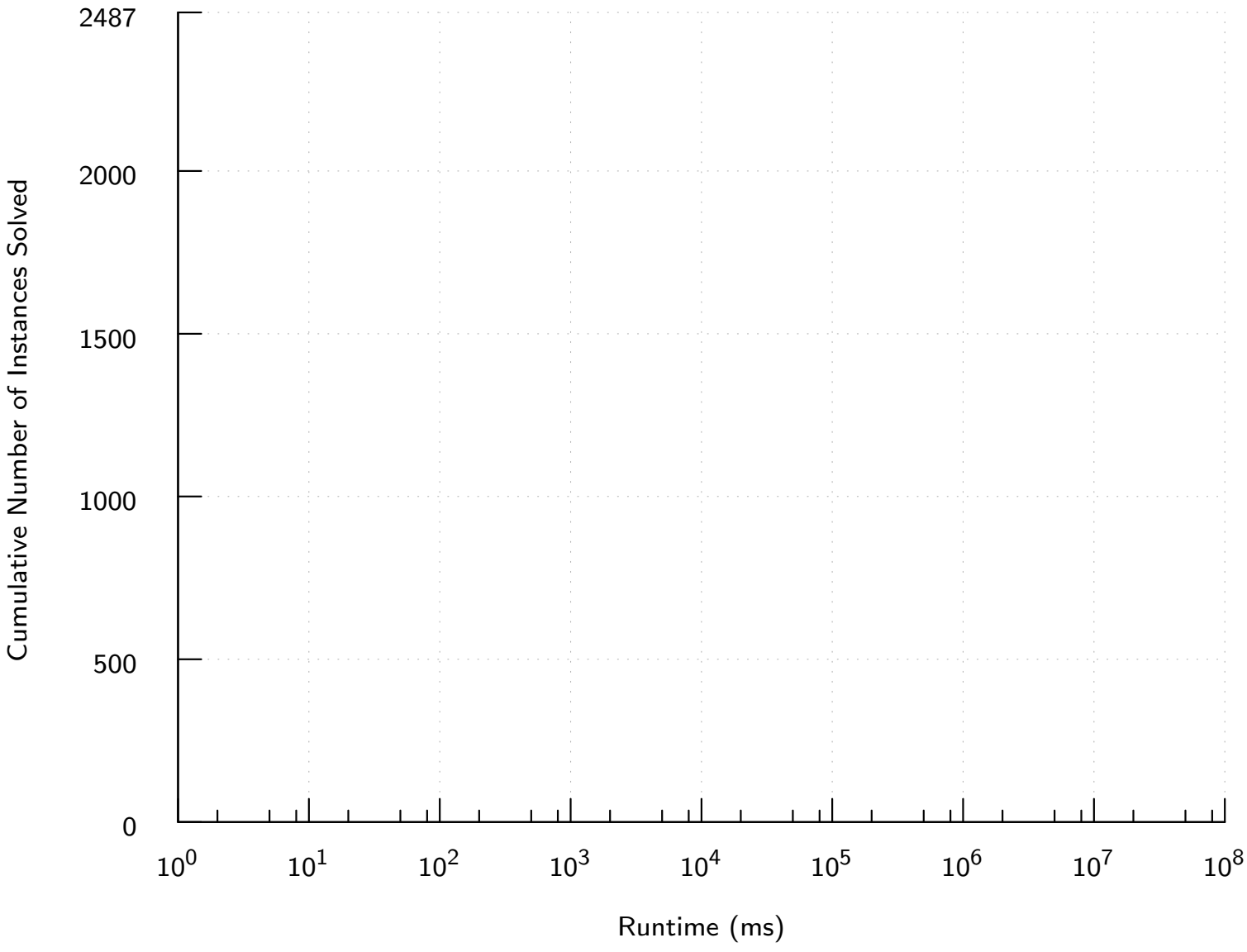
- Rather than backtracking on failure, we can sometimes show it's safe to jump back several levels immediately.
- This can be done with without explicit conflict sets, and without modifying propagators.
- Producing smaller conflict sets from a failed all-different gives better results.

Experimental Setup

- Dual Intel Xeon E5-2640 v2 (Q3'13), 64GBytes RAM. Our code is C++, LAD and VFLib are C, SND is Java.
- 2,487 pairs of instances, selected by other people:
  - Real-world graphs.
  - 2D images and 3D meshes from computer vision applications.
  - Random (simple, scale-free, 4D mesh, bounded degree).
- Up to 900 vertices and 12,410 edges in the pattern, and 5,944 vertices and 34,210 edges in a target. A mix of satisfiable and unsatisfiable instances, although there is a bias towards satisfiable instances...
- Please donate your subgraph isomorphism problem instances:

  `http://liris.cnrs.fr/csolnon/SIP.html`

Cumulative Instances Solved



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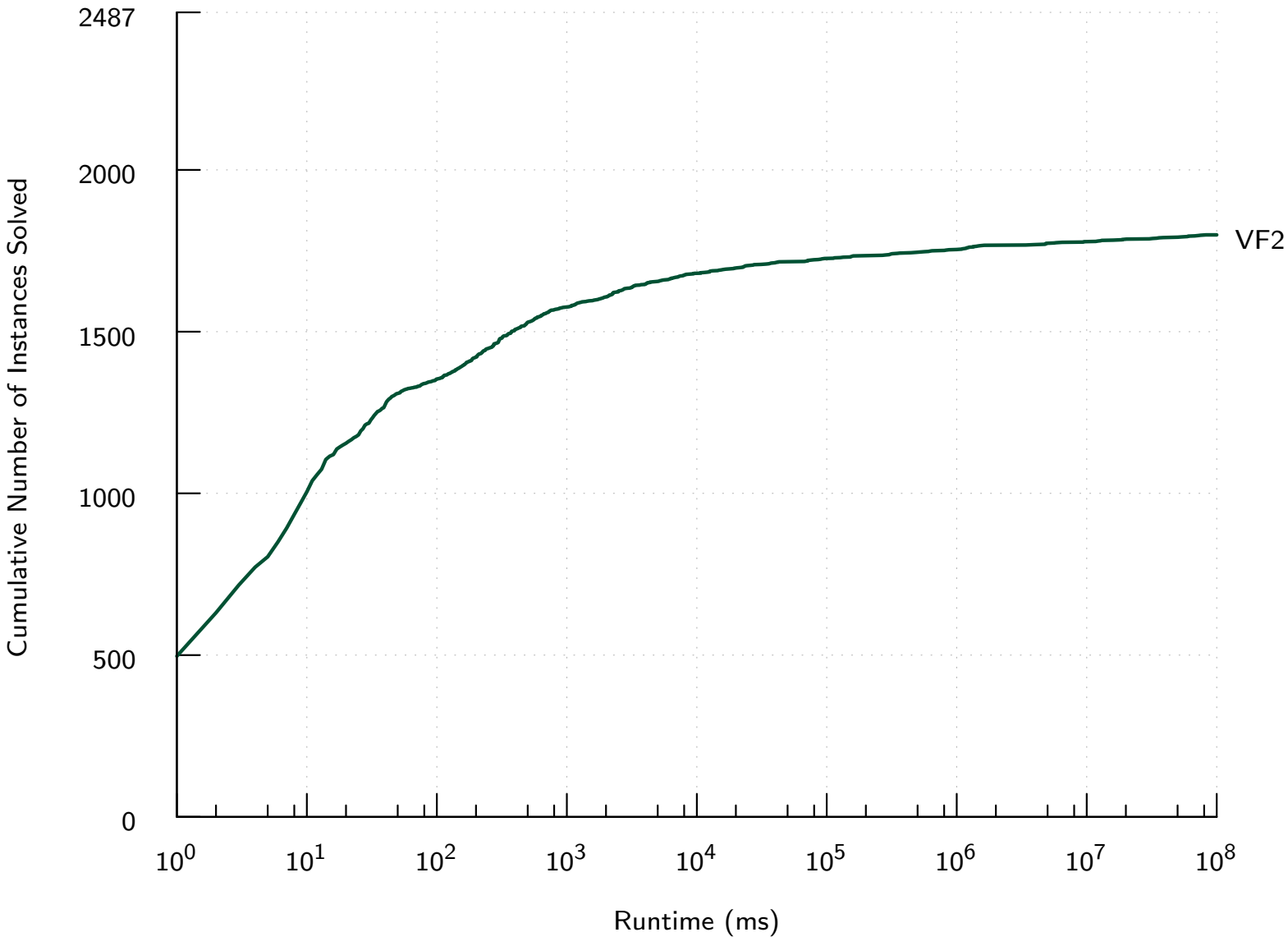

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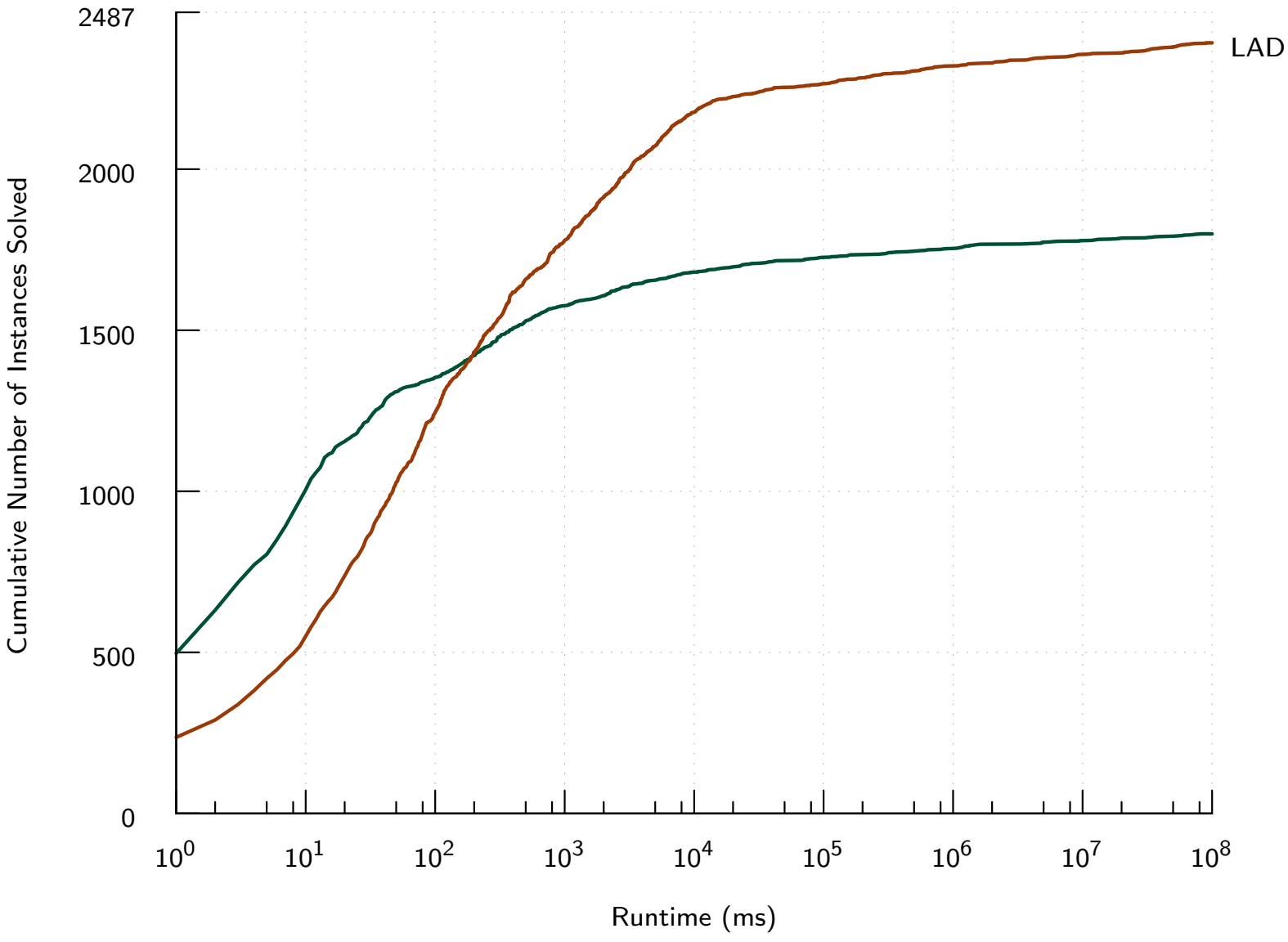

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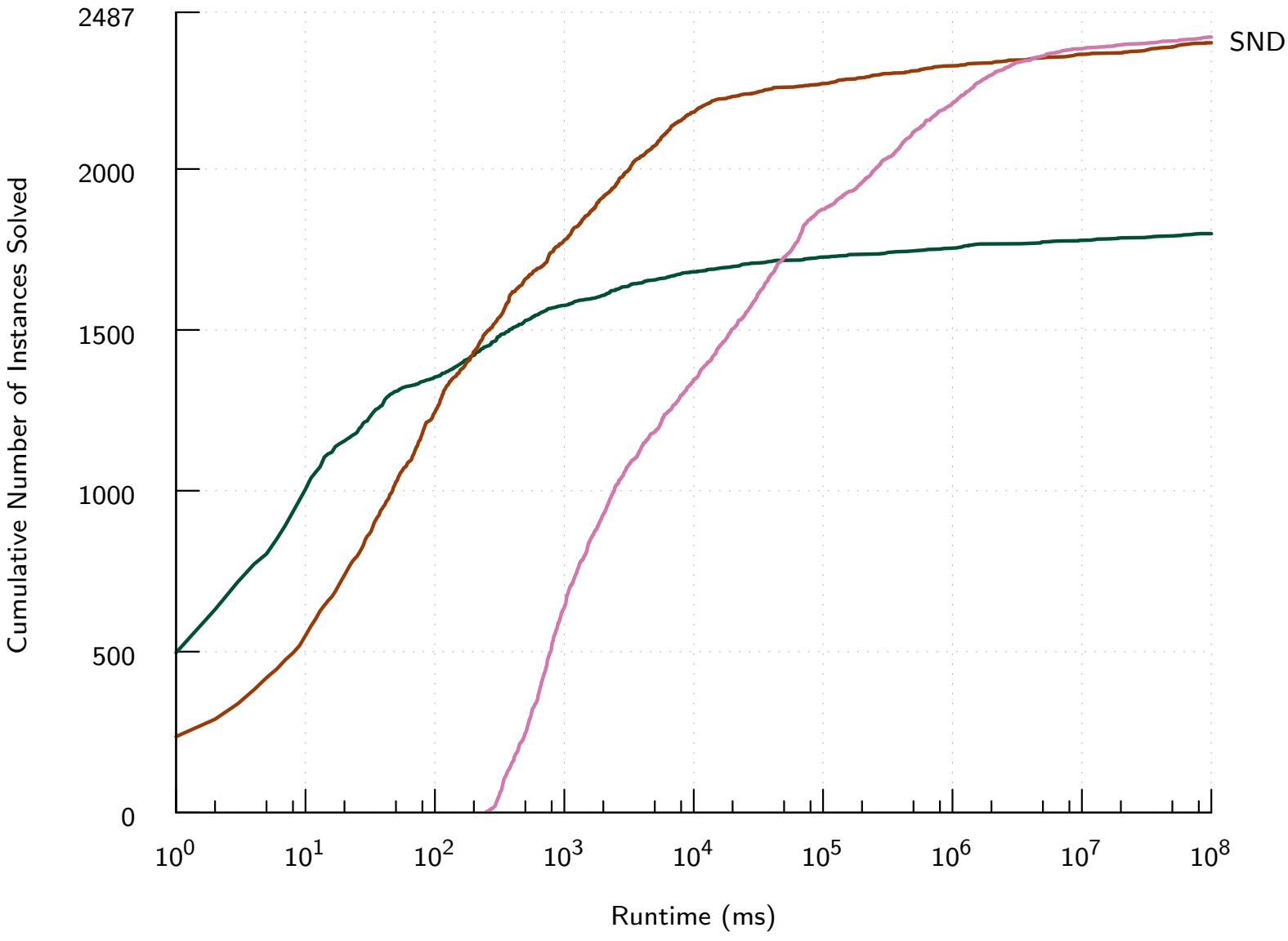

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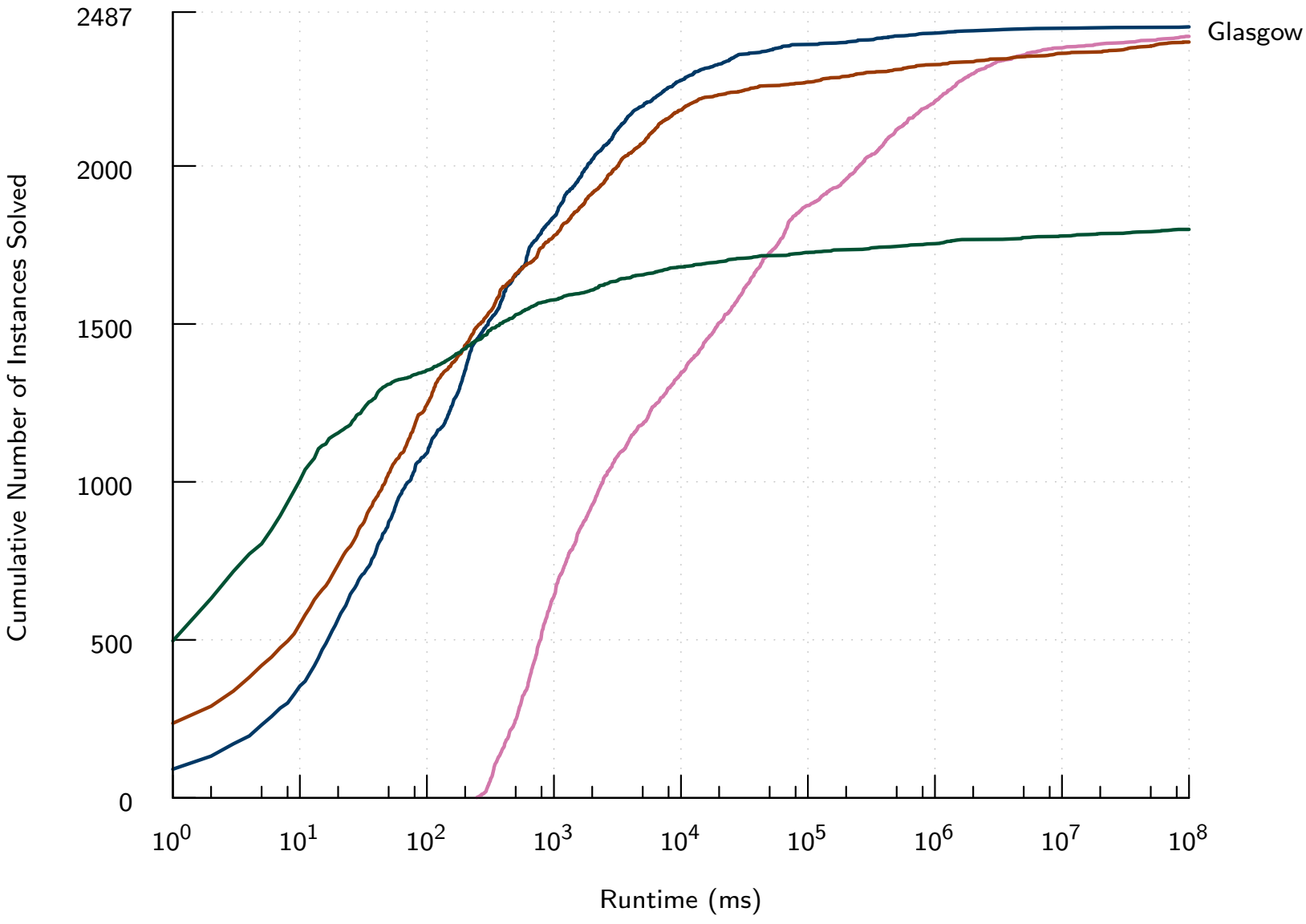

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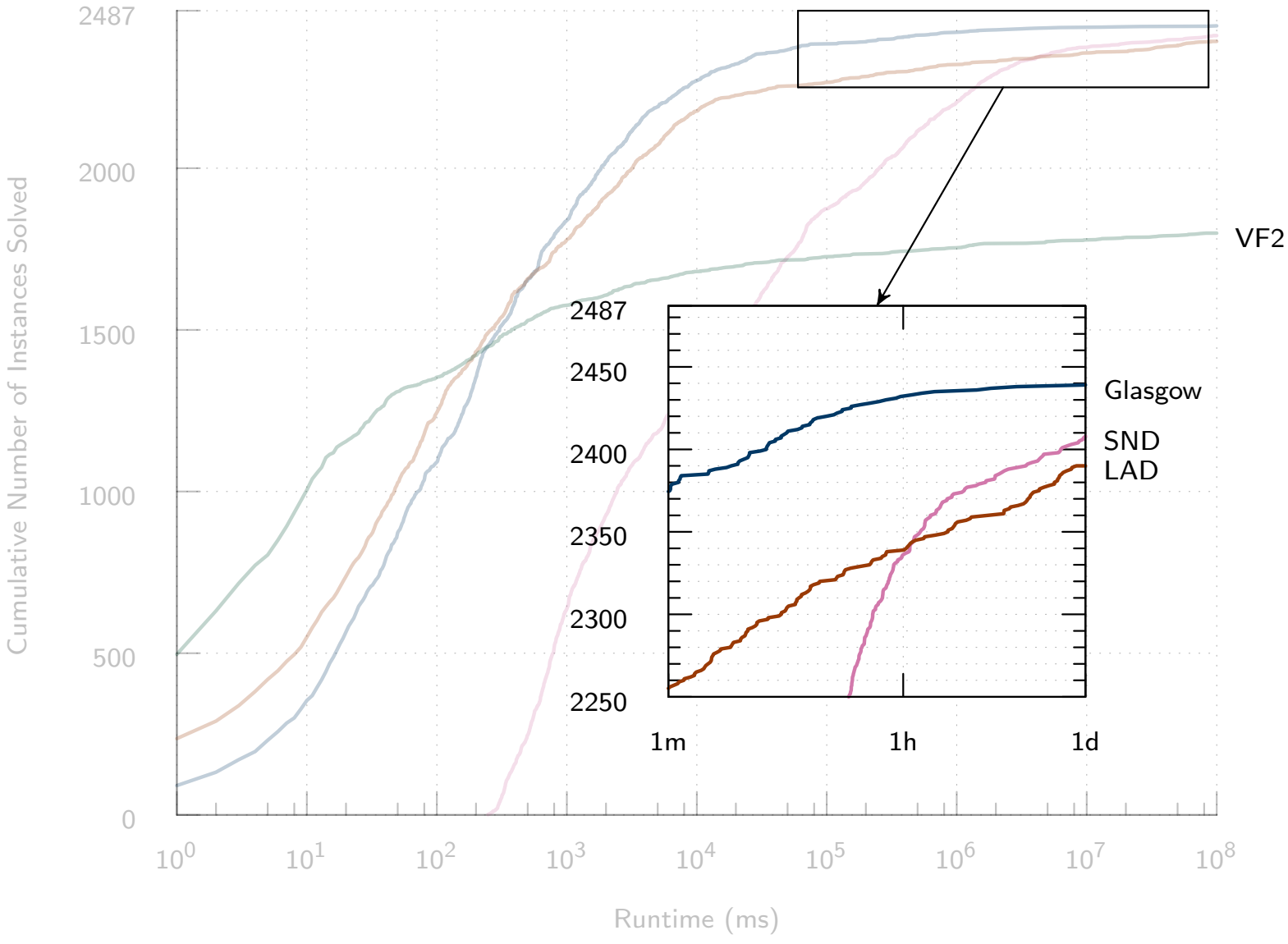
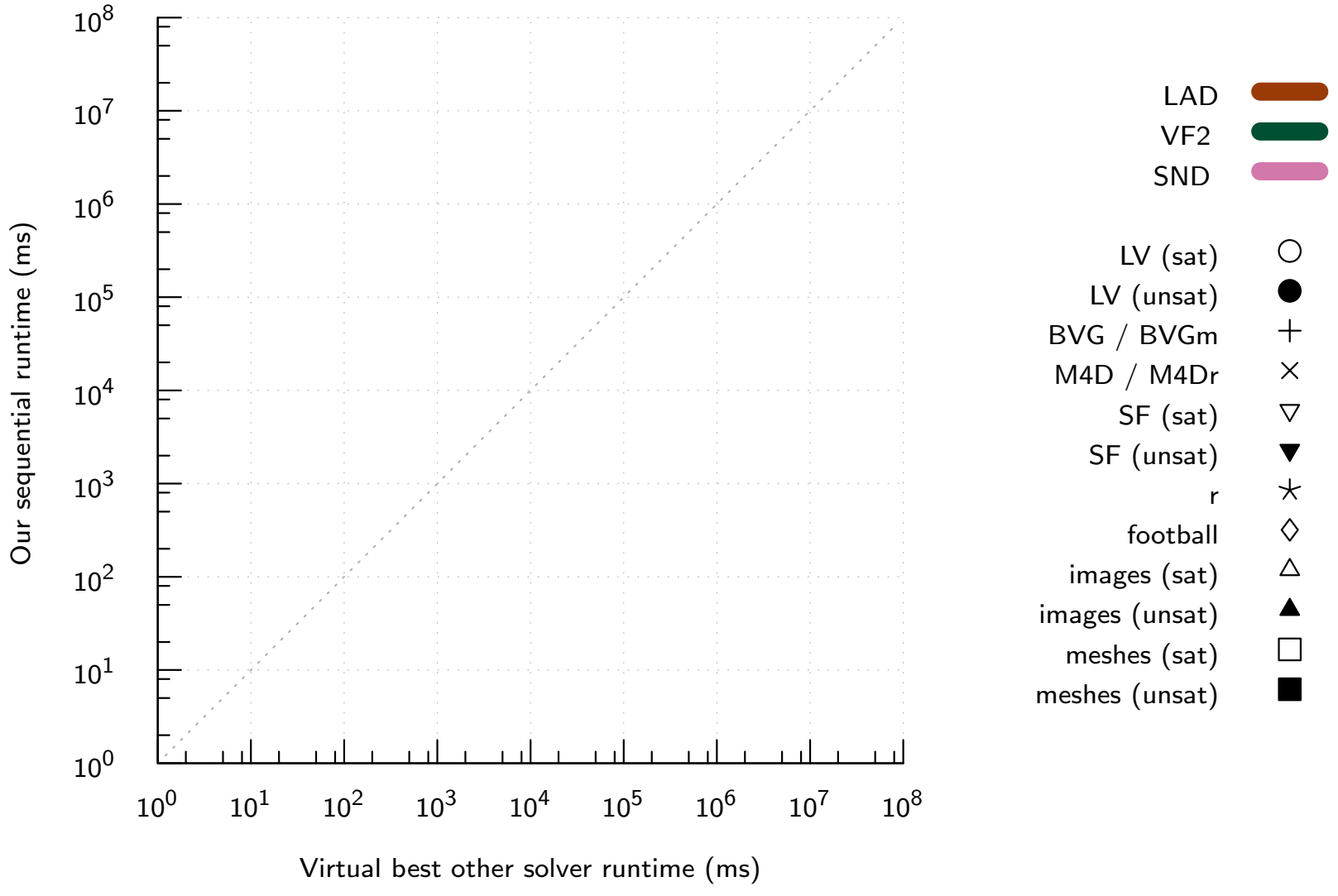


Versus Virtual Best Other Solver



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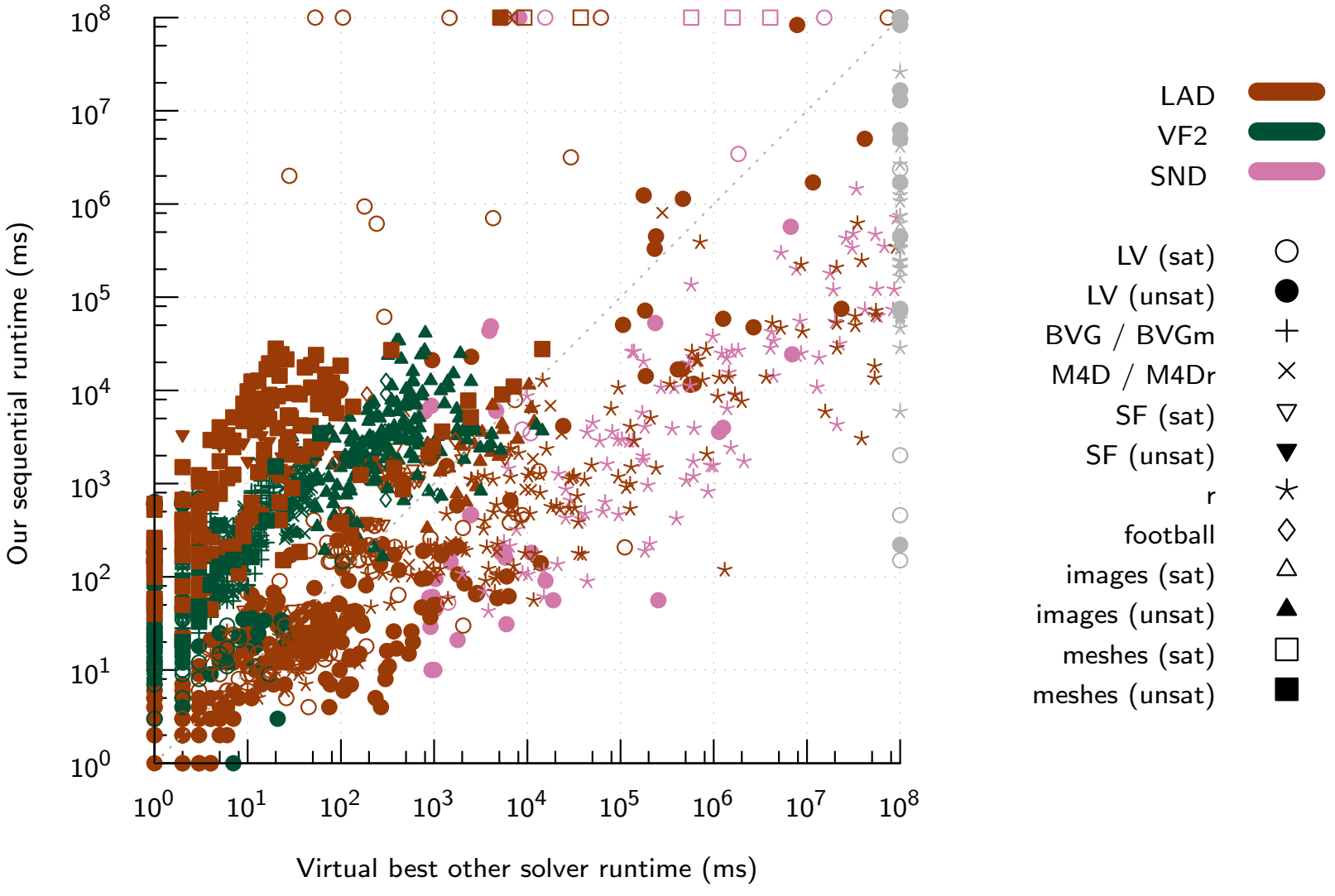
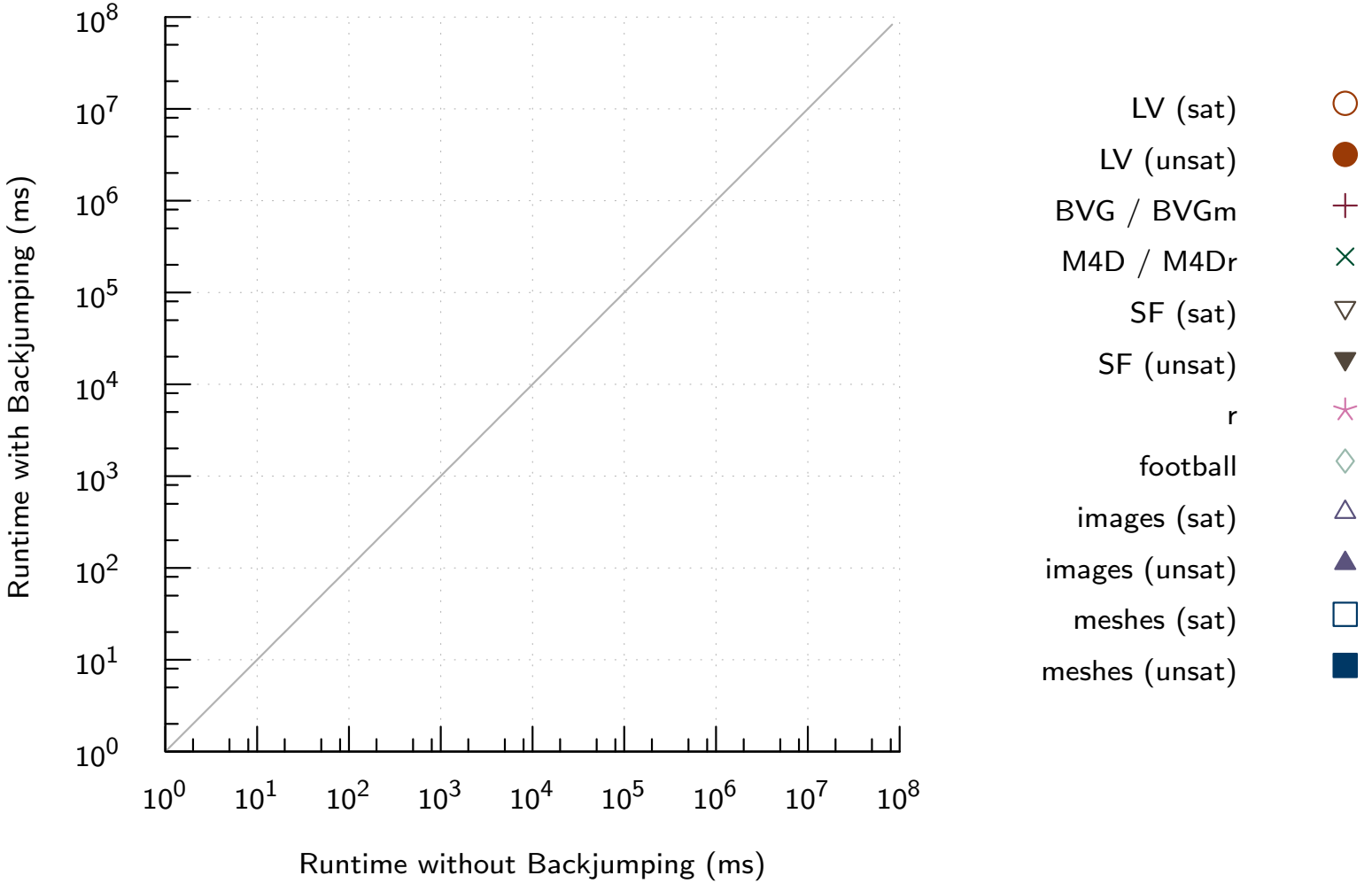


Backjumping?



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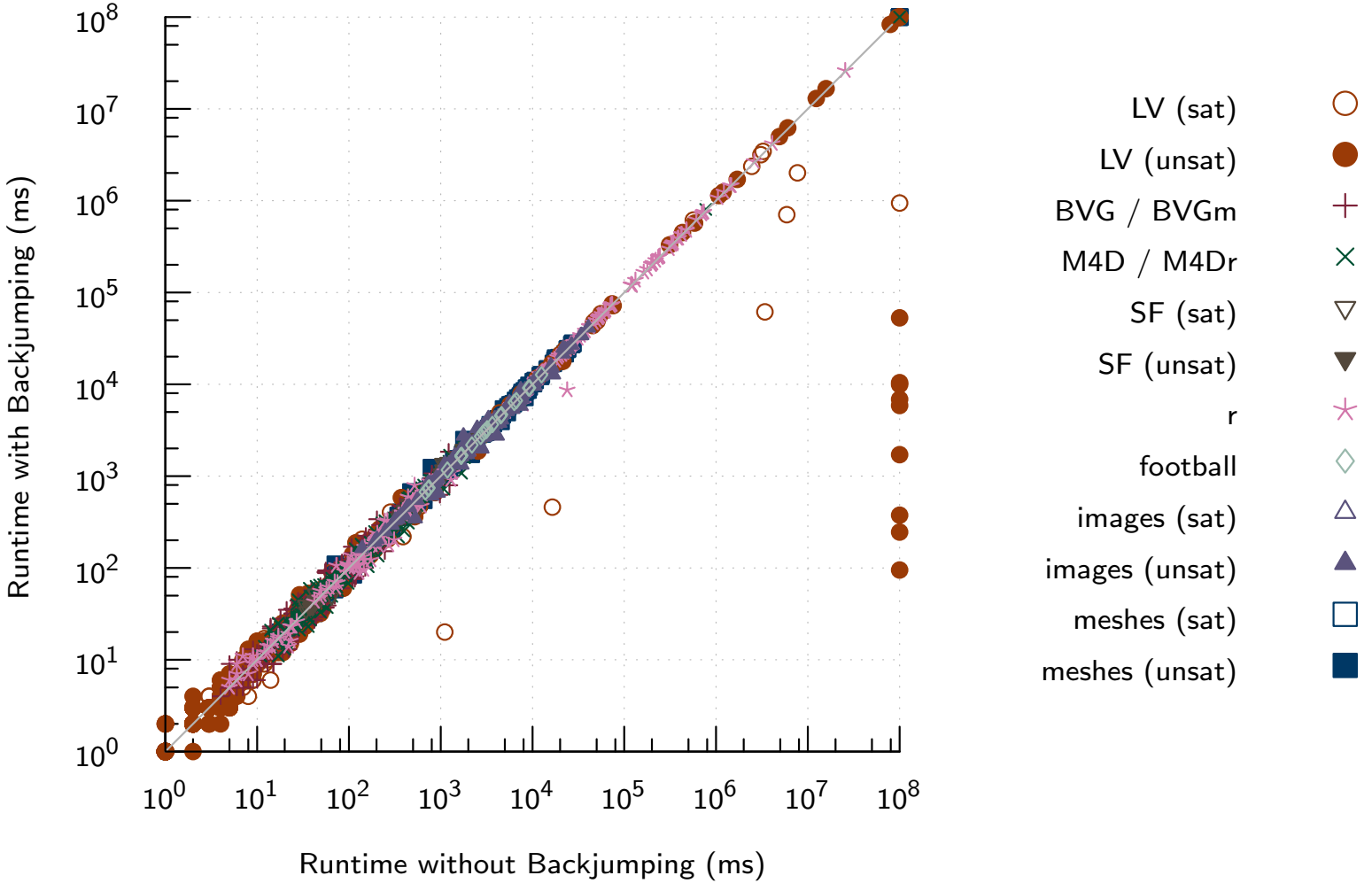
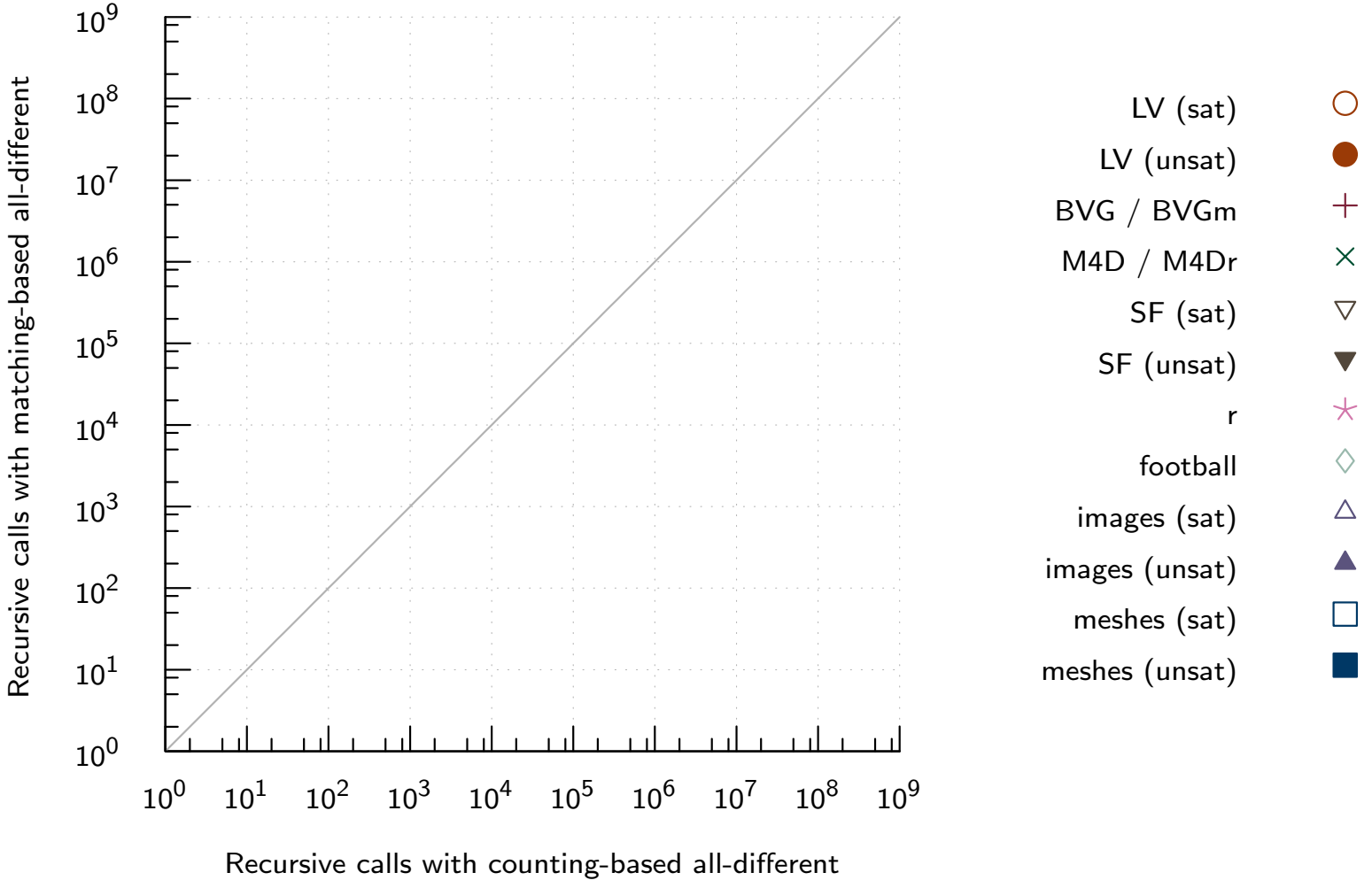


Counting All-Different?



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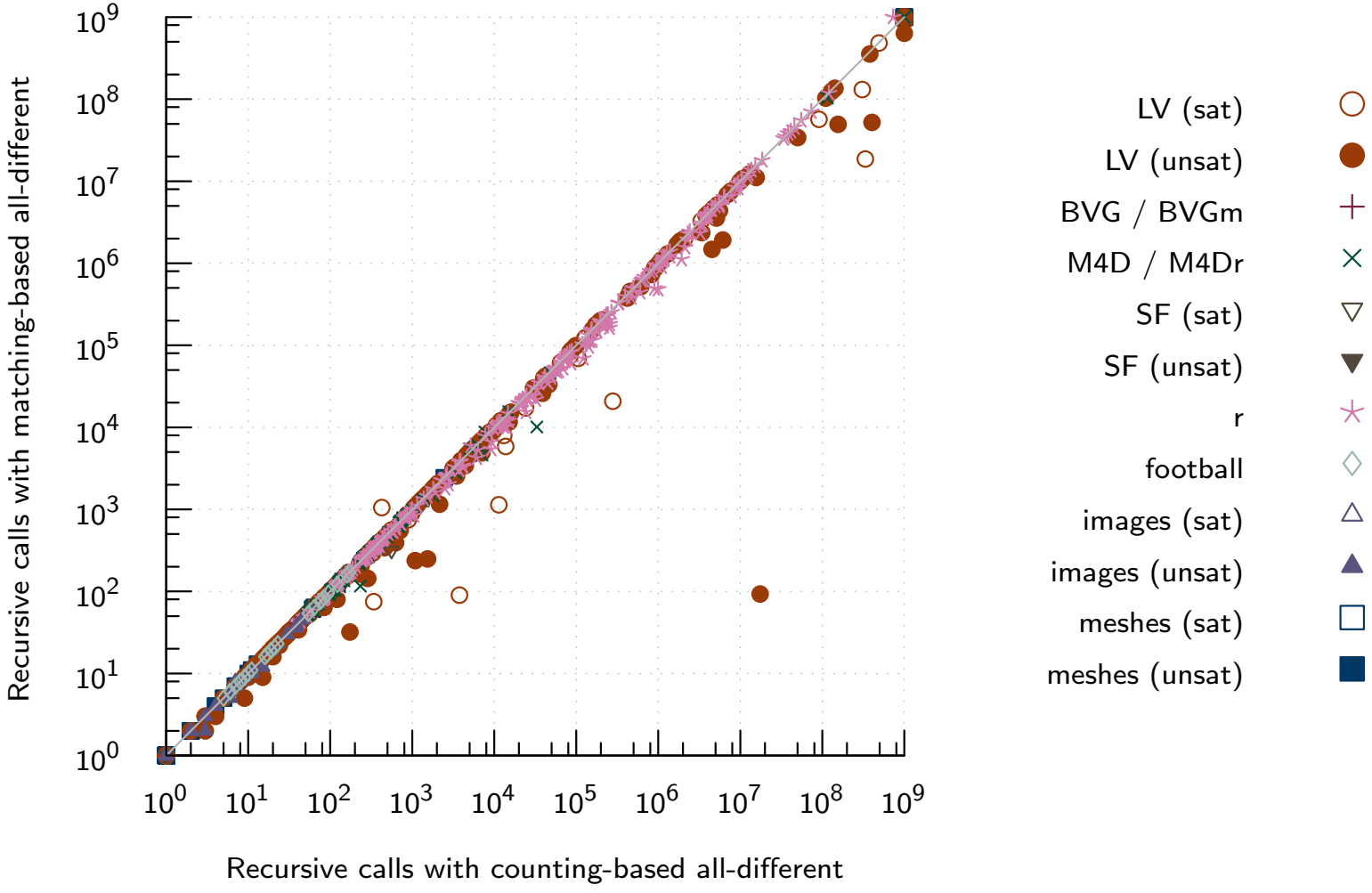

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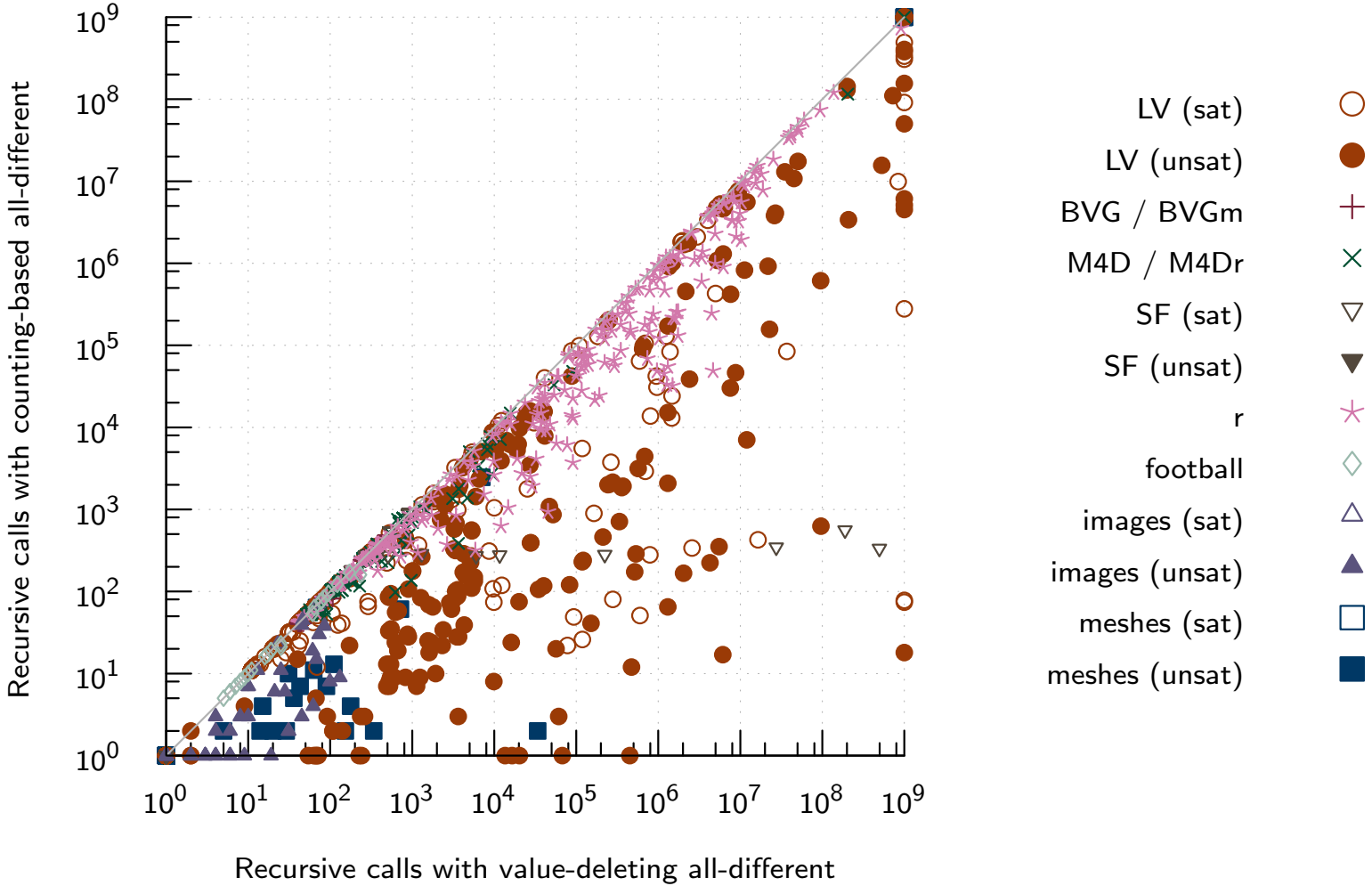
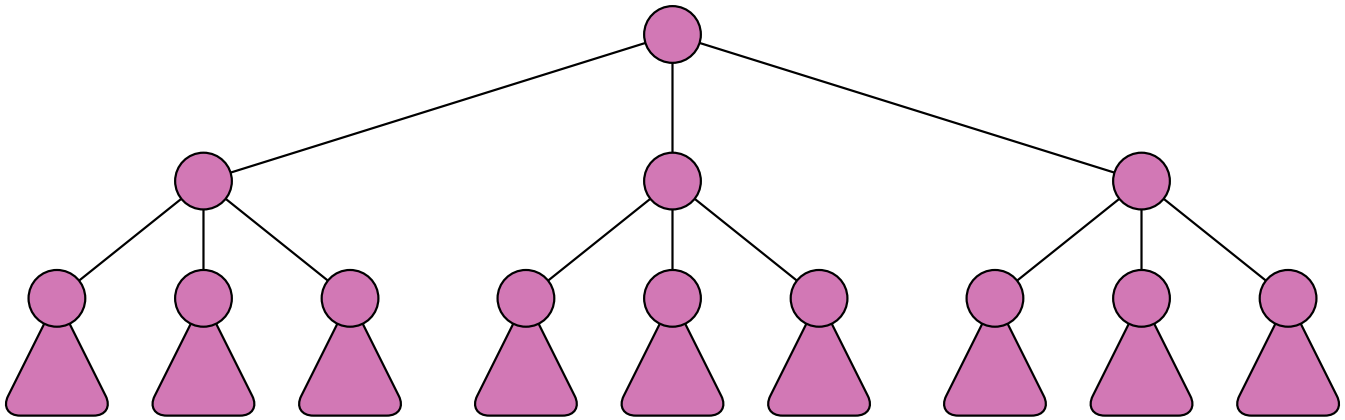


Parallel Preprocessing

- Constructing supplemental graphs is expensive, but we have lots of cores.
- Parallelising the loops is entirely (mostly...) routine and not very interesting, but good at offsetting the cost of supplemental graph construction.

Parallel Tree-Search



Parallel Tree-Search

- Explicit deterministic early-first work stealing, to offset incorrect decisions early in search.
  - There are EHPs, and this gets rid of them more cheaply than discrepancy searches.
- Backjumping is a nested parallel fold with left-zero elements.
- Speculative, so we should not expect linear speedups (even on unsat instances).

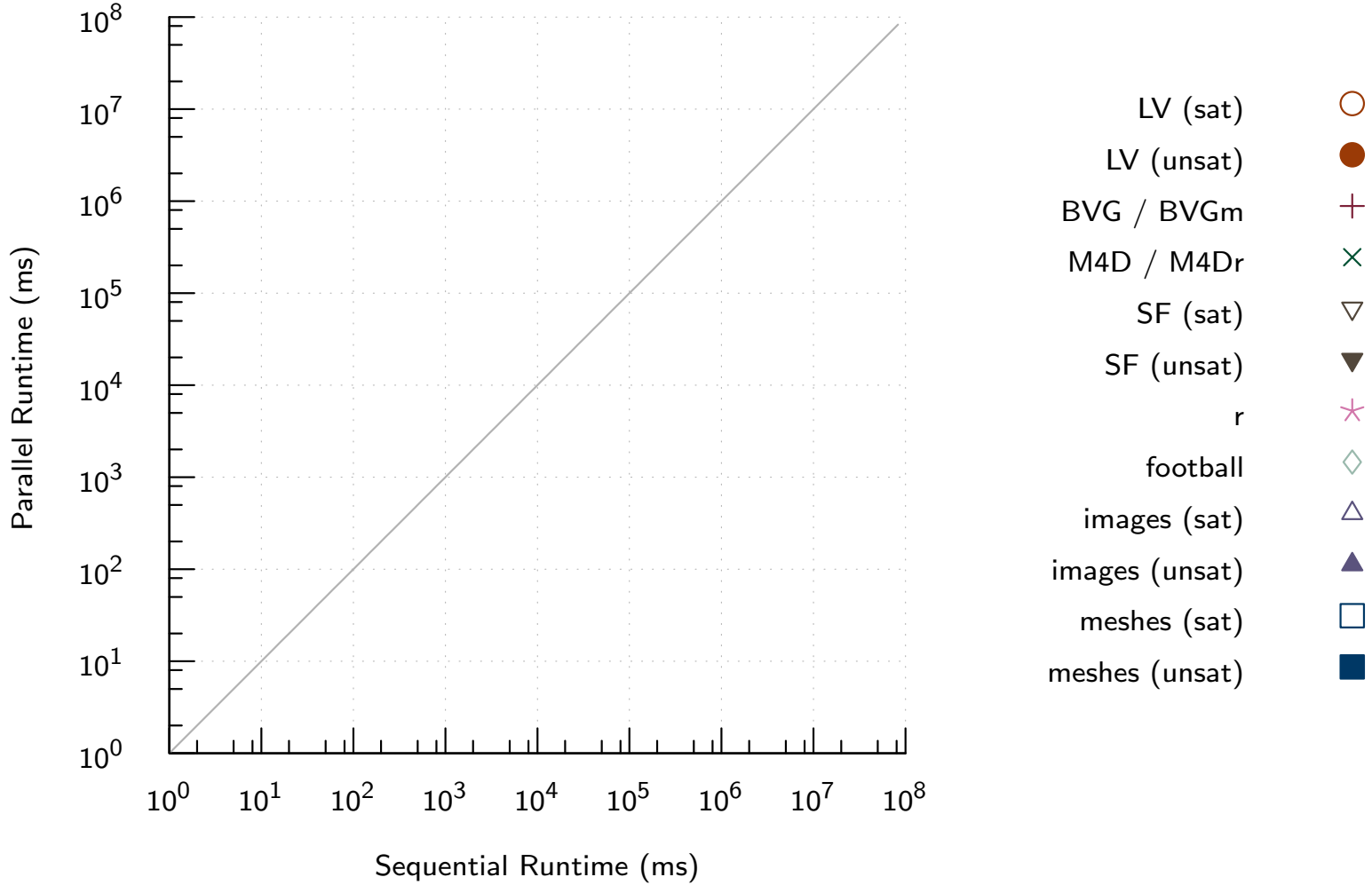
Sidenote on Safety, Parallelism, and Benchmarking

- All comparisons are against a dedicated sequential implementation of a strong algorithm, not a threaded implementation run with one thread.
- The way we do parallelism guarantees:
  - Parallel search will never be substantially worse than sequential search.
  - Adding more cores will never make things substantially worse (excluding hardware weirdness).
  - Running it twice will give more or less the same runtimes.

Parallel Experiments

- 2 CPUs, 8 cores per CPU, hyper-threaded, so 32 software threads.
- We do *not* have 32 times the computation power, and only about twice the memory bandwidth...

Speedups


Parallel Runtime (ms)
Sequential Runtime (ms)
10^0
10^1
10^2
10^3
10^4
10^5
10^6
10^7
10^8
LV (sat)
LV (unsat)
BVG / BVGm
M4D / M4Dr
SF (sat)
SF (unsat)
r
football
images (sat)
images (unsat)
meshes (sat)
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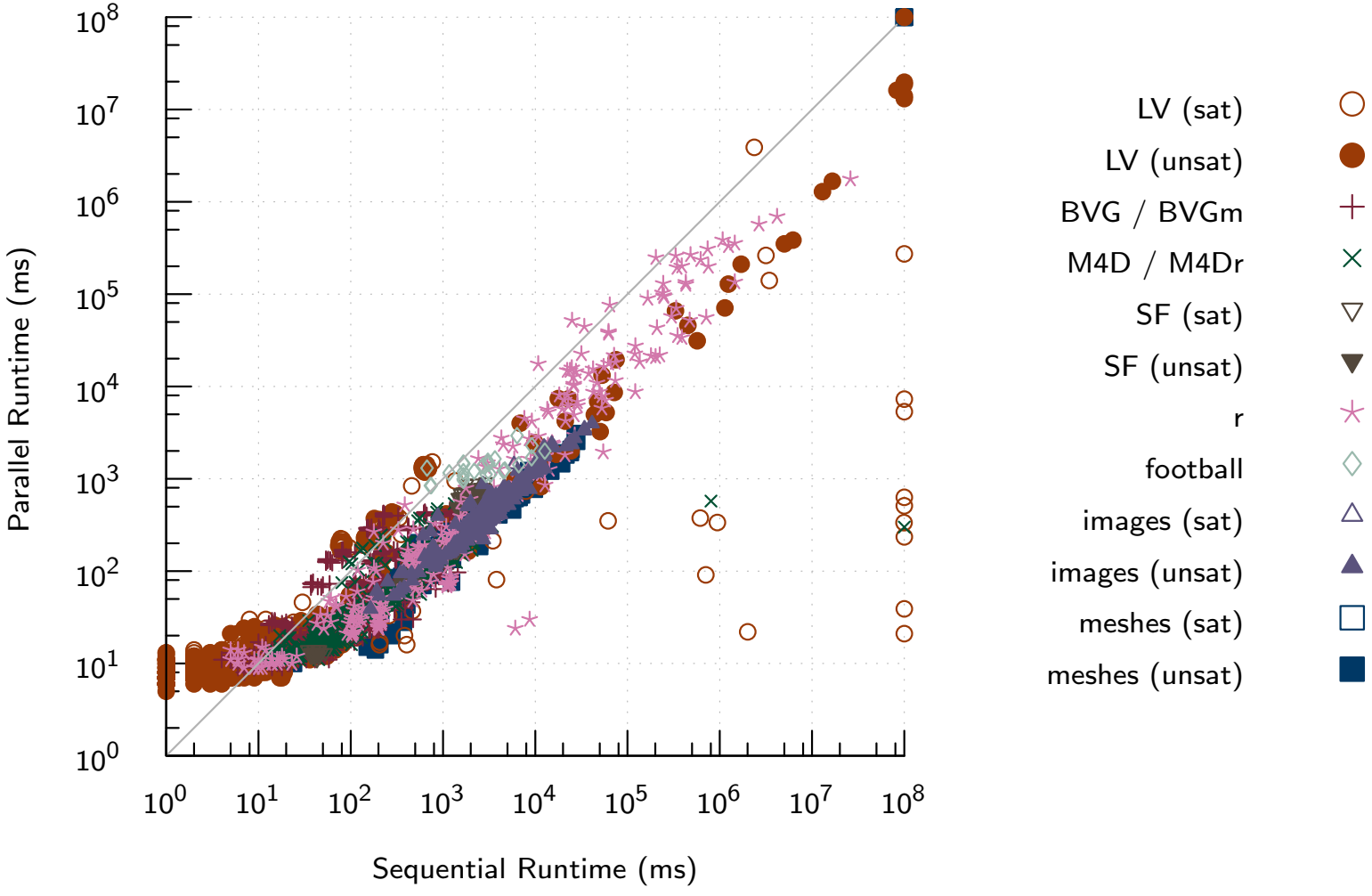
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Is Comparing Sequential and Parallel Solvers Fair?

- Personal view: single-threaded for understanding "inside search" behaviour, but "horse race" benchmarks should be a free-for-all.
- You're paying for multi-core even if you aren't using it.
- Java uses multiple cores even for sequential code, so the sequential benchmarks were unfair.
- You are welcome to disagree, and ignore the following two slides.

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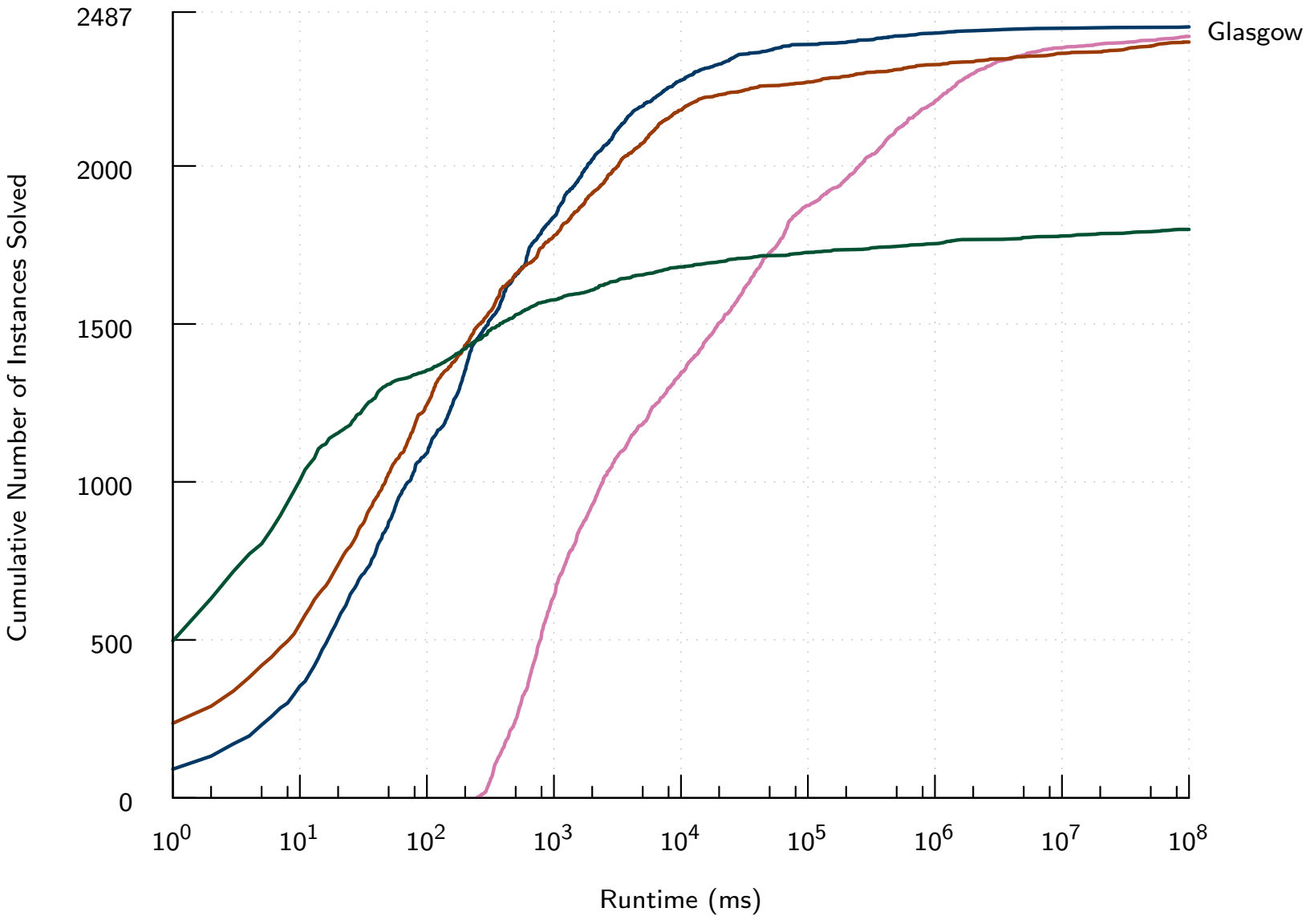

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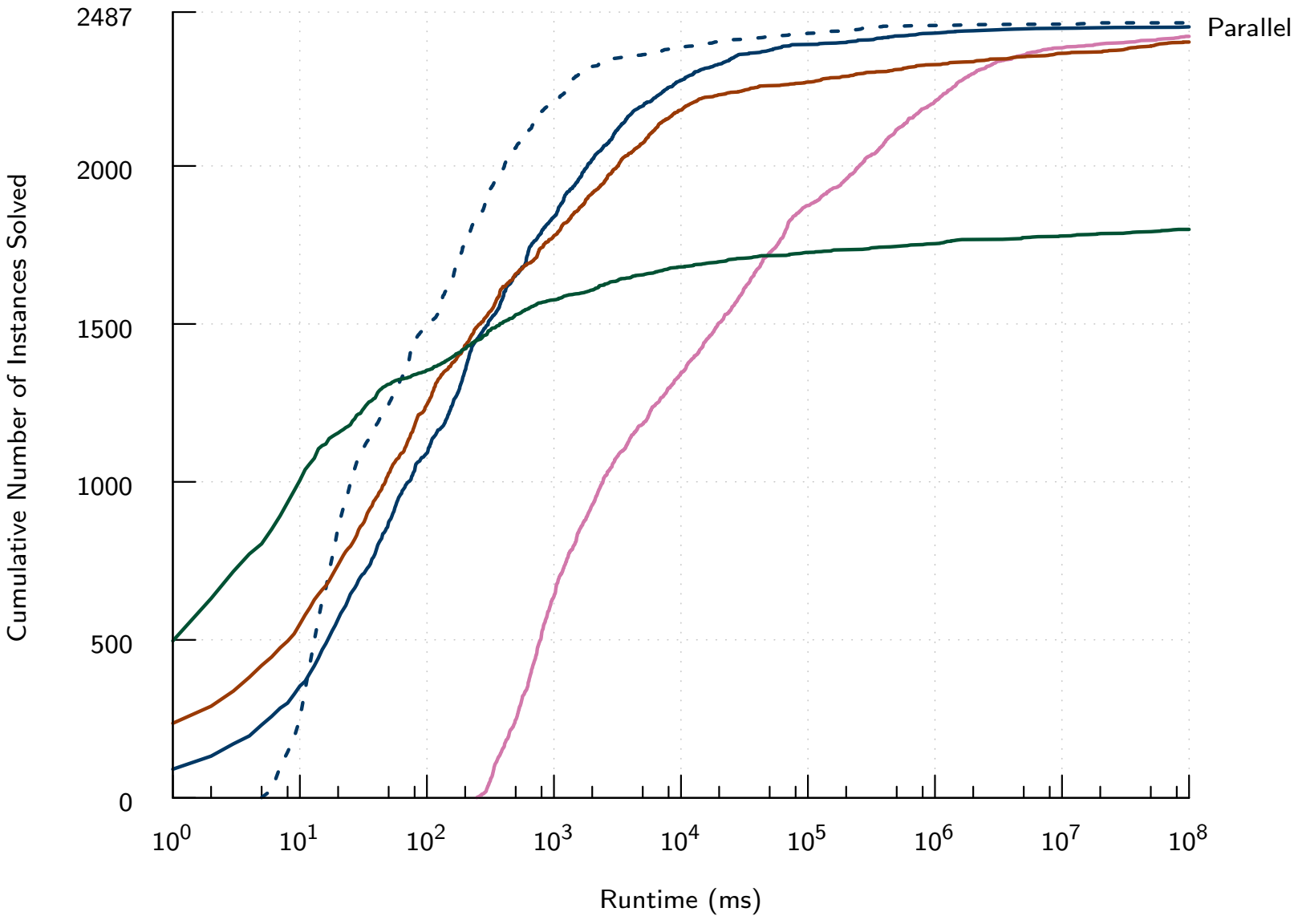

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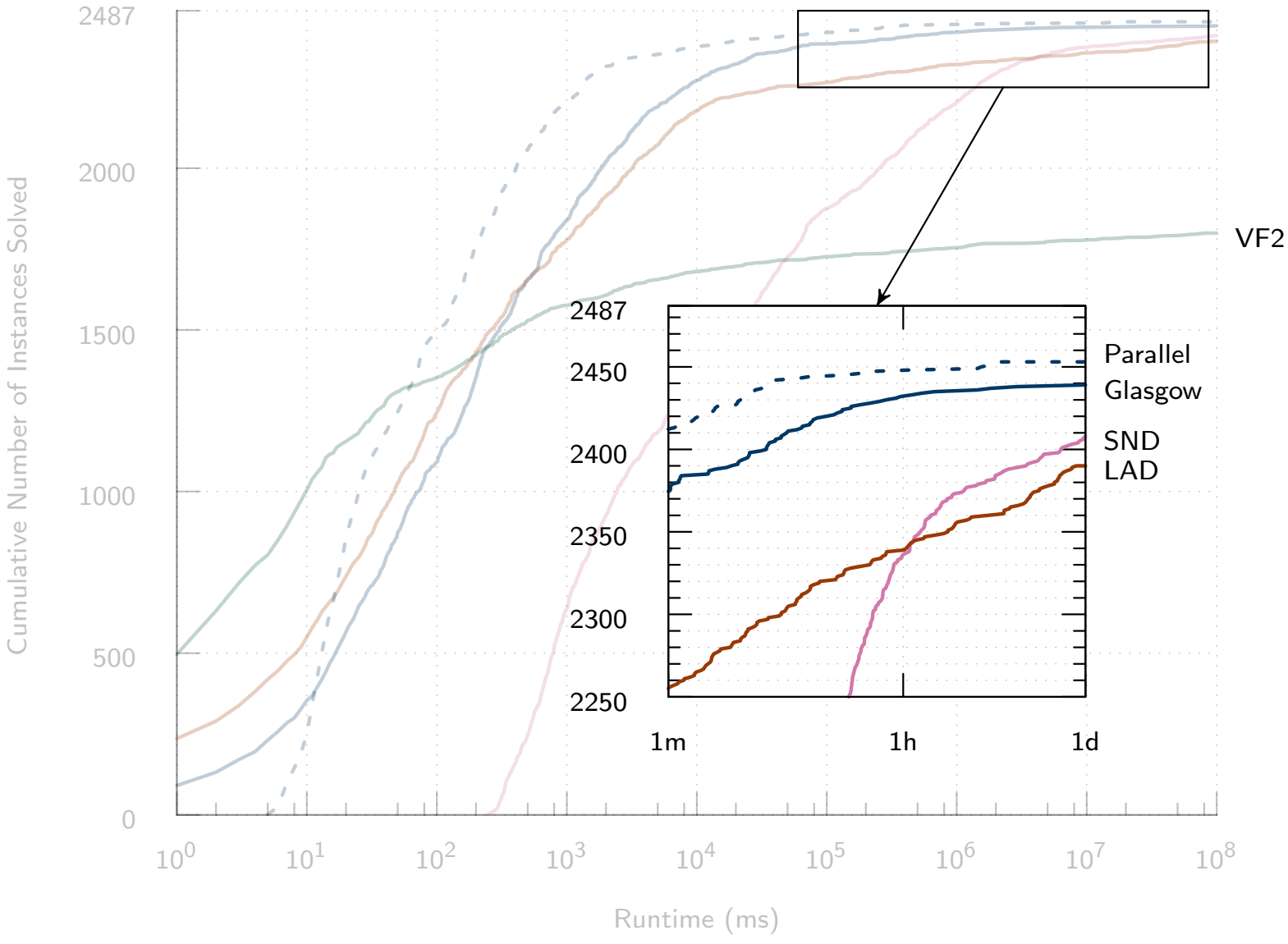

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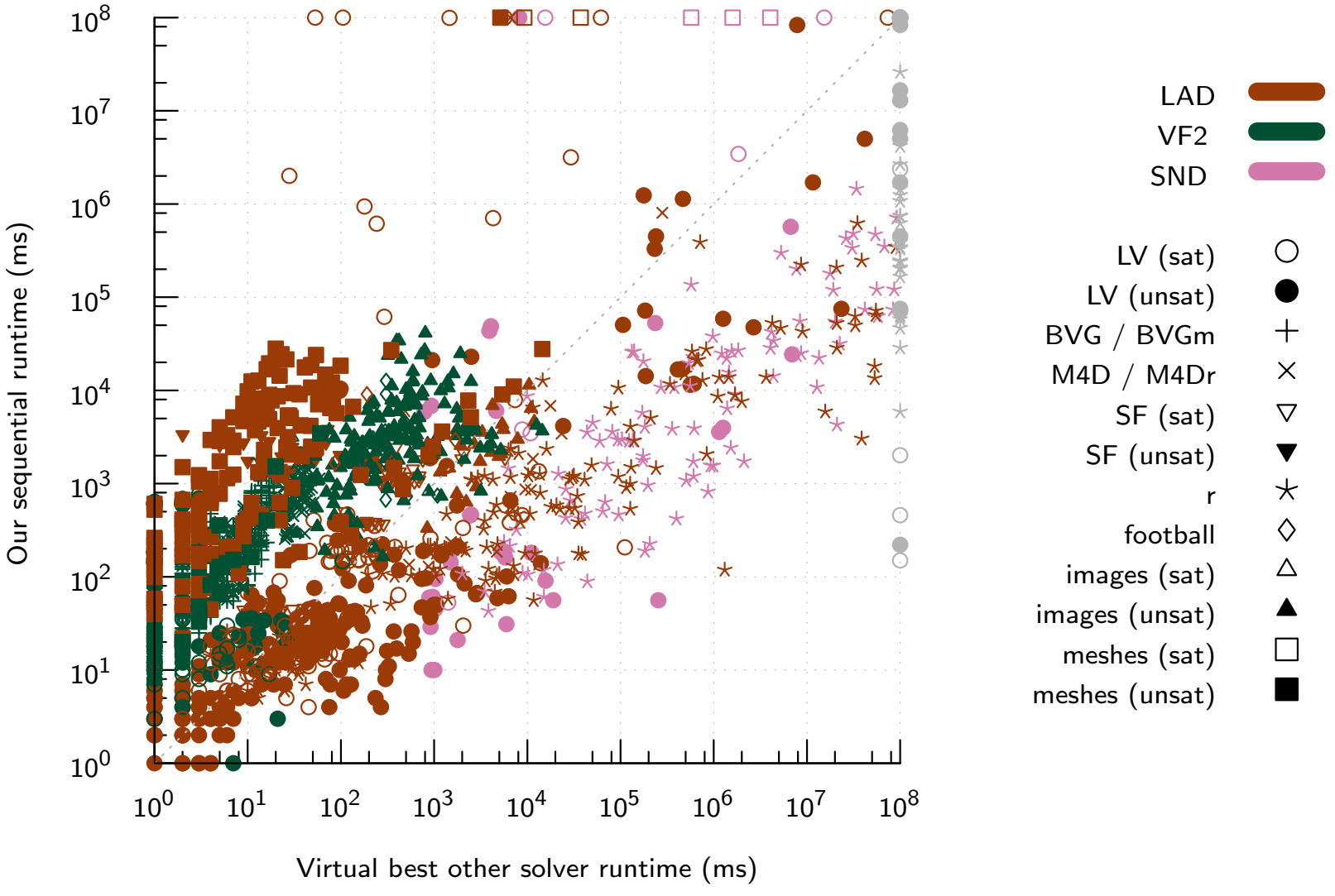

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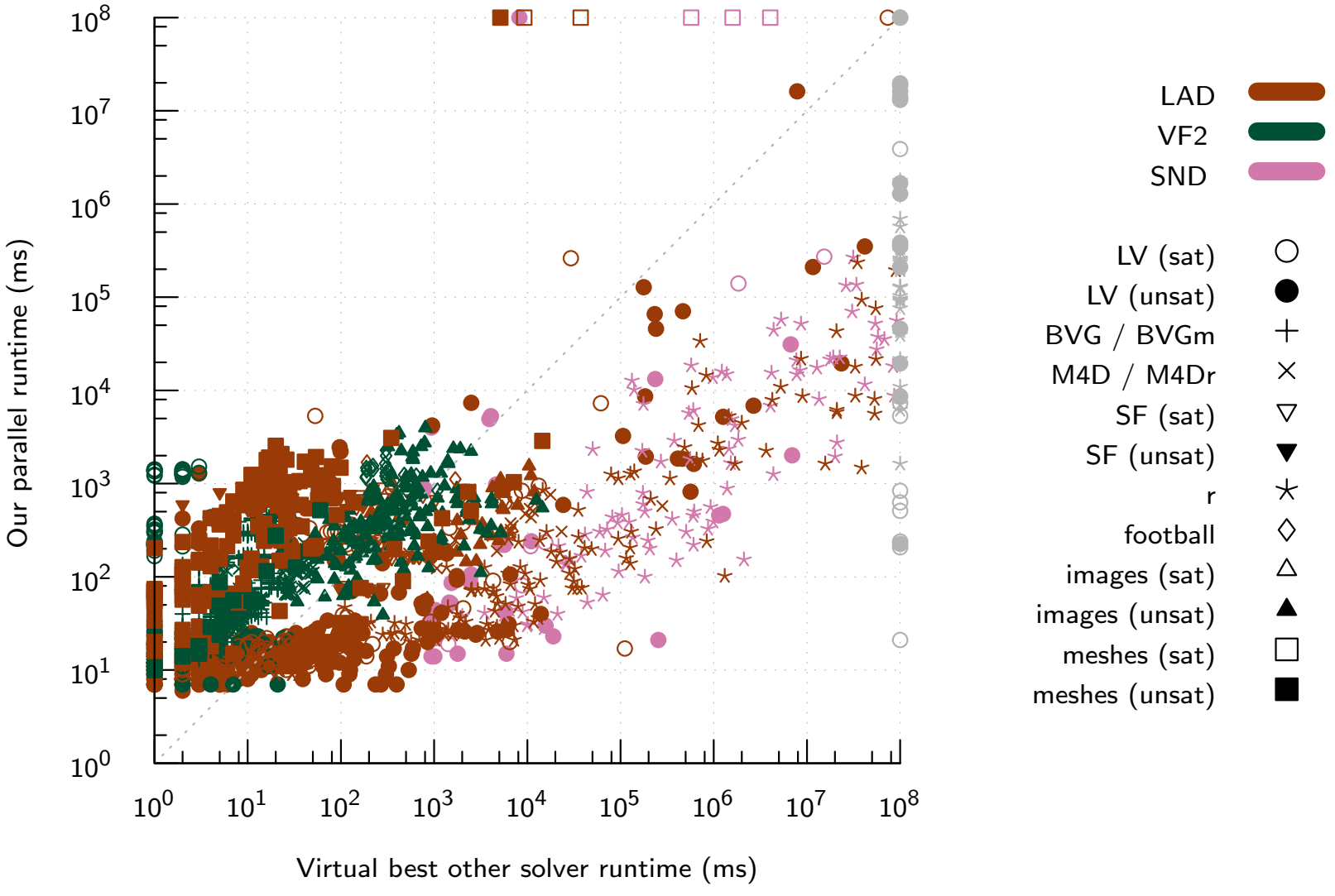

Conspicuously Absent From This Talk

- Labels? Directed edges? Induced? Wildcards?
- Why that choice of supplemental graphs, using only paths of lengths 2 and 3? It seems rather arbitrary. Can we do better with domain knowledge, or with portfolios?
- Caching supplemental targets (graph databases) or patterns (code generation, kidney exchange).
- Why does that counting all-different propagator do so well in practice? Is it only subgraph isomorphism where it helps?

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Tell me whether it worked:
`c.mccreesh.1@research.gla.ac.uk`

Donate your subgraph isomorphism problem instances:
`http://liris.cnrs.fr/csolnon/SIP.html`