LOGICAL MODELS IN INFORMATION RETRIEVAL: INTRODUCTION AND OVERVIEW

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Abstract — *The use of logic to model the information retrieval process has become an established research area. Nevertheless, many people in the information retrieval community do not yet appreciate the work performed in this area, mainly because they do not understand logical formalisms, and hence cannot see the connection between logic and information retrieval. This paper aims at resolving the problem. It introduces the formalisms used in logical models for information retrieval, shows the use of logic to build the models, and presents a brief overview of some of the current logical models in information retrieval.*

1 INTRODUCTION

It has been argued that current *information retrieval* (IR) models offer only simplistic and specific representations of information (Chiaramella and Chevallet, 1992, Nie, 1990, van Rijsbergen, 1989). There is, therefore, a need for the development of a new formalism able to model IR systems in a more generic manner, hence capturing information as it appears in an IR system, and also in any of its inherent forms. It has been suggested (Huibers, 1996, Nie, 1990, van Rijsbergen, 1986a, van Rijsbergen, 1986b) that such formalisms can be both appropriately and powerfully defined within a *logic* (Barwise, 1993, Crossley *et al.*, 1972). The reasons are threefold. Firstly, logical models are believed to be more general than other existing IR models (Bruza and Huibers, 1994, Nie, 1990). Indeed, some logical models are able to represent within a uniform framework various features of IR systems, such as the semantics of information (Nie, 1990), hypermedia objects (Muller and Kutschekmanesch, 1995, Thiel and Muller, 1996) and structured multimedia documents (Chiaramella *et al.*, 1996), the user's knowledge (Nie *et al.*, 1996) and the nature of IR agents (Huibers and van Linder, 1996). This is not to say that these features were not represented in other IR models, but when they were, often it was not as part of the model, but as an extension, sometimes ad hoc. Such representations can lead to heterogeneous and complicated formalisms that lack underlying meaning. Secondly, logic has been used for many years in artificial intelligence to formalize the manipulation of information by intelligent devices in the way they use information to think, infer, conclude, acquire knowledge, make decisions and so forth (e.g., in robotics and natural language processing). A primary aim of an IR system is to capture the manipulation of information, for example, during the retrieval process, or query evaluation. Thirdly, logic makes it possible to reason about an IR model and its properties (Huibers, 1996). This possibility is becoming increasingly important because conventional evaluation methods such as precision and recall, although good indicators of the effectiveness of IR systems, often give results which cannot be predicated or even satisfactory explained. For instance, the use of natural language processing techniques to build IR models has often led to counter-intuitive results (see for example (Sanderson, 1994)).

The use of logic for IR modeling, although showing great potential, does not seem to have been fully recognized as an important research topic by many people in the IR community. One reason is that logical formalisms can be difficult to understand, and hence are intimidating. As a result, many people do not see the connection between logic and IR, and then cannot appreciate how logic may, or may not, provide powerful IR models. This paper aims to remedy the problem by introducing logic in the context of IR modeling. The intention is not to state that logical IR models are better than other IR models, or which logical model is the most appropriate, but to introduce the concepts necessary to understand the recent work.

The general idea of a logical model is as follows. A logic generally consists of objects (sentences, worlds, models, etc) and connectives. One particular connective is the implication, denoted "$\rightarrow$". Given two objects o_1 and o_2, $o_1 \rightarrow o_2$ means that the object o_1 *implies* the object o_2. In other words, o_2 can be *inferred* from o_1. Suppose there is a way to represent the information content of a document by an object d and the information need as phrased in the query by an object q. $d \rightarrow q$ would mean that the query object can be inferred from the document object. To put it in another way, the information captured by d is sufficient to infer the information represented by q. In the IR world, this could be viewed as the document satisfying (or being relevant[†] to) the query.

This paper is organized as follows. First, I describe the most commonly used logic, *Classical Logic*, so that the reader becomes familiar with logical formalisms. They are the basis of many logical IR models. Then, I show how Classical Logic is used to model IR. However, Classical Logic presents some weaknesses in the modeling of IR. I explain

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[†] In this paper, the term "relevance" is used in two contexts, with respect to the user and with respect to the system.

the problems encountered and cite appropriate alternative *non-classical logics*. Finally, I give a brief overview of some of logical models which have emerged.

2 CLASSICAL LOGIC

Under the heading Classical Logic, for simplicity, I shall describe a subclass, namely *Propositional Calculus*. A logic L defines a *vocabulary*, composed of a set of *propositions* $P = \{p, q, r, s, \ldots\}$, as well as *logical connectives*: *negation* ($\neg$), *conjunction* ($\wedge$), *disjunction* ($\vee$), *implication* ($\rightarrow$) and *equivalence* ($\leftrightarrow$). The logic L, then, defines a formal language by its *syntax* and its *semantics*.

2.1 Syntax

The syntax formally specifies a set of well-formed *formulae* (wffs) or *sentences* of the logic L:

(i) if p is a proposition in P, then $p \in L$,

(ii) if $\phi \in L$ and $\psi \in L$, then $\neg\phi \in L$, $\phi \wedge \psi \in L$, $\phi \vee \psi \in L$, $\phi \rightarrow \psi \in L$ and $\phi \leftrightarrow \psi \in L$,

(iii) no other formula belongs to L.

Examples of wffs are $p \vee q$, $\neg p \rightarrow (q \wedge r)$.

2.2 Semantics

Any formula in L has an intended meaning called a *semantic value*. The set of these constitutes the semantics of L. In Propositional Calculus, semantic values are the set $\{0, 1\}$ of truth values *false* and *true*, respectively. The semantics of a formula are defined by the semantics of the formulae that constitute it and the semantics attached to the different logical connectives. In Classical Logic, the semantics attached to $\neg$, $\wedge$, $\vee$, $\rightarrow$ and $\leftrightarrow$ are described in the following *truth table*:

Table 1: Semantics of negation, conjunction, disjunction, implication and equivalence

p	q	$\neg p$	$p \wedge q$	$p \vee q$	$p \to q$	$p \leftrightarrow q$
0	0	1	0	0	1	1
0	1	1	0	1	1	0
1	0	0	0	1	0	0
1	1	0	1	1	1	1

The semantic value of a sentence $\phi \in L$ is denoted $||\phi||$, whose value depends on the propositions and the connectives that appear in ϕ. For example, if $\phi = \neg p \vee \neg q$ and $||p|| = ||q|| = 1$ (last row of the truth table) then $||\phi|| = 0$.

If the set of propositions is $P = \{p, q\}$, four *interpretations* are obtained, one for each row of the above truth table. For example, in the second interpretation, $||p \wedge q|| = 0$ and $||p \vee q|| = 1$. More formally, an interpretation is a structure $I = \langle P, \{0, 1\}, F \rangle$ where F is a function that assigns semantic values to the propositions of P. It then becomes necessary to say that a formula ϕ is *true with respect* to a particular interpretation I, not just that ϕ is true. The semantic value of a formula ϕ with respect to an interpretation I is denoted as $||\phi||^I$. Moreover, $||\phi||^I = 1$ is written $I \models \phi$ and $||\phi||^I = 0$ is written $I \not\models \phi$. The relation $\models$ is read "satisfies". Semantics are therefore re-expressed as follows (p is a proposition of L, and φ and ψ are formulae of L):

(i) $I \models p$ if and only if (iff) $F(p) = 1$

(ii) $I \models \neg\phi$ iff $I \not\models \phi$

(iii) $I \models \phi \wedge \psi$ iff $I \models \phi$ and $I \models \psi$

(iv) $I \models \phi \vee \psi$ iff $I \models \phi$ or $I \models \psi$

(v) $I \models \phi \to \psi$ iff $I \not\models \phi$ or $I \models \psi$

(vi) $I \models \phi \leftrightarrow \psi$ iff either $I \not\models \phi$ and $I \not\models \psi$, or $I \models \phi$ and $I \models \psi$

The fact that a formula ϕ is true in any interpretation is denoted $\models \phi$; the formula

ϕ is said to be *valid* or *logically true*. It is also called a *tautology*.

Often, only a few interpretations of the above four are of interest. Suppose that one wants to represent only the cases in which p is true; one is then only interested in those interpretations that make this proposition true. These interpretations are called *models* with respect to p (these are the last two interpretations in Table 1). So a model for a formula ϕ, or a set of formulae Φ, is any interpretation that satisfies ϕ, or Φ.

Finally, the relation $\models$, when used as follows $\phi_1, \ldots, \phi_n \models \psi$, says that any model of $\phi_1, \ldots, \phi_n$ is also a model of ψ. In such a case, it is said that ψ is a *logical consequence* of $\phi_1, \ldots, \phi_n$.

2.3 Inference System

There is another way to characterize validity for formal languages. Rather than trying to establish whether $\phi_1, \ldots, \phi_n \models \psi$ by enumerating all interpretations, it is possible to define syntactic rules. These are *axioms* which are formulae that are assumed to be true, and also *inference rules*. Classical Logic can be syntactically defined with several axioms and one inference rule called *Modus Ponens*, which states that if both ϕ and $\phi \rightarrow \psi$ are true, then ψ is true.

A *proof* is defined as any sequence of formulae of L such that each formula is either an axiom or follows from one or more of the preceding sentences of the sequences by the application of Modus Ponens. A *theorem* of the language is any sentence ϕ for which there is a proof ending in ϕ.

A *derivability* relationship $\vdash$ is defined between a set of formulae and a formula $\phi_1, \ldots, \phi_n \vdash \psi$ iff there exists a finite sequence of the inference rule that leads $\phi_1, \ldots, \phi_n$ to ψ. The fact that a formula ϕ is a theorem is written $\vdash \phi$. A set of axioms together with all the theorems that can be derived from it is called a *theory*. To finish, the *Deduction Theorem* says that $\phi \vdash \psi$ is equivalent to $\vdash \phi \rightarrow \psi$.

2.4 Soundness and Completeness

Soundness means that only true statements can be proven. That is, if $\phi \vdash \psi$ then $\phi \models \psi$. *Completeness* means that all true statements can be proven. That is, if $\phi \models \psi$ then $\phi \vdash \psi$. Classical Logic is both sound and complete. As a result, there are two ways to prove the truth of a formula, one using $\models$, and the other $\vdash$. The first method is referred to as a *model system* approach and the second as an *inference system* approach. The different notations are summarized in the table below*:

Table 2: Model System and Inference System

Model System		Inference System	
Valid sentence	$\models \phi$	Theorem	$\vdash \phi$
Logical Consequence	$\phi_1, \ldots, \phi_n \models \phi$	Deduction	$\phi_1, \ldots, \phi_n \vdash \phi$

The concepts thus far defined are the basis of most logics. In the next section, Classical Logic is used to express the relevance of a document to a query. This will allow the reader to become more familiar with the notations just described, and at the same time to identify some features that a logic should possess for the appropriate modeling of IR.

3 MODELING INFORMATION RETRIEVAL WITH CLASSICAL LOGIC

Given a logic, let d and q be the sentences, in that logic, representing the information content of a document and the information need phrased in a query, respectively. One way to express the relevance of the document to the query is through the implication $d \rightarrow q$. In this case, determining the relevance consists of deciding whether $d \rightarrow q$ is valid, meaning that the implication holds for all interpretations of the logic†. As explained in the previous section, evaluating the validity of $d \rightarrow q$ in Classical Logic is equivalent

* In some logics, the correspondence between the inference system and the model system does not exist.

† An excellent discussion on the difference between truth and validity of $d \rightarrow q$ in the context of IR can be found in (Sebastiani, 1995).

to asserting $d \models q$, $\models d \rightarrow q$, $d \vdash q$ or $\vdash d \rightarrow q$. Here, for simplicity, $d \models q$ is used, which consists of establishing whether any model of d is a model of q.

Here is a working example. Suppose that the vocabulary consists of the propositions $\{t_1, t_2, t_3\}$. They could, for instance, represent the terms of a document collection. Let one document be represented by $d = t_1 \wedge t_2$, meaning, that the terms represented by the propositions t_1 and t_2 are present in the document. The term represented by the proposition t_3 is not present in the document. This does not imply, however, that t_3 is not part of the document's information content, for it may be implicit in the document. This is represented by having two models of d, one in which t_3 is true, and the other where t_3 is false*:

Table 3: The models of the document d in the Classical Logic

t_1	t_2	t_3	d
1	1	0	1
1	1	1	1

Five queries are defined: $q_1 = t_1$, $q_2 = t_3$, $q_3 = t_1 \wedge t_3$, $q_4 = t_1 \vee t_3$, and $q_5 = t_1 \wedge t_2$. Their evaluations, with respect to the models of d, are given in the following truth table:

Table 4: Evaluation of different queries in Classical Logic

t_1	t_2	t_3	q_1	q_2	q_3	q_4	q_5
1	1	0	1	0	0	1	1
1	1	1	1	1	1	1	1

From the fourth column, it can be seen that $d \models q_1$; the document is relevant to the query. From the fifth column, it can be seen that $d \not\models q_2$; the document is not relevant to the query. It seems to work so far!

From the sixth column, $d \not\models q_3$. However, one would have considered the document represented by the formula d to be more relevant to q_3 than it is to q_2 because the

* This is different from the Boolean model where the *closed-world assumption* holds; that is, the document would be represented as $d = t_1 \wedge t_2 \wedge \neg t_3$.

document, though not *exhaustively* relevant, is nevertheless *partially* relevant to the query q_3. The problem is that $\models$ is too rigid a relation and cannot express partial relevance.

From the seventh and the eighth columns, $d \models q_4$ and $d \models q_5$. One would have expected the document represented by d to be more relevant to q_5 than to q_4. This counter-intuitive result is due to the semantics attached to disjunction. If ϕ is true, then any sentence of the form $\phi \vee \psi$ is also true even if ϕ and ψ are the representations of information items that are not related.

To finish, with the queries q_1 and q_5 (fourth and eighth columns), the outcomes are $d \models q_1$ and $d \models q_5$. One would have considered the document to be more relevant to q_5 than to q_1, for all the information items in d concern q_5, whereas fewer are related to q_1. That is, the document is more *specific* to q_5 than it is to q_1[†].

This simple example shows how logic is used to model an IR system. It also picks out some of weaknesses produced by taking Classical Logic as the basis for the model. The question is now whether *non-classical logics* can be used instead.

4 CHARACTERISTICS OF A LOGIC FOR INFORMATION RETRIEVAL

Although Classical Logic has provided a simple model for IR, it has not modeled all the intuitively desirable attributes of the IR process. A logic that encompasses the attributes is required. In this section, some of the fundamental characteristics of such a logic are presented in turn, although they do overlap somewhat. Each characteristic is examined in the light of the inadequacy of its representation in the simple classical model; and finally, non-classical alternatives that possess some of the required characteristics are cited[‡].

[†] Nie (Nie, 1990) proposed a formulation of the specificity by evaluating the inverse implications, respectively $q_1 \rightarrow d$ and $q_5 \rightarrow d$. Using truth tables as above, the outcome is $q_1 \not\models d$ and $q_5 \models d$. This formulation can reveal specificity of the document. However, this formulation is still too strict since, in most cases, a document is composed of many conjuncts, whereas a query contains very few conjuncts, and thus very few implications are valid.

[‡] The list of references is not exhaustive.

4.1 Significance

Some information in a document might be considered more predominant than other information. For instance, the fact that an item of information appears many times usually indicates that the item is a *significant* part of the document information content. The formula $\phi \wedge \phi$ can be used to mean the information item represented by the formula ϕ appears twice. However, in Classical Logic, because of the truth-based interpretation of the conjunction connective, $\phi \wedge \phi \leftrightarrow \phi$ is a tautology. Therefore, the fact that an information item appears many times, thus showing its significance, cannot be captured.

An example of a logic that refutes the above tautology is *linear logic* (Girard, 1987). Another way to capture significance is to use a weighting mechanism in parallel to the logic. Many conventional IR models use (successfully) weighting mechanisms, and hence empirical results already established by the implementation of these models can be exploited. Quantitative theories, such as *Probability Theory* (Pearl, 1988) or any statistics-based measures, may be used to a define weighting mechanism.

4.2 Information containment

In intuitive reasoning, two items of information are connected because one contains information about the other. In Classical Logic, the connection is expressed by an implication $\phi \rightarrow \psi$ where ϕ and ψ represent information items. However, in Classical Logic, $\phi \rightarrow \psi$ is equivalent to $\neg\phi \vee \psi$, implying that $\phi \rightarrow \psi$ is true whenever $\neg\phi$ is true. For example, "$2 + 1 = 5 \rightarrow Mounia\ likes\ to\ swim$" is a valid sentence because "$2 + 1 = 5$" is always false. This rule is rather inadequate in representing everyday reasoning. For most people, asserting an implication $\phi \rightarrow \psi$ (to be true) means that ψ cannot be false if ϕ is true. Furthermore, if both ϕ and ψ are valid then $\phi \rightarrow \psi$ is valid as well. Yet, one might hesitate to say that $\phi \rightarrow \psi$ is valid since one would expect some informational connections between ϕ and ψ. For example, the sentence

"$2 + 2 = 4 \rightarrow Banana\ is\ a\ fruit$" is valid in Classical Logic because both sentences "$2+2 = 4$" and "$Banana\ is\ a\ fruit$" are valid, although there is no connection between them. These examples show that implications as defined in Classical Logic do not always capture *information containment*. Therefore, the implication $d \rightarrow q$ as defined in Classical Logic does not adequately represent the relevance of a document to a query, since it does not necessarily mean that the document contains information pertinent to the query.

Logics concerned with the representation of informative relationships are *Conditional Logic* (Harper *et al.*, 1981, Nute, 1980) and *Situation Theory* (Devlin, 1991). In the former, an implication $p \rightarrow q$ is only evaluated in those (closest) worlds (a notion explained in section 5.1) in which p is true. In the second, an implication, referred to as a constraint, links two items of information only if they are informationally related.

4.3 Intensionality

Contexts often contribute to the meaning of information. For example, the polysemic word "bank" can refer to the river context or the money context. Hence, the substitution of "bank" by one of its synonym must respect the context in which the word is used. The phenomenon of the context-dependence in the meaning of information is referred to as *intensionality* (Zalta, 1988). In Classical Logic, the fact that two terms are synonymous is symbolized by the validity of $\phi \leftrightarrow \psi$, where ϕ and ψ are the formulae representing the two terms. It follows that every instance of ϕ in a formula can be replaced by ψ. Such a substitution is not always correct if ϕ represents a polysemic word. Hence, Classical Logic cannot handle intensionality.

A logic that deals with intensionality is *Intensional Logic* (Dowty *et al.*, 1981, van Benthem, 1985). There, the meaning (semantics) of an information item, in one or several contexts, is represented with indices, expressing, for example, times and locations.

4.4 Partiality

Many items of information are not originally identified as part of a document's information content, though they are implicit in the document. The representation of a document is only *partial*; it can grow when the implicit information becomes available, for example, by means of the flow of information (explained in section 4.5). This characteristic is referred to as the *partiality*[*] of information (Barwise, 1989, Landman, 1986).

The representation of partiality in Classical Logic would need to express that the truth value of a formula may not be known at some point, but may become known at some later stage. The representation of partiality would also need to express that the truth value of a formula may change. In Classical Logic, the representation of the unknown truth of a proposition p necessitates at least two models, one in which p is true and one in which p is false. If models symbolize a document, a set of models may be involved in representing the document. The representation of a change of truth values also requires that models are related. Nonetheless, the notions of growth of information and related models are foreign to Classical Logic, for models are distinct and unrelated entities. Therefore, Classical Logic cannot capture partiality.

There are many logics that deal with partiality. For example, *Three-valued Logic* (Kleene, 1967), *Belief Revision Systems* (Gardenfors, 1988) and *Data Semantics* (Landman, 1986) all of which postulate an initial stage of representation of information, knowledge, or belief that may vary by acquiring new information, or by refuting old information. Other logics dealing with partiality are *Modal Logic* (Hughes and Cresswell, 1968) and *Situation Theory* (Devlin, 1991). In these, different stages of representation are fixed, and acquiring new information consists of going from one stage to another.

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[*] The terminology is not to be confused with partial relevance, mentioned earlier in this paper.

4.5 Flow of information

A text document possesses an information content. Part of it corresponds to the meaning of the sentences of which the document consists, whereas another part goes beyond this meaning. Moreover, the content of a document conveys information in two forms: explicitly, one can read it; or implicitly, one can deduce or infer it (this also applies for non-textual documents). For example, the information item "Scandinavian sports" is often implicitly contained in any reference to "cross country skiing". As a result, a document about "cross country skiing" may be relevant to a query about "Scandinavian sports", even if the latter is not explicit in the document.

The phenomenon of information containment defines the *flow of information* (Dretske, 1981). In general, the flow of information is defined between two objects as the information an object contains about another object. If the first object is a text, an image, or a video and the second object is a query, then one can see that the exploitation of such a phenomenon is the task of an IR system.

Classical Logic cannot model information flow properly because related information items are symbolized by formulae, which are truth-based rather than information-based. Therefore, many relationships are erroneous. A logic that captures the flow of information is based on *Situation Theory* (Devlin, 1991).

4.6 Uncertainty

The inference (or derivability) of the query from the document may be *uncertain*. For example, let a document be about "animals living in the South Pole" and a query be about "penguins". The inference from "animals living in the South Pole" to "penguins", although conceivable, is uncertain because not all animals living in the South Pole are penguins. Uncertainty can also arise due to partiality of information, as there may not be enough information to perform the inference. For example, suppose that the word "bat" is contained in the document. If it is not known whether the word refers to a baseball bat or a flying mammal, then inference with respect to that word is uncertain.

The above two examples shows that the inference process in IR is often uncertain. In the case that the relevance of a document to a query depends on an inference, the more uncertain the inference, the less relevant the document. The correspondence cannot be expressed in Classical logic.

Examples of logics that capture uncertainty in the inference process are those dealing with *non-monotonic reasoning* (Krauss *et al.*, 1990, Reiter, 1980). However, the approach often adopted in logical IR modeling is to numerically express the correspondence between relevance and uncertainty. The correspondence is done by attaching a number to each inference and using the values as the basis of the computation of the degree of relevance. This has the advantage of yielding a numerical evaluation of relevance, which can be used to rank documents according to the extent to which they are considered relevant to a given query. In this case, the logic used to model the IR system must be quantitative. Since this is not a characteristic of Classical Logic or many other logics, other frameworks are used in parallel to the logic. Appropriate frameworks are those that are based on what is described as *theories of uncertainty*, for example, *Probability Theory* (Pearl, 1988), *Bayesian networks* (Duda *et al.*, 1976), *Theory of Evidence* (Shafer, 1976), and *Fuzzy Set Theory* (Zadeh, 1987).

4.7 Conclusion

Various characteristics of a logic that encompasses important attributes of the IR process have been discussed. Classical Logic does not capture well, and in some instances, does not capture at all many of these characteristics. Similar weaknesses in the use of Classical Logic were observed in other domains (e.g., natural language processing, robotics), and as a result alternative non-classical logics have been developed. Several of these logics have been examined and used to construct logical IR models. The next section describes some of the models so far developed.

5 OVERVIEW OF LOGICAL MODELS OF INFORMATION RETRIEVAL SYSTEMS

The choice of the appropriate logic has been one of the main research debate in logical IR modeling. One consensus is that a logic to model IR should not yield only true or false values for the practical reason that basing relevance on the validity of $d \rightarrow q$ leads to either too few or too many documents being retrieved. It does not permit the documents that are only partially relevant to the query to be retrieved as well as those directly relevant. It also fails to capture the uncertainty that often accompanies the inference process. Two directions are possible: the first is to make the evaluation of $d \rightarrow q$ numerical; the second is to keep the evaluation qualitative as opposed to quantitative and to use concurrently a *theory of uncertainty* to embody partial relevance and uncertainty.

The second of these directions is not uncommon, as (Saffioti, 1987) says:

> "Many of these solutions [of representing uncertainty] share the attitude of viewing the knowledge and the uncertainty about it as two different entities, and so treating them by means of two distinct loosely-coupled processes: the reasoning process handles knowledge as if it were exact, while a "parallel uncertainty inference" process accompanies it, computing the uncertainty affecting each newly arrived fact. This uncertainty is in turn usually based on the uncertainty affecting the facts used to derive the new fact."

Most researchers are following this second direction to develop logical models of IR: the objective is to determine an appropriate logic and a theory of uncertainty, and to develop a method to combine them in a consistent fashion. This choice has the advantage that it leads naturally to a numerical expression of relevance. Also, many of these logical models are based on the *Logical Uncertainty Principle* proposed by(van Rijsbergen, 1986b), where the connection between logic and IR modeling was for the first time explicitly made:

> "Given any two sentences x and y; a measure of the uncertainty of $y \to x$ relative to a given data set, is determined by the minimal extent to which we have to add information to the data set, to establish the truth of $y \to x$."

The process of adding information is represented by a logic, whereas the involved uncertainty is modeled by a theory of uncertainty.

A number of logical models have been developed based on the Logical Uncertainty Principle. The models adopt different variations of the principle, and are briefly covered from section 5.1 to section 5.6. Variations stem from the fact that the addition process, original to the principle, has been generalized to one of *transformation*, which can also be a deletion or a modification of information. Furthermore, not only the data set can be transformed, but also documents and queries. In the next sections, only the logic is described[†]. For recent work on the modeling of uncertainty in logical IR models, see (Crestani and Lalmas, 1996, Lalmas, 1995).

Also based on logic, meta-models of IR systems were developed. These models propose a logical framework in which IR systems can be represented and their properties and results can be formally proven. IR meta-models are briefly described in section 5.7.

In the following sections, the descriptions are given for the reader to acquire an idea of the work in the area; I would like to recommend the interested reader to refer to the cited literature for the details of the various models.

5.1 Models based on Modal Logic and conceptual graphs

Modal Logic (Hughes and Cresswell, 1968) adopts the notion of *possible–worlds* (Kripke, 1963) which correspond to the interpretations defined in section 2, but which are connected

[†] Fuzzy Logic (Zadeh, 1987) has been used in IR (for example, in (Kracker, 1991)). However, its use was quantitative (it was used as an alternative to Probability Theory, for example). The use of "quantitative logic" in IR is not described in this paper.

to each other via an *accessibility relation*. The evaluation of the truth of a proposition is with respect to a possible world, and may involve the evaluation of the truth of the proposition or others in connected worlds.

Modal Logic was used to develop a logical model for IR (Nie, 1989, Nie, 1990). Documents are worlds, and queries are formulae. A document represented by a world d is relevant to a query represented by a formula q if q is true in d, or if it is true in a world d' accessible from d. The accessibility relation captures the transformation of documents; the fact that the world d is connected to the world d' is interpreted as d being transformed into d'. For example, d' contains terms that are synonymous to those contained in d.

The accessibility relationship can have different properties. For example, transitivity, meaning that if a world d is related to a world d', which is itself related to a world d'', then the world d is also related to the world d''. Consider the example of a hypertext system. Using Modal Logic, worlds can represent texts and the accessibility relation can represent the links between the texts. Let d be a world text, linked to a second world text d', itself linked to a third world text d''. If d does not contain information relevant to a query, but d' does, we may still want to retrieve d. It could be that only d'' contains information relevant to the query. Do we still want to retrieve d? This decision can be formally represented by allowing the accessibility relation to be transitive or not. This example gives an indication on how it is possible to reason about the type of system needed.

The model also allows the transformation of both the query and the data set. One query can be transformed into another using, for example, thesaural information. Query transformation is not a new approach in IR (e.g., query expansion). The novelty is that the transformation process can be formally represented, and hence reasoned about. Transforming a data set can capture the modeling of a user's state in the retrieval process. The data set can be transformed until it reaches one that reflects the user's state.

A variant of this model was proposed in (Chevallet, 1992). The logic was instantiated by the formalism of *conceptual graphs* (Sowa, 1984), which are graphs built out of concepts and their associated semantics. Documents and queries are represented by conceptual graphs, and the transformation process is instantiated by operations performed on the graphs. For example a graph could be transformed to another one, where one concept in the initial graph is replaced by one more general. This formalism has been used for instance to represent medical documents and software components. The problem, however, is the automatic construction of graphs from documents or queries.

5.2 Models based on Imaging

Imaging (Harper *et al.*, 1981, Lewis, 1976, Nute, 1980) is also a framework based on the notion of possible-worlds. The truth value of the implication $p \rightarrow q$ in a world w depends on two cases. If p is true in w, then $p \rightarrow q$ is true (false) in that world if q is true (false) in that world. If p is not true in w, then the implication is evaluated in the worlds that differ *minimally* from w and in which p is true. The worlds in which p is true are referred to as $p-$ worlds.

The set of worlds comes with a probability distribution P, reflecting the probability of each world. The probability of a proposition p is the summation of the probability of those worlds in which p is true. The computation of the probability of $p \rightarrow q$ involves a shift of probability (the imaging process) from non $p-$worlds to their closest $p-$worlds. A new distribution probability, written P_p, is constructed. It is proved that (Lewis, 1976):

$$P(p \rightarrow q) = P_p(q)$$

The imaging process seems compatible with the notion of minimal transformation. If d represents the document and q models the query, then the relevance of the document to the query can be evaluated as

$$P(d \rightarrow q) = P_d(q)$$

Two logical models have been developed on the concept of Imaging. In the first one (Crestani and van Rijsbergen, 1995a, Crestani and van Rijsbergen, 1995b), worlds model terms, and propositions model documents and queries. A term t "makes a document true" if that term belongs to that document. Imaging with respect to d gives the closest term to t that is contained in d. It is t if t is contained in the document. Imaging consists then of shifting the probabilities to the terms contained in d (i.e., the terms that make d true). The evaluation of the relevance takes into account the semantics between terms by shifting to those (semantically) closer terms contained in the document. This model has been successfully implemented on some standard test collections (Crestani and van Rijsbergen, 1995a, Crestani and van Rijsbergen, 1995b). Large-scale testing and implementations on TREC are ongoing (Crestani *et al.*, 1996).

A second model based on Imaging (Nie *et al.*, 1996) includes user's knowledge in the evaluation of the relevance of a document to a query. In the model, both the document and the query are propositions. Worlds model different possible states of knowledge that can be held by users. The document d is true in a world w if the document is compatible with the state of the knowledge associated with the world. Imaging with respect to d gives the closest world to w in which d becomes true. In those closest worlds, relevance is evaluated. This model has the advantage that user modeling is formally included.

5.3 Models based on Situation Theory

Situation Theory (Devlin, 1991) is a theory of information that provides an analysis of the concept of information and the manner in which intelligent organisms (referred to as cognitive agents) handle and respond to the information picked up from their environment. The theory defines the nature of information flow and the mechanisms that give rise to such a flow. Information items are represented by *types*. For example, $\varphi = [\dot{s}|\dot{s} \models \ll Swimming, Mounia; 1 \gg]$ represents the information item that Mounia is swimming[‡].

[‡] Types are build upon infons, which are the basic items of information. In the above example, the infon is $\ll Swimming, Mounia; 1 \gg$.

Nothing is said about the truth of this type; a type is just the representation of an item of information. What makes a type true is the *situation* (a partially defined world) from which the information represented by that type is extracted. Situation Theory models the notion of “make true” by the *support* relation, denoted $\models$. If s is a situation that makes the information “Mounia is swimming” true, then one can write $s \models \varphi$, which should be read as “s supports φ”.

A logical model was developed based on Situation Theory (Lalmas, 1996, Lalmas and van Rijsbergen, 1993). A document is a situation s and the query is a type φ. The document is relevant to the query if there exists a flow of information from a situation s to a situation s' such that $s' \models \varphi$. The nature of the flow depends on the so-called *constraints* which capture semantic relationships (e.g., the relationships that many people attach to white wine and Australian wine, the relationships based on synonymy, etc.). More formally, constraints are defined between types. Let φ and ψ be two types that constitute the constraint $\varphi \rightarrow \psi$. The application of this constraint to a situation s is possible if first $s \models \varphi$ and then informs of the existence of a situation s' such that $s' \models \psi$: the fact $s \models \varphi$ carries the information that $s' \models \psi$. A flow of information circulates between the situations s and s', and the nature of the flow is defined by the constraint $\varphi \rightarrow \psi$.

Flows of information do not always materialize because of the unpredictable nature of situations; thus flows are often uncertain. In Situation Theory, an uncertain flow is modeled by a *conditional* constraint of the form $\varphi \rightarrow \psi | B$, which highlights the fact that $\varphi \rightarrow \psi$ holds if some *background conditions* captured within B are met. If the background conditions are satisfied, the corresponding flow arises. The use of background conditions in an IR model acknowledges the important fact that information is seen to be dependent on a context. For example, background conditions can represent context with respect to polysemic words.

Situation Theory satisfies many of the features of a logic for IR modeling (see

(Huibers *et al.*, 1996, van Rijsbergen and Lalmas, 1996) for a discussion on this). It also allows the representation of uncertainty (via background conditions), although only qualitatively (Lalmas, 1995). This means that most components of an IR system can be integrated into one model, which implies that further reasoning about the different components of the model are possible.

5.4 Models based on Plausible Reasoning

Other derivation relations have been proposed which are more flexible than that of Classical Logic. An example is one that captures *plausible reasoning*, which has been used to develop a logical IR model (Bruza, 1993, Bruza and van der Weide, 1992). The purpose of the model is to capture syntactically related information. The documents and queries are represented by *index expressions* defined upon noun-phrases. For example, the two noun-phrases “penguin in South Pole” and “wine tasting” are represented by the index expressions “$penguin$ in $South \circ Pole$ ” and “$wine \circ tasting$”, respectively. The inference process is based on a *strict* derivation and a *plausible* derivation mechanisms, examples of which are, respectively:

$$wine \circ tasting \vdash wine$$

$$wine \circ tasting \mid\!\sim wine \text{ in } cellar$$

The first derivation is strict because “wine tasting” is related to “wine”. The second is plausible because “wine tasting” may be done in a “cellar” and hence may be related to “wine in cellar”. Each type of derivation comes with its own set of rules and axioms. Relevance occurs if, given two index expressions d and q, representing the document and the query, respectively, at least the following $d \mid\!\sim q$ can be proven[§]. Obviously, if $d \vdash q$, the relevance is higher.

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§ The proof may involve strict derivations; as soon as one of the derivation is not strict, the overall reasoning is then plausible.

5.5 Models based on Terminological Logic

Terminological Logic provides an object-oriented flavored knowledge representation. The primary syntax starts with terms, which are either individuals or relations. Concepts are defined on top of those. The logic was used in the model proposed in (Meghini *et al.*, 1993, Sebastiani, 1994). Documents are represented by individual constants, whereas a class of documents is represented as a concept. The fact that a particular individual is an instance of a concept is written as an assertion. For example, the following

(and paper
(func appears-in (sing IP&M))
(all author (func affiliation (sing GLASGOW)))
(c-some deals-with logic)) [paper666]

means that the individual document, named paper666, belongs to the concept that denotes the class of all those papers that appear in IP&M where all authors are from Glasgow and that deal with logic.

Concepts come with a partial order $\dot{\leq}$ which stands for *conceptual containment*. The fact that two concepts are ordered with respect to $\dot{\leq}$ constitutes an axiom that describes some thesaural knowledge. Queries are described as concepts. Given a query represented by a concept C, the retrieval task is to find all those documents i such that $C[i]$. The evaluation of $C[i]$ uses the set of assertions which describe documents, and the set of axioms which describe thesaural knowledge, and the notion of subsumption (hierarchical domination) defined with $\dot{\leq}$. The evaluation of $C[i]$ is defined as in Classical Logic. However, the semantics of the terms go beyond true or false. For example, the semantics of the concept "author" will be the set of individuals that are authors.

5.6 Models based on Abductive Logic

Abduction is a way of explaining observations, expressed as formulae, by minimally extending a theory with some added hypotheses. More formally, given a theory T, and a formula p that needs to be explained in terms of T, abduction leads to a set of hypotheses Φ such that $T \cup \Phi \vdash p$ where $\vdash$ is a derivability relation in a logic. The hypotheses are also referred to as *abductive sentences*.

A logical model based on abductive reasoning has been developed in (Muller and Kutschekmanesch, 1995, Thiel and Muller, 1996) to build a hybrid system, where IR and hypertext facilities are combined. p is a query, and T is a knowledge base which captures semantic relationships (e.g., synonymy). The abductive sentences correspond to information related to documents (e.g., author, topic, etc.). The abduction process yields a structured proof, which is used to compute a solution space (a formula can be explained in different ways, so several sets of hypotheses, or solutions, can be found). Each solution generates a model that constitutes a starting point for a user to browse through, either to access relevant documents, or related documents (e.g., same authors). This work is particularly attractive because it defines a logical framework that integrates IR and hypertext.

5.7 Meta-models of information retrieval systems

The goals of developing a meta-models for IR is to formally study the properties and the characteristics of IR systems within a uniform framework. The advantage is that it will be then possible to compare IR systems, and this not only with respect to their effectiveness. For example, an application may require a system that retrieves all relevant documents (recall-oriented systems), or that retrieves only relevant documents (precision-oriented systems).

The use of logic to formally conduct proofs for IR purposes originated in (Nie, 1990) where it was showed that a logical model is a general form of many other IR models. The idea was later thoroughly investigated in (Bruza and Huibers, 1994, Huibers, 1996), where a framework was proposed in which different models of IR systems could be theoretically expressed, and then formally studied and compared. The framework was developed within a logic, thus allowing formal proofs to be conducted. Their idea of the *meta-model* is based on the work of (Krauss *et al.*, 1990) and (Devlin, 1991).

The framework defines the *aboutness* relationship, denoted $\mid\!\sim$, thus capturing the notion of information containment primary to IR. Given two objects a and b, $a \mid\!\sim b$ means that object a is about object b. Axioms are defined that represent eventual properties of IR systems. Examples of axioms include:

(i) Reflexivity: $a \mid\!\sim a$

(ii) Symmetry: $if \ a \mid\!\sim b$ then $b \mid\!\sim a$

(iii) Transitivity: $if \ a \mid\!\sim b$ and $b \mid\!\sim c$ then $a \mid\!\sim c$

Most IR models seem to satisfy reflexivity because if objects are documents, then if a document is submitted as a query, then this document should be retrieved. The IR models satisfying symmetry are the vector space model, and those based on overlap measures such as Jaccard, or Dice. The models based on overlap measures were shown to not satisfy transitivity.

To be formally studied, an IR model must be expressed within the above framework. The objects and the aboutness relationship must be instantiated. Then, determining which property is satisfied by that model can be carried out based on these instantiations. In the example of the Boolean model, objects are formulae and $\mid\!\sim$ is the material implication. The resulting model is one based on Classical Logic with the closed-world assumption. It is easy to prove that this model satisfies reflexivity but not symmetry (in the original papers there are other examples and their formal proofs).

6 SUMMARY AND CONCLUSION

This paper provides an introduction of the use of logic for the modeling of IR systems. The concepts necessary to understand the work carried out in the so-called "logical models" were presented, and the relation between logic and IR was highlighted. The aim of this paper is to encourage the reader to learn more about this research area.

Several logical models that are currently being developed were mentioned. One obvious question is which of these models leads to the most effective IR systems, or is the most expressive. Maybe none of these models is the best, but some deal well with particular attributes of the IR process, such as described in sections 3 and 4. Maybe the best model is that satisfying most of these attributes; but, are these attributes essential in practice? For this reason, it is currently important that various formalisms are examined because further investigations are required before attempting to answer this question.

The use of logic for IR modeling is still in its early stages. Enormous progress has been made, but further investigation and development are required before the effectiveness of logical models can be established. For instance, the implementation of logical models can be complex, and when possible, often only small document collections can be handled. However, implementations on standard test collections have been recently attempted with some positive results (Crestani and van Rijsbergen, 1995a, Crestani and van Rijsbergen, 1995b). Well-known large-scale experiments, such as TREC, are ongoing (Crestani *et al.*, 1996). Nevertheless, more experimental work is necessary to demonstrate the effectiveness of logical models.

I believe that one major strength of logical models is that they allow for the formal study of IR models and their properties. This is becoming increasingly important because the performance and behavior of IR models needs to be justified. Furthermore, this study may help in implementing logical models as some attributes may be identified as less important than others, and hence their implementations can be simplified. Finally, it may also identify which logic leads to the most effective IR systems.

In conclusion, the following quote from (Smeaton, 1996) summarizes my beliefs in such work:

“While currently much of this work [the use of logic in IR modeling] can be regarded as an interesting theoretical exercise because of the small scale of actual experimentation, if any, it is without doubt that if there is ever to be a really significant breakthrough in information retrieval, it will come from this kind of fundamental and basic work."

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