Applications of bigraphs with sharing

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Abstract

Bigraphs are a mathematical model for systems of interacting agents (real or virtual) introduced by Milner [7]. A bigraph consists of two independent structures: a set of nodes that can be nested one inside another, and a set of hyper-edges linking the nodes. The intended interpretation is that nodes represent locality, i.e. the spatial placement of agents, while links encode connectivity, i.e. their communication capabilities. Bigraphs are a fully graphical process algebraic formalism, capable of representing both the position in space of agents and their inter-connections. However, they assume a topology based on sets of trees and thus cannot represent spatial locations that are shared among several entities in a simple or intuitive way. This is a problem, because shared locations are often a requirement, for example, when modelling scenarios in the physical world or in modern complex computer systems such as wireless networks and spatial-aware applications in ubiquitous computing.

We propose *bigraphs with sharing*, a generalisation of the original definition of bigraphs in which places may have several parents. This leads to a formulation of place graphs based on acyclic binary relations instead of the acyclic functions in Milner's definition. We demonstrate the new formalism can be defined in the general framework of bigraphical theories and wide reactive systems, as originally devised by Milner. We do so by defining a categorical interpretation of bigraphs with sharing and an axiomatisation derived from the equations of a bialgebra over finite ordinals. Two new elementary place graphs are required to express algebraically bigraphical terms: $0 : 1 \to 0$ and $\mathit{split} : 1 \to 2$. They are the dual bigraphs of $1 : 0 \to 1$ and $\mathit{join} : 2 \to 1$, respectively. They are essential to represent orphans and shared places. An additional operator capable of expressing sharing is required. We define share expressions as

$$\text{share } F \text{ by } \phi \text{ in } G = G \circ (\phi \otimes \mathsf{id}_X) \circ F$$

where all the operations are assumed defined.

We show that bigraphs with sharing can be used realistically in a production environment by describing the implementation of a graph theoretic algorithm to solve the bigraph matching problem [1]. The two key features of the algorithm are: native support for both standard bigraphs and bigraphs with sharing; capability to enumerate all the distinct occurrences of a pattern in a target. The latter is essential in a stochastic setting for the computation of reaction rates [5]. The algorithm is defined as a reduction to the sub-graph isomorphism problem. In more detail, the first phase of the algorithm finds the isomorphisms between the patterns underlying Directed Acyclic Graph (DAG) and sub-graphs of the targets underlying DAG. In the next phase, the algorithm discards the isomorphisms obtained in the previous phase that do not satisfy the following compatibility conditions: node controls are preserved, patterns sites and roots allow for a valid decomposition of the target, and no node in the context has an ancestor in the pattern. In the final phase, a mapping between the patterns and targets link graphs is constructed for every compatible isomorphism. We implemented the matching engine by encoding in SAT [1] the matching algorithm described above. The solutions are then obtained by passing the resulting SAT instances to the MiniSat solver [4].

[1] Boolean satisfiability problem.

We illustrate how sharing is essential for modelling overlapping localities by presenting two example case studies in the field of wireless networking.

The first application is a model of 802.11 RTS/CTS that supports overlapping signals and arbitrary network topologies [2]. The model consists of a sorting that specifies the kinds of bigraphs used to encode wireless networks and a set of stochastic reaction rules describing how a network evolves during the execution of the protocol at transmission time. A distinguishing characteristic of this approach is that the network topology is expressed explicitly by representing overlapping wireless signals with sharing nodes. Therefore, locality determines collision probability. In order to avoid the definition of parameterised reaction rules, the reaction rules of our model are organised into priority classes. This not only leads to a more compact and readable specification of the model but also allows for a more efficient construction of the reaction relation.We also describe a general method to reduce significantly the size of the state space by employing instantaneous reaction rules and discarding the intermediate interleavings obtained by their applications (if confluent). Quantitative analysis is carried out by using the probabilistic model checker PRISM [6] and **CSL** predicates expressing properties of a network.

The second application is real-time generation and analysis of bigraphical models of domestic wireless networks. Our approach is to define a sorting that specifies the structure of the admissible bigraphs used to represent wireless networks and a set of reaction rules to encode network and policy events. However, the fundamental difference here is that the model generation process is event-driven, i.e. instead of computing the whole state space of the Bigraphical Reactive System (BRS), sequences of reaction rules are applied in real-time to update only the current model of the network according to the events captured by the Homework router [8]. The analysis process consists of verification of system properties expressed as **BiLog** [3] predicates. These include detecting configurations that violate user-invoked access control policies.

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