Deleting Edges to Save Cows: Using Graph Theory to Control the Spread of Disease in Livestock

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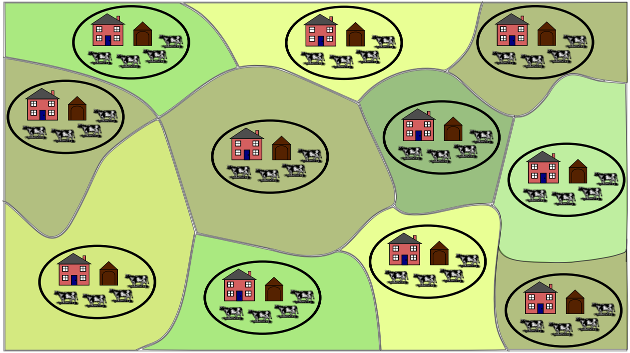
Joint work with Jessica Enright (University of Stirling)

The animal contact network



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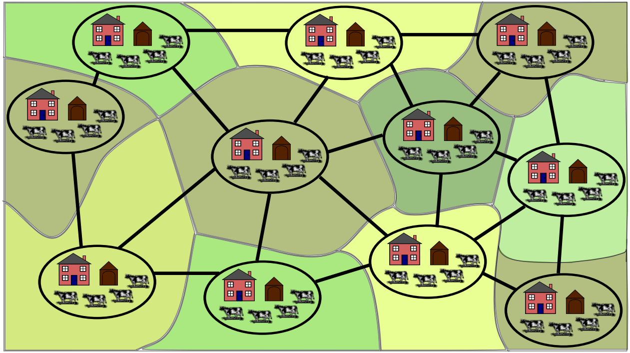

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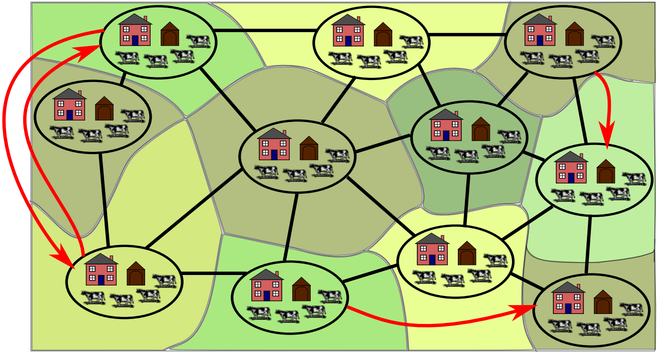

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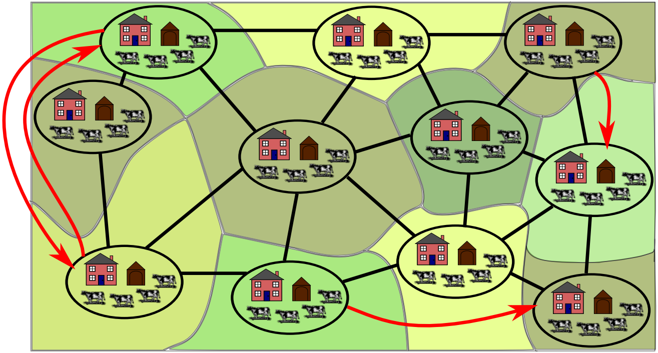

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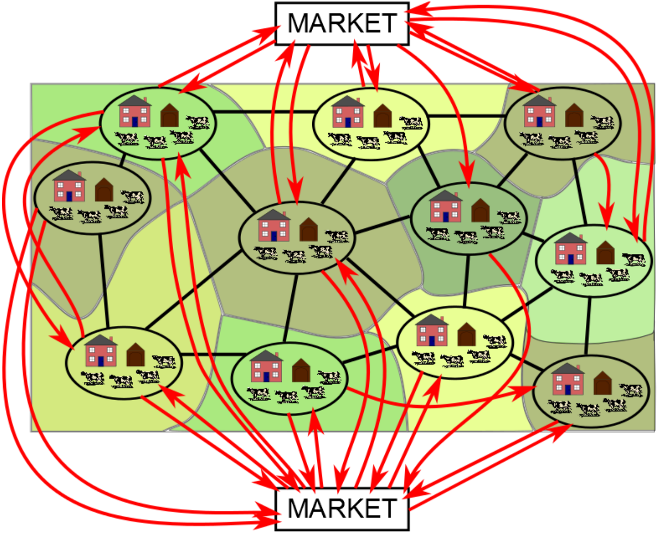

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Modifying the network

Vertex-deletion

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- Testing or quarantine for animals on a particular trade route

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Cost of modifications The cost of deleting individual vertices/edges may vary; this can be captured with a weight function on vertices and/or edges.

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- consider the total number of animals in e.g. a connected component, rather than just the number of animal holdings
- place more or less strict restrictions on individual animal holdings

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- Component Order Edge Connectivity

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PROBLEM: There is no polynomial-time algorithm to solve this problem in general unless P=NP (even if $h = 3$).

Using structural properties of the input

- There is an efficient problem to solve this problem on trees (Gross, Heinig, Iswara, Kazmiercaak, Luttrell, Saccoman and Suffel, 2013).

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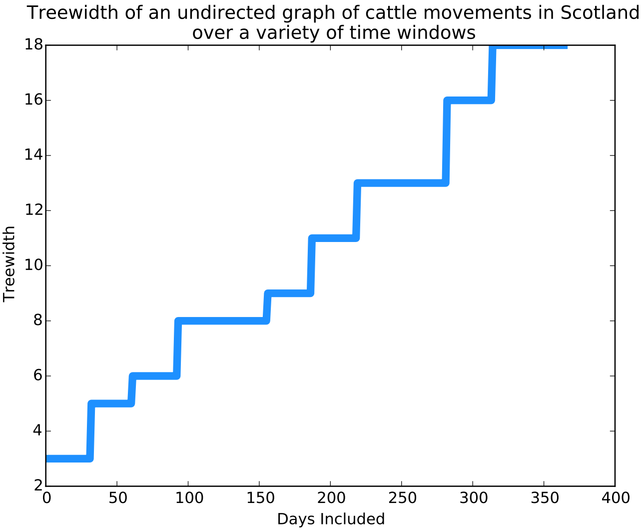
- The **treewidth** of a graph is a measure of how "tree-like" a graph is, in a specific sense. Trees have treewith equal to 1, and cycles have treewidth 2.
- Often algorithmic problems can be solved more efficiently on graphs with small treewidth.

(Some) cattle trade networks have small treewidth



New results

Theorem (Enright and M., 2015)

Suppose we are given a (weighted) graph G on n vertices which has treewidth w. We can determine the least costly set of edges to delete, so that the remaining graph has no connected component with more than h vertices, in time $O((wh)^{2w}n)$.

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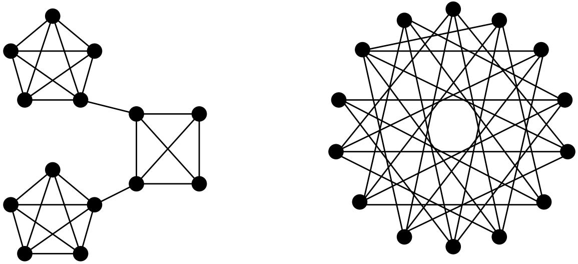

New results

Theorem (Enright and M., 2016+)

Suppose we are given a (weighted) graph G on n vertices which has treewidth w. We can determine the least costly set of edges to delete, so that the remaining graph contains no graph from the set $\mathcal{F}$ as a subgraph, in time $2^{O(|\mathcal{F}|w^r)}(n + 2^r)$, if no element of $\mathcal{F}$ has more than r vertices.

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Thank you