Using Mathematical Models to Guide the Simulation of Improvised Explosive Devices in Public Spaces

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Abstract
Previous terrorist attacks, for example in Madrid and London, have increased concern over the threat that Improvised Explosive Devices (IEDs) poses to public safety. Insurgent groups in Iraq and Afghanistan have developed relatively sophisticated tactics, including the use of synchronised attacks with multiple devices that have not yet been witnessed in Europe or North America. Some of these approaches specifically target the fire and rescue services that are deployed in response to an initial detonation or warning. Computer simulations provide tools that can be used to plan for potential attacks. They can be used to work through a range of scenarios so that emergency personnel minimise their vulnerability and mitigate the threat that these attacks pose to the general public. However, it can be difficult to simulate the range of human behaviours that are seen in the aftermath of terrorist attacks. Similarly, it is unclear how to develop appropriate blast and fragmentation models that capture a range of future Improvised Explosive Devices. The following pages present a brief overview of a range of mathematical models that are being integrated into simulation tools that are intended to help emergency services and counter terrorism agencies plan for future attacks in public places.

1. Introduction
Software simulations have been widely used to model evacuations from fire or structural collapses [1]. However, very few have been created to help analyse other hazards such as terrorist attacks. It is possible to extend some of the techniques used to create evacuation simulations to model some of the characteristics of a terrorist attack including crowd motion, blast and fragmentation patterns from explosive devices, and to help predict casualty levels in a given situation [2]. An important benefit of these tools is that they can be used in table-top exercises to work through scenarios for coordinated terrorist attacks. The threats posed by these incidents are illustrated by a suicide bombing attack on Mustansiriyah University in Iraq during January 2007. A car bomb was detonated at one of the two principle entrances to the site. This led to a partial evacuation that drew crowds to the other exit where a suicide bomber detonated their device. This is not an isolated incident. Hours before, another coordinated bombing took place in a second-hand motorcycle market in the Shia Bab al-Sheik neighbourhood of Baghdad. The first blast drew onlookers and the emergency services, who were then hit by a second explosion moments later. In these incidents, the use of 'standard' evacuation procedures that were designed to protect the public from localised fires created opportunities or vulnerabilities that were exploited by the terrorists. Building occupants, spectators and the emergency services gathered at common assembly points that were the target for secondary devices. Software simulation tools can be developed, for example, to identify ways in which evacuations can be synchronised to help disperse the crowds that otherwise create significant opportunities for terrorist attacks.

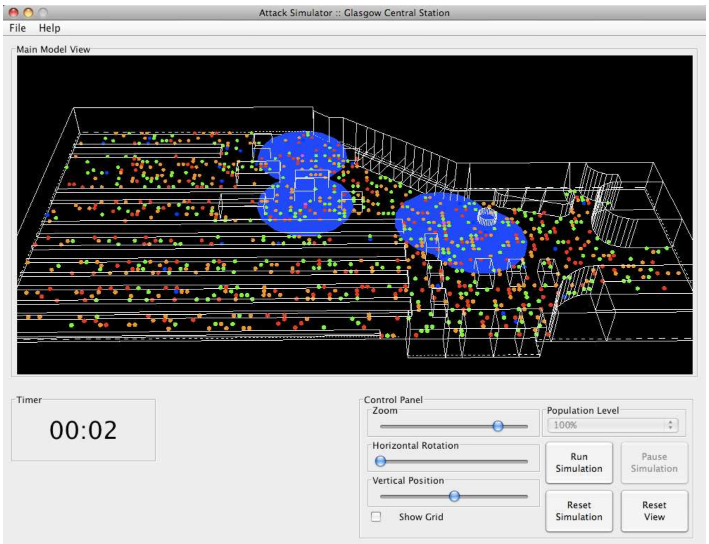

Figure 1: Interface to the Glasgow IED Simulator

Figure 1 illustrates the interface to a prototype system; the ellipses represent the extent of a potential blast as suicide bombers move inside a three dimensional model of a major UK railway station. The number of people inside these zones changes as each bomber and the other passengers move throughout the station concourse in real-time. The size of the blast and fragmentation areas can be varied to allow for larger and smaller devices given the type of explosive used. These tools can provide important information for the security services that *in extremis* may be forced to coordinate their response to several suspected bombers as they move through public spaces. The simulator does not account for concussive force of the blast on the people and structures in the surrounding area, although this is an obvious area for further development. However, fire and rescue services can also use the simulator to help identify scenarios in which they themselves might become the target for secondary devices as they respond to an initial incident. These potential benefits depend upon the development of mathematical models that can be used to drive the simulations. In particular, software engineers must consider potential crowd behaviours both before and after the detonation of any devices. They must also provide some idea of the potential impact of an IED. Unless appropriate approximations can be derived then there is a danger that these software tools will provide insights that hold little relation to the potential situations that must be addressed by counter terrorism agencies and emergency services.

2. High Level Models of Crowd Motion

It is important to stress that simulators provide high-level predictions that act as a starting point for wider decision making and planning activities. They are approximations rather than 'exact models' for complex human behaviours under extreme situations. One reason for this is the number of individuals that can be involved in a potential IED attack. For example, at peak times the railway station illustrated in Figure 1 can contain between 5,000 and 15,000 passengers.

In simulations of terrorist attacks on public spaces, the behaviour of individual agents is arguably of less interest than that of the overall crowd which emerges from their interactions with each other and their environment. Within the simulation behaviours such as flocking, following and collision avoidance can be used to mimic observations of crowd behaviours in a public space. For example, Helbing et al [3] present a summary of research into crowd motion from the analysis of video footage. The most pertinent points from this include: pedestrians show an aversion to detours and moving away from their intended destination even when the direct route is busy, and will normally choose the most efficient route; pedestrians keep a comfortable distance from other pedestrians and obstacles, but this distance will decrease if they are hurrying or the crowd is dense; crowd density will increase around attractive areas of the environment such as exits and shop windows; and finally about half of the individuals in a crowd are part of groups of varying sizes and these groups tend to act as a single entity. Musse et al [4] encapsulate many of these observations in their three rules for group behaviour: 1. Members of groups walk at the same speed; 2. Members share the same goals; 3 Members of groups will wait for each other if they are separated. These observations about the behaviour of crowds in public spaces can be used to inform the development of mathematical models that can, in turn, be incorporated into software simulations. Most of these models can be categorised in terms of three different approaches.

Cellular Automata. In this approach each person becomes an automata within the system. Their environment is, typically, represented in terms of a grid or fixed lattice of cells. Individual movement is then governed by a set of behavioural rules that specify permitted moves between the cells in the grid. Cellular automata models are fast and easy to implement, but have several disadvantages. The floor space in the environment is discrete. Automata can only move between adjacent free cells. In the basic form of the approach, this creates a fixed minimum distance between people. It limits the range of possible crowd interactions. This approach is much more focussed on the behaviour of the individual rather than the overall emergent behaviour of the crowd.

Fluid dynamics. It has been observed that the flow of a crowd is similar to that of a fluid; for example both a crowd and a fluid will follow the path of least resistance through a space. Henderson demonstrated the use of fluid and gas dynamics to model crowd flow in 1971 [5]. His pioneering work used the Navier-Stokes equations that describe the motion of fluids according to Newton's second law.

Particulate Models can be interpreted as a specialisation of the more general fluid dynamic models. Each agent in the simulation is represented as an individual particle that navigates the environment according to its own perceptions and behavioural rules. The individual behaviours interact to form an overall crowd behaviour. Hence an individual's movements will both shape and be shaped by the behaviours of those around them. Particular models have also modelled the environment as continuous rather than discrete space. This frees the agents from the constraints of a fixed movement lattice.

A variant of particulate modelling was used to drive the simulation illustrated in Figure 1. Helbing and Molnár introduce the idea of "social forces" between individuals in a crowd. These forces include a desire to keep an acceptable distance from other. Social forces also attract them to groups as they navigate toward a shared destination [6]. This model has more recently been extended to include the physical forces that may occur in high density crowds such as pushing and friction acting upon the agents [7]. The social-physical force model accounts for several forces that act upon an agent: 1. Acceleration – the velocity of an agent varies over time as it attempts to reach its optimum speed while avoiding obstacles. 2. Repulsion – there is a repulsive force between agents and between an agent and an obstacle. 3. Attraction – agents may be attracted to other people, for example other members of their group, or by objects such as shop windows, exits etc. 4. Pushing – in dense crowds pedestrians may collide and influence each other's movement. 5. Friction – in dense crowds pedestrians can exert direct contact forces upon other pedestrians. The problem then is to take these general observations of crowd behaviours and develop mathematical abstractions that can be used to drive simulation software. Let

A = magnitude of repulsive/attractive force, set to 2000N

B = the fall-off length of the social repulsive force, set to 0.5m
k = spring constant, set to $1.2*10^5$ kg/s^2
κ = co-efficient of sliding friction, set to 1
R_{ij} = $r_i + r_j$, the sum of the radii of the pedestrians
d_{ij} = the distance between their centres
n_{ij} = the normal vector between *i* and *j*
t_{ij} = the tangential vector to n_{ij}

$$\eta(x) = \begin{cases} x, & x \geq 0; \\ 0, & x < 0 \end{cases}$$

The values for the constants A, B, k and κ are taken from Lakoba et al [8]. The force equations 1-4 are described in Helbing et al [7]. One or more vectors are used to describe each force in the model. A directional vector made up of three components can describe the repulsive force between two pedestrians *i* and *j*:

$$\begin{aligned} \vec{f}_{ij} &= \vec{f}_{social\ repulsion} + \vec{f}_{pushing} + \vec{f}_{friction} \\ \vec{f}_{social\ repulsion} &= Ae^{(R_{ij}-d_{ij})/B}\ \vec{n}_{ij} \\ \vec{f}_{pushing} &= k\eta(R_{ij}-d_{ij})\ \vec{n}_{ij} \\ \vec{f}_{friction} &= \kappa\left|\vec{f}_{pushing}\right|\vec{t}_{ij} \end{aligned} \qquad (1)$$

The form of η(x) ensures that when *i* and *j* are not in contact the pushing and friction forces are 0, and hence have no impact on the repulsive force. The equation for the force between a pedestrian, *i*, and a solid obstacle, *o*, has a similar form to this first equation.

$$\vec{f}_{io} = Ae^{(r_{io}-d_{io})/B} + k\eta(r_i - d_{io})\vec{n}_{io} + \kappa k\eta(r_i - d_{io})\vec{t}_{io} \qquad (2)$$

The force of attraction to another object in the environment, including another agent or an object, can be given by reversing the social repulsion force in (1) i.e.:

$$\vec{f}_{attraction} = -Ae^{(R_{ij}-d_{ij})/B}\ \vec{n}_{ij} \qquad (3)$$

Where *i* is the individual under consideration and *j* is the attracting person or object. The force of acceleration in the simulation is modelled by an equation for personal velocity, which changes over time. It also takes into account the velocity of the crowd in the immediate vicinity of the individual, the "collective velocity", which can impact on the preferred velocity e.g. a slow moving crowd will slow down a faster individual. The ability to avoid some objects and also to model the attractive forces of other individuals is particularly important for an IED simulator where we would like to model the intervention of counter terrorism agencies and also of the fire and rescue services. The opening sections have described recent coordinated suicide attacks can target the crowds that gather in assembly points after an initial evacuation. These potential risks can be mitigated by the marshalling strategies after suspected terrorist attacks. The impact of these techniques can be modelled using the attraction and repulsion forces in this variant of the particulate models.

$$\begin{aligned} \vec{f}_{preferred} &= -m\frac{\vec{v} - \vec{v}_0}{\tau_i} \\ \vec{v}_0 &= (1-p)\,v_0\,\vec{e}_i + p\left\langle \vec{v}_j \right\rangle_i \end{aligned} \qquad (4)$$

Where

m = mass of *i*
$\vec{v}$ = current velocity
τ = reaction time
$\vec{v}_0$ = the preferred velocity
v_0 = the desired isolated velocity of *i*
$\vec{e}_i$ = the unit vector in *i*'s direction of motion
$\left\langle \vec{v}_j \right\rangle_i$ = the average velocity that *i* perceives in a 2-3m radius around themselves
p = determines the weights of the "own" and "collective" velocities

At any given point in the simulation, at least one and most likely several of the forces described above will be acting upon an individual. In the IED simulator the 'driving' force that acts on each person is derived from the attractive force that acts to move them towards their current goal. This could be another agent, such as a first responder, or an exit or any point on a preferred route through their environment.

3. Movement, Collisions and Group Behaviour

The particle motion approach to crowd modelling embedded within the IED simulator can be broken down into three broad tasks - movement, collision avoidance, and group behaviours.

Movement forces direct the progress of an agent through their environment. Each individual is assumed to have a starting point and their motion is mainly driven by an attraction to a destination. The route between a starting point and a destination can be broken down into a series of way-points or 'sub-goal'. Some of these way-points may lead into 'dead ends'. There may also be obstacles to avoid, see below. Perfect knowledge of the environment is not assumed, therefore the complete route is not calculated. An agent moves towards their destination using a directed search algorithm, this can be depth first, breadth first or hybrid techniques depending on the implementation. Some way-points have actions associated with them, for example if a shop is a goal then the agent may incur a waiting time to simulate shopping before being assigned a new goal. This is again important for counter-terrorism models where people are not simply walking between points in the environment when a device is detonated. Previous attacks have been focussed on shopping areas, markets and on political rallies where significant portions of a crowd are not moving at the moment of initial detonation [2].

Collision Avoidance. As individuals navigate through their environment they will have to avoid collisions with other agents and with static objects. Collision avoidance is achieved by a system of detection of relevant obstacles by means of an 'area of influence', followed by an alteration of the agent's trajectory to avoid them. The area of influence needs to be properly sized to allow for detection to occur while there is still enough time to alter the agent's path, and is therefore contingent on the velocity of the agent. It should also only detect relevant objects i.e. those that will impact on the pedestrian, which means that the size of an agent's 'personal space' also needs to be accounted for. This aspect of the model is essential in the moments after an IED attack given that individuals and groups may have to navigate around an environment that is very different from the one immediately before an explosion. They may also be forced to move around emergency personnel as they provide assistance in the aftermath of an attack.

The avoidance of static objects is relatively simple, as they do not move. The trajectory of an individual *i* can be altered by the application of an avoidance force [9] to miss obstacle *o:*

$$\vec{f}_{avoidance} = \frac{(d_{ik} \times v_i) \times d_{ik}}{|(d_{ik} \times v_i) \times d_{ik}|} \qquad (5)$$

Avoiding collisions with other pedestrians is more complicated. The simplistic approach could be more correctly termed 'collision detection'; when two agents are about to collide one will simply stop moving, generally the slower or less aggressive one, and allow the other to pass. This approach does not produce an adequate simulation of crowd motion. The second approach is similar to that of the static obstacle avoidance and involves altering the trajectory of one agent to avoid the other entirely. The 'area of influence' mentioned above is used to detect approaching agents and once again an avoidance force is calculated to alter the trajectory if there is sufficient time to do so. Otherwise the simplistic method is used and one agent must stop in order to avoid the collision.

Several parameters affect the avoidance forces on an individual. These include the distance to obstacles, the direction vector of the colliding agent relative to the current direction vector of the avoiding agent, and the density of the crowd in the area. The rational of the equation used to obtain the avoidance force is described in detail in Pelechano et al [9] but the tangential force that will steer pedestrian *i* to avoid pedestrian *j* is given by:

$$t_j = \frac{(d_{ij} \times v_i) \times d_{ij}}{|(d_{ij} \times v_i) \times d_{ij}|} \qquad (6)$$

This tangential force is then multiplied by two scalar weights to obtain the agent avoidance force that will alter the trajectory:

$$\begin{aligned} \vec{f}_{ij} &= t_j w_i^d w_i^o \\ w_i^d &= (d_{ij} - D_i)^2 \\ w_i^o &= \begin{cases} 1.2, & (v_i \cdot v_j) > 0; \\ 2.4 & otherwise \end{cases} \end{aligned} \qquad (7)$$

w_i^d is due to the distance between the agents, and will increase in weight as the agents get closer because the trajectory will have to be more radically altered to avoid collision. w_i^o is due to the differences in orientation of the agents' velocity vectors (the driving force), the weight is greater if the colliding agent is travelling in the opposite direction. D_i is the detection distance of the pedestrian and controls the size of the detection area. This allows the density of the crowd to be factored in; the distance at which collisions can be detected decreases as density increases, therefore the value of D_i should decrease as density increases.

Group Behaviours. 'Flocking', can be described as the tendency of people to form coherent groups which act together. Some have argued that approximately half of any crowd consists of isolated individuals while the rest is made up of 'flocking' groups of varying sizes [3, 4]. The strength of this collective influence on individual decision making is a controversial topic, like many areas of crowd behaviour. However, flocking is arguably one of the most easily observed group influences. For the purposes of our prototype IED simulator, a group consists of between 2 and 10 people. Each group has a randomly allocated 'leader' that, all other things being equal, will be followed by the other members of the group via an attractive force, and a group identifier. This method accounts for the formation of predetermined groups. It does not allow for the spontaneous formation of groups e.g. friends who run into each other. These collective behaviours can be incorporated into future versions of the simulator. However, as mentioned earlier, considerable care must be taken to insure that any increase in the complexity of the underlying models also yields corresponding benefits in terms of the benefits that it provides to potential counter terrorism applications. The introduction of spontaneous group formation might be appropriate in the aftermath of a blast in a nightclub district, such as the area affected by the Bali blasts, where individuals struggle to find their friends in the remains of other clubs and bars. Spontaneous group formation may, arguably, be less relevant in an attack on a railway station where most groups of travellers will be close together immediately before any incident.

'Following' has also been observed in crowds [3]. Individuals and groups are influenced to follow a path that has already been traversed by others. This is distinct from 'flocking' because individuals will also follow some after a group or another person has already taken a particular route. This type of behaviour results in the formation of lanes of pedestrians through a crowd. These lanes are often the only way that people can get through a congested area such as a narrow corridor or dense crowd. Again, this is important in the context of counter terrorism given that such congested areas may themselves be the targets of secondary devices. 'Following' behaviour can be modelled using residual direction fields; as an individual moves through an area, an imprint of their direction is left which exerts a small force on the direction of the next pedestrian to move through the area. The imprint fades over a fairly short time span of only a few seconds if not replaced by that of another pedestrian, but the effect can be cumulative over time to have a greater influence over agents to follow the path.

'Following' behaviours can combine with obstructions to create queues. These form at popular points where bottlenecks form and individuals are forced to slow down. Fans are created when multiple queues are created by different 'lanes' towards a common obstruction or 'pinch point'. The social force model recreated fans and queues as a by-product of the components described in previous sections. A key concern for the end-users of our system is to train first responders to minimise the delays created by the formation of these group structures. However, queues often form in other areas of a public building when staff members are used to move individuals and groups from particular bottlenecks. Simulations can help senior personnel to gain an overview of the interactions between their staff and these different crowd behaviours that can present opportunities and targets for subsequent terrorist attacks.

This section has described how a variant of the particulate model can be used to simulate individual and group behaviours during IED attacks. We have identified those components of the approach that provide significant benefits for counter terrorism applications. These include the different forces of attraction and repulsion that can be used to mimic staff intervention or the desire to avoid the site of a primary device. They also include the delays that arise as groups struggle to navigate between obstacles and other debris in the aftermath of an explosion. As mentioned, there are several other alternate approaches and further work is required to validate our choice of this approach. In contrast, the following section goes on to identify techniques for predicting the impact of an Improvised Explosive Device on the individuals and groups in a public building.

3. Blast and Fragmentation

For any explosive device it is possible to calculate the radius of the blast, the effective range of fragmentation, and the distance from the device at which specific "overpressures" will occur. The pressure in excess of the normal atmospheric pressure, called overpressure, is used to measure blast effect in pounds per square inch (PSI). The effects of ranges of overpressure on people and buildings have been observed, so the damage at a certain range can be predicted with some accuracy. However, damage to structures is highly contingent on materials and construction. A host of other factors can also be included in more sophisticated models, for example to account for the effects of structures on the extent of any over pressure, and this remains an active area of research in its own right. In contrast, the focus of our work is on crowd behaviours and the impact of Improvised Explosive Devices. Several software tools can be used to derive approximations for the distance at which specific overpressures will occur for a given mass of explosive material [10]. For example, an over pressure of 0.04 PSI can result in a form of sonic boom (over 140dB). In the range from 1-3 PSI damage can include the partial demolition of houses and warping in metal panels. There may be damage to eardrums and some fatalities can be expected. In areas where the overpressure reached 10 PSI there will be specific tissue damage to the lungs. At double this level, very few people will survive the blast.

The construction of improvised explosive devices varies greatly. The raw materials and technical expertise used to construct IEDs has changed over time. The system illustrated in Figure 1 can be used to simulate the impact of car bombs and of devices carried by pedestrians. However, vehicle devices arguably create less effective fragmentation than IEDs that are carried by individual suicide bombers as they mingle with a crowd. Even relatively small amounts of explosive can be used to considerable effect. It is for this reason that most of our modelling work has focused on devices that are worn as a belt or vest. These garments are packed with cylinders or plates of explosive surrounded by metal fragments, such as ball bearings, nails or screws. Shrapnel is responsible for around 90% of the casualties from these devices.

The explosive used in a device is limited by availability. Industrial explosives such as C4 and TNT (Trinitrotoluene) are popular because they are easy to handle. However, there can be considerable risks for terrorists in both obtaining and securing the explosives prior to any attack. Other 'homebrew' substances such as TATP (Triacetone Triperoxide) are relatively easy to make from commonly available ingredients, however, the risks are different. In this case, the explosives are often extremely unstable. TATP is very sensitive to heat, friction and shock. Due to its highly volatile nature it is often used as an initiator rather than the main explosive in a device. A much more stable homemade compound similar to Ammonal (an industrial explosive) can be made out of ammonia nitrate (fertilizer) mixed with coal and aluminium dust. This compound is commonly used as the bulk component detonated with a small amount of TATP. The simulation shown in Figure 1 models attacks using this type of hybrid TATP-Ammonal device. We assume a suicide belt may hold around 4.5kg of explosives, a vest 9kg, and a briefcase around 23kg. These default values can, however, be varied for each run of the simulator. The particular mass of explosive that a bomber could carry is only limited by their strength and the volume which can be concealed on their person.

Table 2 presents approximations for the blast effects in our simulations. More information about the interpretation of these values and the methods used to obtain them can be obtained from [1]. Damage to structures within the environment is not currently part of the simulation. This is a specialised area of research; data regarding the materials and construction methods used in the building would be required to properly predict it and could be incorporated into future versions of the software.

A "cookie cutter" damage function is often used to predict casualties in conventional combat. This assumes that everyone within a defined area is considered to be a casualty. However, this is inappropriate for more complex urban environments such as those modelled in our simulations. The 'cookie cutter' approach does not take into account the ways in which blast and fragmentation can be mitigated by other people in the crowd who can come between an individual and the effects of an IED. Similarly, these simplistic approaches to casualty prediction often ignore the impact of obstacles that can come between the bomber and the target. For the likely mass of explosive contained in a suicide device, most blast casualties will be within a 10 metre radius, assuming there are no protective structures between them and the explosive [10]. The railway station illustrated in Figure 1 is, however, full of permanent and semi-permanent structures that could offer some degree of protection to individuals in the model. These include concession booths, benches, large signs etc. Following the PEAC-WMD model, we assume that there is a 1% probability of fatality for individuals within an overpressure area of 2.5 PSI, and a 99% fatality rate at 20 PSI [10]. The expected casualty rate between these values is difficult to determine.

Kress has recently developed mathematical models that can be used to refine these approximations by developing probability distributions for the number of casualties based on the density of the crowd around the suicide bomber at the time of detonation [11]. This approach considers the impact of shrapnel, which can be determined by the number of fragments packed into the device, the distance over which it is effective, the dispersion angle of the fragments, the density of the crowd surrounding the bomber etc. Our implementation of the Kress model makes a number of further simplifying assumptions; each piece of shrapnel can only injure one person, fragments do not become less deadly with distance, fragments have a uniform distribution and there is total crowd blocking. These assumptions are justified because the current implementation is a prototype or proof of concept. Further work is required to weaken these constraints that yield 'conservative' estimates of potential casualties.

The *arena* of an Improvised Explosive Device can be defined to be the effective range of the fragments that it can produce. The Kress model [11] assumes that the radius of the arena, R_0, is the same size as the fragmentation range [10]. This radius is larger than the effective range of the blast, supporting the assertion that the fragmentation is more dangerous than the blast itself. The arena is divided into a sequence of M concentric circles, each the width of b = a person's diameter, giving $M= R_0/b$. The Kress model [11] was originally developed to provide a static indication of casualty numbers from Improvised Explosive Devices given a particular crowd density and device. The innovative feature of our implementation is that we can continuously update the Kress values in real-time as the simulated crowd moves within a potential target area. As people walk between different areas of the railway station concourse, the number of potential casualties will change as the crowd densities change. Hence the user of the simulation can assess the impact of potential group behaviours and different occupancy levels on the casualties from a coordinated terrorist attack. Our tools enable the simultaneous application of the Kress model to more than one attacker. Let

μ_m = number of people in a ring, observable from simulation
$\alpha(m)$ = probability that an exposed target in ring *m* is exposed
N = number of effective fragments
L = number of people in the arena
A = the exposed area of a person
K(M) = the maximum number of people possible in the arena
$P_H(m)$ = probability that exposed person in ring *m* will be hit
β = dispersion angle of fragments

Assuming total crowd blocking, meaning that anyone whose position is blocked will not be injured, E_m casualties can be expected in the m^{th} ring of the arena, with E_m given by:

$$E_m = \mu_m \times \alpha(m) \times P_H(m) \qquad (8)$$

Therefore the total number of casualties E(M) due to fragmentation can be assumed to be:

$$E(M) = \sum_{m=1}^{M} E_m \qquad (9)$$

The three terms of equation 1 can be calculated from information available from the simulation and from data provided by the user. The first term μ_m is the number of people in the m^{th} ring, which is observable. The second term $\alpha(m)$, is calculated by:

$$\alpha(m) = \begin{cases} \prod_{l=1}^{L-1}\left(1 - \frac{m-1}{K(M)-l}\right), L < K(M) - m + 2 \\ 0 \end{cases} \quad (10)$$

Where K(M) is given by:

$$K(M) = \sum_{m=1}^{M} \frac{\pi}{\arcsin(\frac{1}{2m})} \quad (11)$$

It is unlikely that the arena around the bomber will be completely packed with people so it can be assumed that this is always going to be found by the top term, thereby avoiding the unnecessary calculation involved in the condition check. The third term in equation 8 is for the probability of an exposed person being hit by a fragment. This is the term in which the number of effective fragments in the device N, and their dispersion angle β, becomes important. It is assumed that only half of the fragments will be effective, the rest being expended on the suicide bomber, so whatever value the user specifies is halved to give N. A typical value for β is not known from empirical data, but Kress uses both 10° and 60°. These parameters can be supplied by the user during interaction with the prototype simulator. $P_H(m)$ is given by:

$$P_H(m) = 1 - e^{-A\sigma_m} \quad (12)$$

Where $A\sigma_m$ is given by:

$$A\sigma_m = N \frac{\min\{2m\tan(\frac{\beta}{2}), c\}}{4\pi\sin(\frac{\beta}{2})\, m^2} \quad (13)$$

The value E(M) in equation 9 gives the total number of projected casualties within the arena due to fragmentation. The equations used in the simulation do not allow for secondary injuries, although Kress also produced formulae to allow for the possibility of persons directly behind exposed persons also being injured. It is difficult to say whether a hit from a fragment will result in injury or death therefore the two are not distinguished in the simulation.

The Kress model focuses primarily on injuries from fragmentation and does not address the blast related injuries that we derive from the over-pressure models in previous sections. We, therefore, combine the expected casualties from the blast and the probable casualties due to fragmentation to create an overall casualty projection in our simulations. Further work is required to determine the best means of combining these two approximations. Summing the casualties produced from blast and fragmentation would produce an over-estimate given that some victims may be injured by both over-pressure and by projectiles produced in the aftermath of any detonation.

4. Conclusion

Previous terrorist attacks, for example in Madrid and London, have increased concern over the threat that Improvised Explosive Devices (IEDs) poses to public safety. Insurgent groups in Iraq and Afghanistan have developed relatively sophisticated tactics, including the use of synchronised attacks with multiple devices. Some of these approaches specifically target the fire and rescue services that are deployed in response to an initial detonation or warning. Computer simulations provide tools that can be used to prepare for potential attacks. They can be used to work through a range of scenarios so that emergency personnel minimise their vulnerability and mitigate the threat posed to the public. It is difficult to simulate the range of human behaviours that are seen in the aftermath of terrorist attacks. Similarly, it is unclear how to develop appropriate blast and fragmentation models that capture the effects produced by Improvised Explosive Devices. This paper has presented a range of mathematical models that are being integrated into simulators. Emergency services and counter terrorism agencies can use these software tools to increase our resilience to future attacks in public places.

The work described in this paper represents initial steps towards the development of counter-terrorism simulation tools. It is particularly important to validate predicted injury patterns and crowd behaviours from more complex, mathematical models. This can be done, at least in principle, by analysing the consequences of terrorist incidents around the globe. Regrettably there is no shortage of data. Improvised Explosive Devices have become a defining characteristic of asymmetric warfare. However, it can be difficult to gather detailed information about fragmentation and blast patterns in the aftermath of terrorist attacks when the priority is to protect the public and tend to casualties Even at this early stage in our work, however, it is apparent that the patterns of attack are changing rapidly in response to recent changes in strategy by emergency services and military organisations. Children and the disabled are being used in the synchronised deployment of IEDs. Multiple devices are being used to support political assassinations of as individual targets move through crowds of supporters. Counter terrorism agencies must take considerable pains to guard against these forms of attack without imposing unnecessary restrictions on civil liberties. Unless we develop tools that reflect the flexibility and ingenuity of those who deploy these devices then there is little prospect that we will be able to protect the public from future terrorist attacks.

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