Comparing Deadlock-Free Session Typed Processes

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This work

A **formal comparison** between two behavioural type systems that enforce deadlock freedom for π-calculus processes

We consider two fundamentally different systems:

- Session types based on linear logic [Caires, Pfenning et al; Wadler]
- Usage types [Kobayashi et al]

We study $\mathcal{L}$ and $\mathcal{K}$, the deadlock-free languages they induce

Outline

Deadlock Freedom

Intuition: Processes do not "get stuck"

Examples (in CCS)

- A deadlocked process:

$$(\nu a)(\nu b)(\overline{a}.b.\mathbf{0} \mid \overline{b}.a.\mathbf{0})$$

- Two deadlock-free processes:

$$(\nu a)(\nu b)(b.\overline{a}.\mathbf{0} \mid \overline{b}.a.\mathbf{0}) \quad \text{and} \quad (\nu a)(\nu b)(\overline{a}.b.\mathbf{0} \mid a.\overline{b}.\mathbf{0})$$

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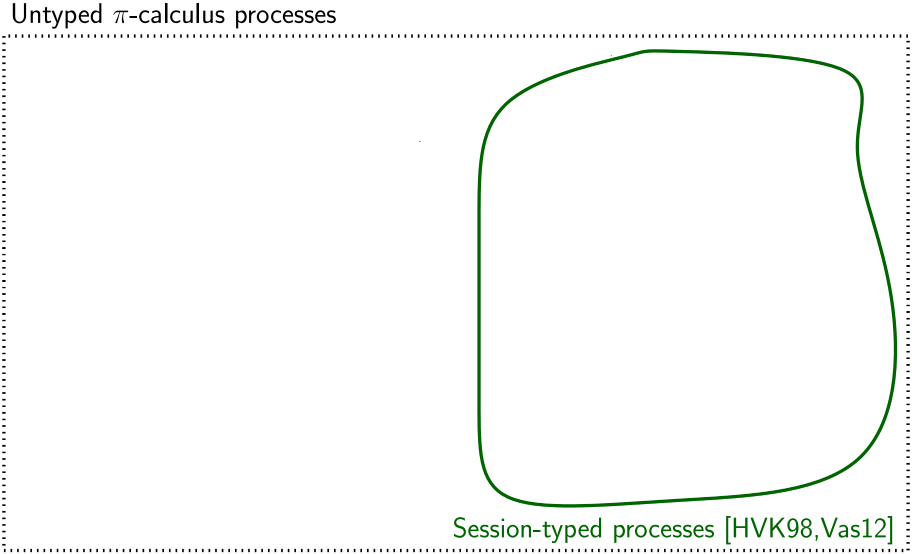
- A liveness property: "A process is deadlock-free if it can always reduce until it eventually terminates, unless it diverges"
- Relevant in practice. Many type systems proposed [cf. Kobayashi et al; Dezani, de' Liguoro & Yoshida; Carbone & Debois]
- Our motivation: clarifying how these different systems are related

The landscape of (un)typed processes



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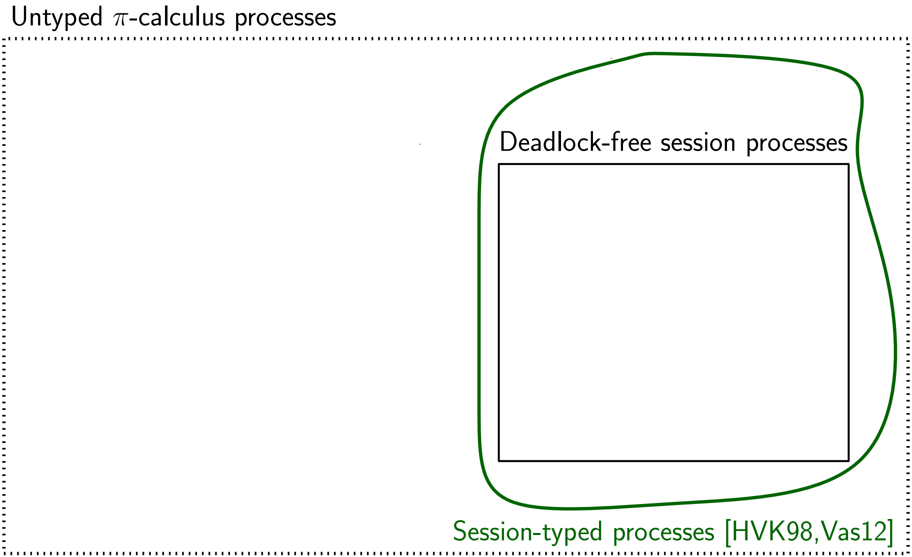

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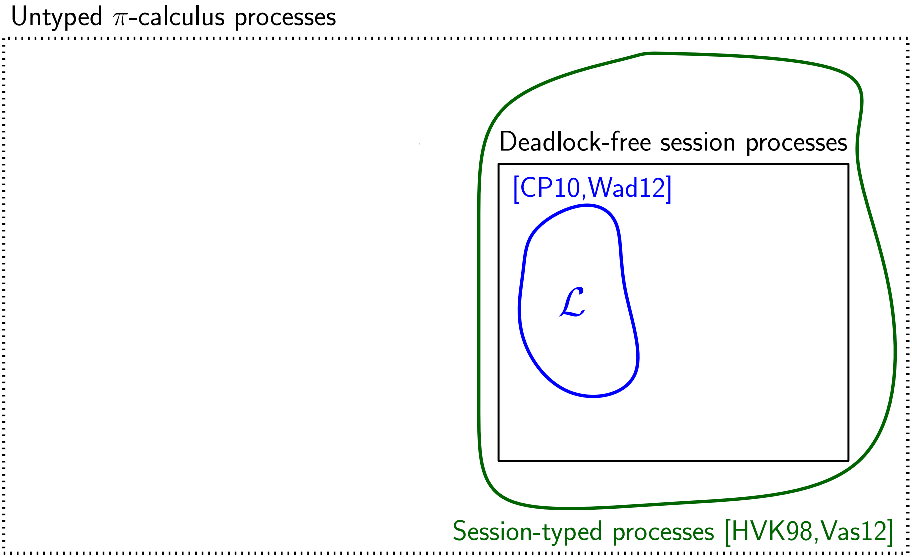

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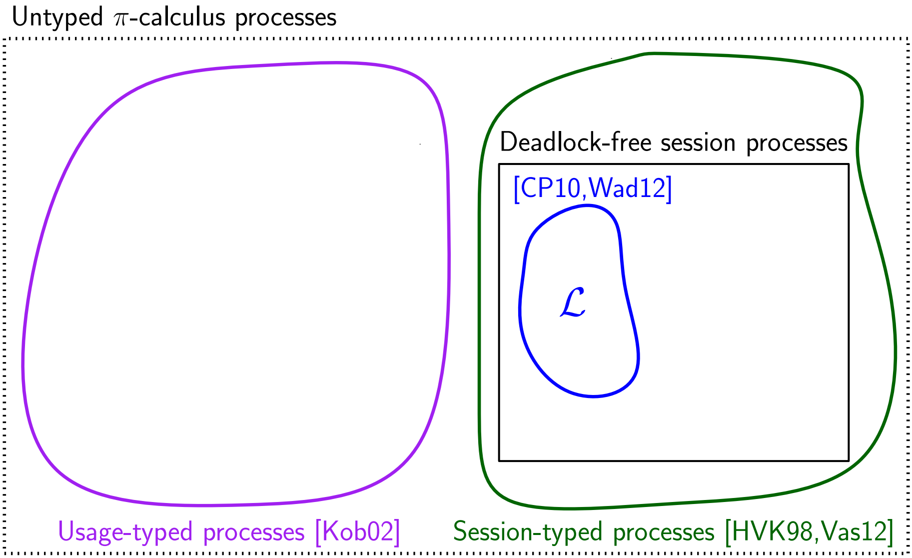

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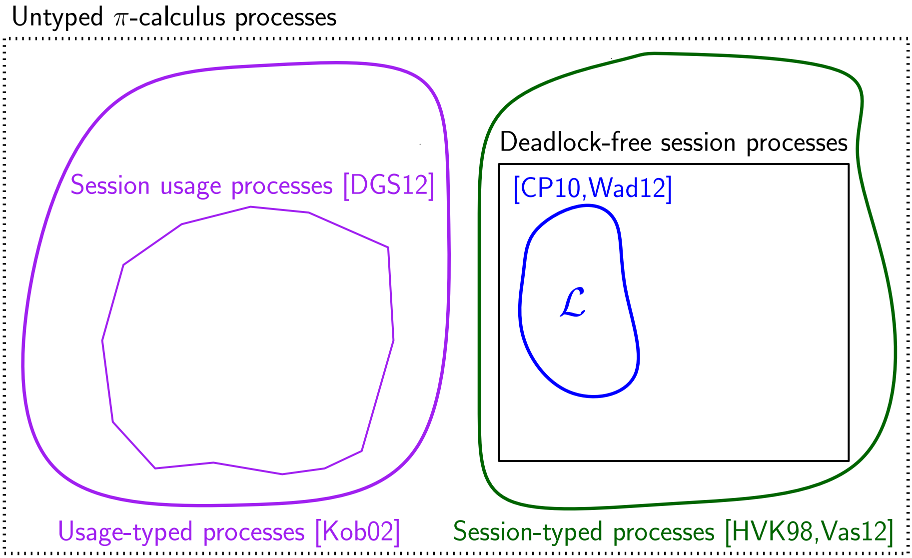

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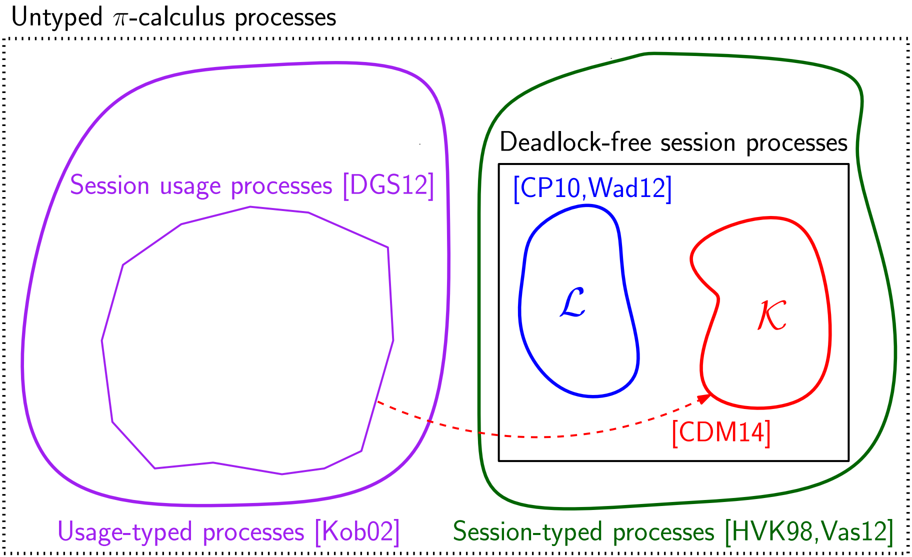


The typed languages $\mathcal{L}$ and $\mathcal{K}$

$\mathcal{L}$: session processes obtained from the correspondence of linear logic propositions as session types [CP10, Wad12, Caires14].

$\mathcal{K}$: session processes are deadlock-free by combining a usage-based type system [Kob02] with the encodability result in [DGS12]

Notice

- $\mathcal{L}$ is **canonic**: due to its deep logical foundations, this is the most principled yardstick for comparisons
- $\mathcal{K}$ is **general**: it strictly includes classes of processes induced by previous type systems for deadlock-free sessions

The degree of sharing

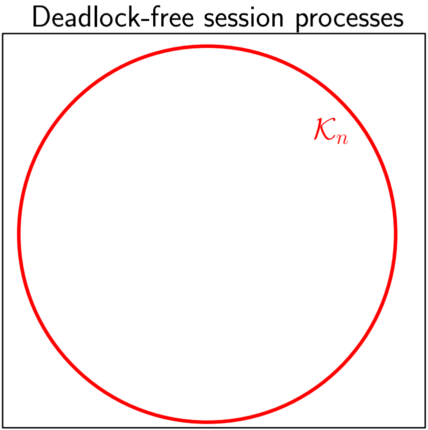
$\mathcal{K}$ is a **family** of classes of deadlock-free processes: $\mathcal{K}_0, \mathcal{K}_1, \ldots, \mathcal{K}_n$

The family is defined by the **degree of sharing** between parallel components:

- $\mathcal{K}_0$ is the subclass with independent parallel composition: for all $P \mid Q \in \mathcal{K}_0$, P and Q **do not share** sessions
- $\mathcal{K}_1$ is the subclass which contains $\mathcal{K}_0$ but also processes with parallel components that share **at most** one session

In general, $\mathcal{K}_n$ $(n \geq 0)$ contains deadlock-free session processes whose parallel components share at most n sessions

Our contributions



$\mathcal{K}_n$ deadlock-free processes by encodability [DGS12]

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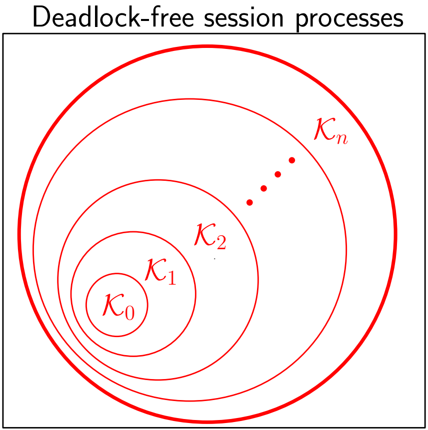

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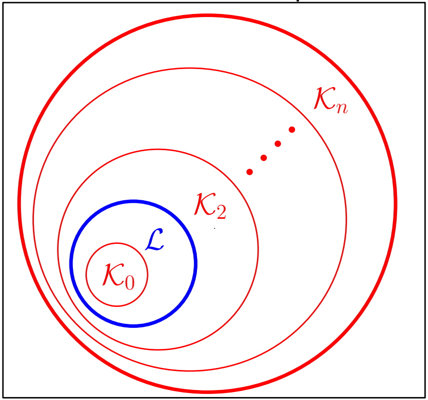
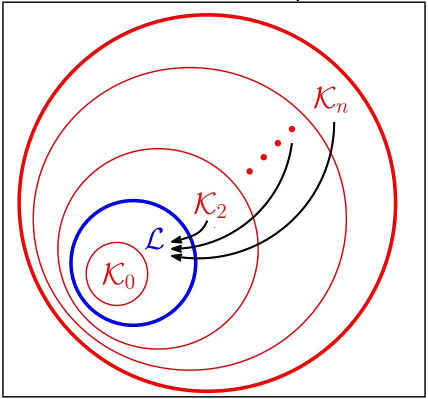


$\mathcal{K}_n$ deadlock-free processes by encodability [DGS12]

$\mathcal{L}$ deadlock-free processes based on linear logic [CP10, Caires14]

Our contributions

Deadlock-free session processes



$\mathcal{K}_n$	deadlock-free processes by encodability [DGS12]
$\mathcal{L}$	deadlock-free processes based on linear logic [CP10, Caires14]
$\longrightarrow$	type-preserving rewriting procedure

Outline

Technical Ingredients

Type systems:

$\Gamma \vdash_{\mathtt{ST}} P$	Simple session types
$P \vdash_{\mathtt{CH}} \Delta$	Linear logic propositions as session types [CP10]
$\Gamma \vdash_{\mathtt{KB}}^{n} P$	Usage type system [Kob02] w/ degree of sharing $n \geq 0$

Encodings/transformations:

$[\![\cdot]\!]_f$	Session π-processes into standard π-processes
$[\![\cdot]\!]$	Session types into usage types (defined in [DGS12])
$[\![\cdot]\!]_{\mathsf{c}}$	Session types into linear logic propositions
$\{\!\{\cdot\}\!\}$	Handling syntactic differences in processes (e.g. restriction)

The two deadlock-free session languages, formally

$$\mathcal{L} = \{P \mid \exists \Gamma. \ (\Gamma \vdash_{\mathtt{ST}} P \ \wedge \ \{\!\{P\}\!\} \vdash_{\mathtt{CH}} [\![\Gamma]\!]_{\mathsf{c}})\}$$

$$\mathcal{K}_n = \{P \mid \exists \Gamma, f. \ (\Gamma \vdash_{\mathtt{ST}} P \ \wedge \ [\![\Gamma]\!]_f \vdash_{\mathtt{KB}}^{n} [\![P]\!]_f)\} \quad (n \geq 0)$$

Session π-calculus

Syntax

$$
\begin{array}{rlll}
P, Q ::= & \overline{x}\langle v\rangle.P & & \text{(output)} \\
& x(y).P & & \text{(input)} \\
& x \triangleleft l_j.P & & \text{(selection)} \\
& x \triangleright \{l_i : P_i\}_{i\in I} & & \text{(branching)} \\
& (\boldsymbol{\nu} xy)P & & \text{(session restriction)} \\
& P \mid Q & & \text{(composition)} \\
& \mathbf{0} & & \text{(inaction)}
\end{array}
$$

Write **n** to denote a channel that cannot be used.

Reduction semantics

$$
(\boldsymbol{\nu} xy)(\overline{x}\langle v\rangle.P \mid y(z).Q) \to (\boldsymbol{\nu} xy)(P \mid Q[{}^{v}/{}_{z}])
$$

$$
(\boldsymbol{\nu} xy)(x \triangleleft l_j.P \mid y \triangleright \{l_i : P_i\}_{i\in I}) \to (\boldsymbol{\nu} xy)(P \mid P_j) \quad j \in I
$$

Session Types

$$
\begin{array}{rll}
T, S ::= & \textbf{end} & \text{(terminated protocol)} \\
& ?T.S & \text{(input value of type } T\text{, continue as } S\text{)} \\
& !T.S & \text{(output value of type } T\text{, continue as } S\text{)} \\
& \&\{l_i : S_i\}_{i \in I} & \text{(external choice)} \\
& \oplus\{l_i : S_i\}_{i \in I} & \text{(internal choice)}
\end{array}
$$

The typing system $\vdash_{\mathrm{ST}}$

- Ensures communication safety and session fidelity, but it does not exclude deadlocked processes.
- Example: process $(\boldsymbol{\nu} x_1 x_2)(\boldsymbol{\nu} y_1 y_2)(\overline{x}_1\langle \mathbf{n} \rangle.\overline{y}_1\langle \mathbf{n} \rangle \mid y_2(t).x_2(s))$ is typable in $\vdash_{\mathrm{ST}}$ but deadlocked. In contrast, the typable process

$$(\boldsymbol{\nu} x_1 x_2)(\boldsymbol{\nu} y_1 y_2)(\overline{x}_1\langle \mathbf{n} \rangle.\overline{y}_1\langle \mathbf{n} \rangle \mid x_2(s).y_2(t)) \text{ is deadlock-free}$$

Linear Logic Session Types [Caires14]

Syntax

- As before, but with standard restriction operator $(\boldsymbol{\nu} x)P$ and the **forwarding process** $[x \leftrightarrow y]$, which "fuses" names x and y
- We have the reduction rule: $(\boldsymbol{\nu} x)([x \leftrightarrow y] \mid P) \rightarrow P[y/x]$

Types (= linear logic propositions)

$$A, B ::= \underbrace{\perp \mid \mathbf{1}}_{\text{Terminated}} \mid \underbrace{A \otimes B}_{\text{Output}} \mid \underbrace{A \parr B}_{\text{Input}} \mid \underbrace{\oplus\{l_i : A_i\}_{i \in I}}_{\text{Int. Choice}} \mid \underbrace{\&\{l_i : A_i\}_{i \in I}}_{\text{Ext. Choice}}$$

Linear Logic Session Types [Caires14]

Some Typing Rules for $\vdash_{\mathtt{CH}}$

$$\text{(T-cut)}\ \frac{P \vdash_{\mathtt{CH}} \Delta, x{:}A \qquad Q \vdash_{\mathtt{CH}} \Delta', x{:}\overline{A}}{(\nu x)(P \mid Q) \vdash_{\mathtt{CH}} \Delta, \Delta'} \qquad \text{(T-mix)}\ \frac{P \vdash_{\mathtt{CH}} \Delta \qquad Q \vdash_{\mathtt{CH}} \Delta'}{P \mid Q \vdash_{\mathtt{CH}} \Delta, \Delta'}$$

Notable points:

- **Composition plus hiding** thanks to rule (T-cut)
- Rule (T-mix) types independent parallel composition
- **Properties**: Type preservation (subject reduction) and progress

Usage Types [Kob02]

Syntax

- **Polyadic communication** and standard restriction
- **Case construct case** v **of** $\{l_i\_x_i \triangleright P_i\}_{i \in I}$ with **variant value** l_j_v

Usage Types

$$
\begin{array}{rlll}
U ::= & ?^o_\kappa.U & \text{(used in input)} & \emptyset & \text{(not usable)} \\
 & !^o_\kappa.U & \text{(used in output)} & (U_1 \mid U_2) & \text{(used in parallel)} \\
T ::= & U[\widetilde{T}] & \text{(channel types)} & \langle l_i : T_i \rangle_{i \in I} & \text{(variant type)}
\end{array}
$$

Obligations o and **capabilities** κ describe channel dependencies.

- An obligation of level n must be fulfilled by using only capabilities of level **less than** n
- For an action with capability of level n, there must exist a co-action with obligation of level **less than or equal to** n

A usage U that satisfies these conditions is **reliable**, noted $\mathrm{rel}(U)$

Usage Types [Kob02]

Some Typing Rules for $\vdash^n_{\mathtt{KB}}$

$$\frac{\Gamma_1 \vdash^n_{\mathtt{KB}} P \quad \Gamma_2 \vdash^n_{\mathtt{KB}} Q \quad |\Gamma_1 \cap \Gamma_1| \leq n}{\Gamma_1 \mid \Gamma_2 \vdash^n_{\mathtt{KB}} P \mid Q}\ (\mathrm{T}\pi\text{-}\mathrm{PAR}_n)$$

$$\frac{\Gamma, x : U[\widetilde{T}] \vdash^n_{\mathtt{KB}} P \quad \mathsf{rel}(U)}{\Gamma \vdash^n_{\mathtt{KB}} (\boldsymbol{\nu} x)P}\ (\mathrm{T}\pi\text{-}\mathrm{RES})$$

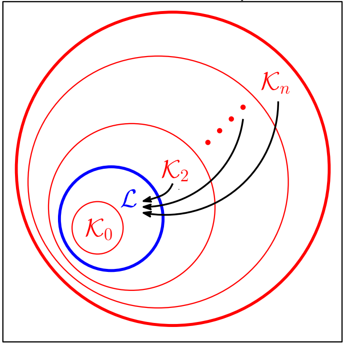
Notable points:

- Separate typing rules for restriction and parallel
- Novelty wrt [Kob02]: rule $(\mathrm{T}\pi\text{-}\mathrm{PAR}_n)$ defines degree of sharing
- Rule $(\mathrm{T}\pi\text{-}\mathrm{RES})$ checks usage reliability
- **Properties**: Type preservation and deadlock-freedom

Outline

Main results

Deadlock-free session processes



1. The inclusion between the constituent classes of $\mathcal{K}$ is **strict**

   We have: $\mathcal{K}_0 \subset \mathcal{K}_1 \subset \cdots \subset \mathcal{K}_{n-1} \subset \mathcal{K}_n$.

2. $\mathcal{L}$ and $\mathcal{K}_1$ **coincide**

   Logical foundations of session types induce the most basic form of concurrent cooperation: sharing exactly one session.

3. **Rewriting** of processes in $\mathcal{K}$ into $\mathcal{L}$

   Sequential prefixes replaced with representative parallel components.
   It enjoys type-preservation, operational correspondence, compositionality.

Inclusion: $\mathcal{K}_n \subset \mathcal{K}_{n+1}$

$\mathcal{K}_2$ contains (deadlock-free) session processes not captured in $\mathcal{K}_1$.

A representative example is:

$$P_2 = (\boldsymbol{\nu} a_1 b_1)(\boldsymbol{\nu} a_2 b_2)(a_1(x).\, \overline{a_2}\langle x\rangle \mid \overline{b_1}\langle \mathbf{n}\rangle.\, b_2(z))$$

P_2 is typable in $\vdash^n_{\mathtt{KB}}$ (with $n \geq 2$) but not in $\vdash^1_{\mathtt{KB}}$, because it involves the composition of two processes which share two sessions

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The idea generalises to show $\mathcal{K}_n \subset \mathcal{K}_{n+1}$, for all $n \geq 0$.

Coincidence: $\mathcal{K}_1 = \mathcal{L}$

Key aspects in the proof:

- Session types always result in "sequential" usages.
- Usages induced by LL session types are always reliable: they respect obligations/capabilities
- Preservation of duality through encodings of types
- Presence of both rules (T-mix) and (T-cut) for composition in $\vdash_{\mathtt{CH}}$

Rewriting $\mathcal{K}_n$ into $\mathcal{L}$

- If $P \in \mathcal{K}_{n+1}$ but $P \notin \mathcal{K}_n$ (with $n \geq 1$) then there is a subprocess of P that needs to be "adjusted" in order to "fit in" $\mathcal{K}_n$
- Such a subprocess of P must become more "parallel" in order to be typable under the smaller degree of sharing n

Rewriting $\mathcal{K}_n$ into $\mathcal{L}$: Intuition

- We propose a **rewriting procedure** that converts processes in $\mathcal{K}_n$ into processes in $\mathcal{K}_1$ (that is, $\mathcal{L}$)
- **Idea**: given a parallel process as input, return as output a process in which one component is kept unchanged, but the other is **replaced** by **parallel representatives** of the sessions in it.
- Using types, such representatives are defined as **characteristic processes** and **catalyzers**
- The procedure is type preserving and satisfies compositionality and operational correspondence

Rewriting $\mathcal{K}_n$ into $\mathcal{L}$: Example

Consider the $\mathcal{K}_2$ process

$$P_2 = (\boldsymbol{\nu} a_1 a_2)(\boldsymbol{\nu} b_1 b_2)(a_1(x).\ \overline{b_1}\langle x \rangle \mid \overline{a_2}\langle \mathbf{n} \rangle.\ b_2(z))$$

Omitting some details, the procedure rewrites P into either:

1. $(\boldsymbol{\nu} a)(\boldsymbol{\nu} b)(a(x).\ \overline{b}(w).([w \leftrightarrow x] \mid \mathbf{0}) \mid \overline{a}\langle \mathbf{n} \rangle.\mathbf{0} \mid\ b(z).\mathbf{0})$
2. $(\boldsymbol{\nu} a)(\boldsymbol{\nu} b)(a(x).\mathbf{0} \mid\ \overline{b}\langle x \rangle.\mathbf{0} \mid \overline{a}\langle v \rangle.([v \leftrightarrow \mathbf{n}] \mid b(z).\mathbf{0}))$

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Concluding Remarks

- The first **formal comparison** between two behavioural type systems that enforce deadlock freedom for π-calculus processes
- We study $\mathcal{L}$ and $\mathcal{K}$, the deadlock-free languages induced by
  - Session types based on linear logic [Caires, Pfenning et al; Wadler]
  - Usage types [Kobayashi et al]
- We identify the **degree of sharing** as a subtle issue that determines new hierarchies in deadlock-free, session processes

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Future Work

- Processes with infinite behaviour
- Semantic characterizations of degree of sharing (e.g., preorders)
- Comparisons/integration with very recent work on static deadlock detection and resolution [Giunti, Francalanza & Ravara, WWV 2015]

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Encodings of Processes and Types [DGS12]

Processes. The structure of a session-typed process is mimicked by sending its continuation as a payload over a channel. E.g.:

$$\begin{array}{rcl} [\![\overline{x}\langle v\rangle.P]\!]_f & \triangleq & (\boldsymbol{\nu} c)\overline{f_x}\langle v, c\rangle.[\![P]\!]_{f,\{x\mapsto c\}} \\ [\![x(y).P]\!]_f & \triangleq & f_x(y,c).[\![P]\!]_{f,\{x\mapsto c\}} \\ [\![(\boldsymbol{\nu} xy)P]\!]_f & \triangleq & (\boldsymbol{\nu} c)[\![P]\!]_{f,\{x,y\mapsto c\}} \end{array}$$

Types. $\bullet$ denotes $\mathbf{1}$ or $\perp$. Let $[\![\Gamma, x : T]\!]_f \triangleq [\![\Gamma]\!]_f, f_x : [\![T]\!]_{\mathsf{su}}$ and

$$\begin{array}{rcl} [\![\mathbf{end}]\!]_{\mathsf{su}} & = & \emptyset[] \\ [\![?T.S]\!]_{\mathsf{su}} & = & ?^o_\kappa[[\![T]\!]_{\mathsf{su}}, [\![S]\!]_{\mathsf{su}}] \\ [\![!T.S]\!]_{\mathsf{su}} & = & !^o_\kappa[[\![T]\!]_{\mathsf{su}}, [\![\overline{S}]\!]_{\mathsf{su}}] \end{array} \qquad \begin{array}{rcl} [\![\mathbf{end}]\!]_{\mathsf{c}} & = & \bullet \\ [\![?T.S]\!]_{\mathsf{c}} & = & [\![T]\!]_{\mathsf{c}} \invamp [\![S]\!]_{\mathsf{c}} \\ [\![!T.S]\!]_{\mathsf{c}} & = & [\![\overline{T}]\!]_{\mathsf{c}} \otimes [\![S]\!]_{\mathsf{c}} \end{array}$$

Lemma. Duality and encoding of session types

(i) $\overline{T} = S$ iff $\overline{[\![T]\!]_{\mathsf{c}}} = [\![S]\!]_{\mathsf{c}}$; (ii) $\overline{T} = S$ iff $\overline{[\![T]\!]_{\mathsf{su}}} = [\![S]\!]_{\mathsf{su}}$.