Type Systems for Distributed Programs: Components and Sessions

by Ornella Dardha
PhD Supervisor: Davide Sangiorgi

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Gentle Intro:
Features of Distributed Systems

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- Systems are heterogeneous and open: pieces using different infrastructures and only partial knowledge of the system.
- Systems are designed as structured composition of computational units called components.
- Giving rise to Component-Based Ubiquitous Systems (CBUS)
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\begin{enumerate}
\item \textbf{Reliability}: Systems need to account for \textit{safe dynamic reconfiguration}, namely changing at runtime the communication patterns.
\item \textbf{Usability}: Components perform \textit{communication} among each-other, following predefined \textit{patterns} or \textit{protocols}.
\end{enumerate}
Problem Description

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1. Guaranteeing consistency of dynamic reconfigurations is a challenging task. It is difficult to ensure that modifications will not disrupt ongoing communications.

2. Guaranteeing safety of communications means a collection of several requirements.
   - privacy
   - communication safety
   - deadlock-freedom
   - livelock-freedom
Aim of the Ph.D. Dissertation

To develop powerful techniques based on formal methods for the verification of correctness, consistency and safety properties related to dynamic reconfigurations and communications in complex distributed systems.
Static analysis based on Types and Type Systems. Why?

1. Types and Type Systems for safety properties.
   - concurrent programming: types for processes in the $\pi$-calculus
   - guarantee deadlock-freedom, livelock-freedom.

2. Types and Type Systems for communication.
   - ranging from standard channel types to behavioural types, like session types.
   - guarantee privacy, communication safety, session fidelity.
Static analysis based on **Types and Type Systems**. Why?

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Approach

Static analysis based on **Types and Type Systems**. Why?

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   - ranging from *standard channel types* to *behavioural types*, like *session types*.
   - guarantee privacy, communication safety, session fidelity.
Contribution of the Ph.D. Dissertation

i) We design a type system for a component-based calculus, to statically ensure consistency of dynamic reconfigurations.

ii) We define an encoding of the π-calculus with session types into the standard typed π-calculus, to understand the expressive power of session types.

iii) We relate the notions of deadlock-freedom, livelock-freedom, progress defined in different calculi via the encoding.
Importance of the Contribution

i) Type System for Components:
   1. Guarantees safe dynamic reconfiguration.
   2. Shifts checks from runtime to compile time.

ii) Encoding of Session $\pi$-calculus:
   1. Reusability of existing theory of the standard typed $\pi$-calculus.
   2. Robustness by subtyping, polymorphism, HO and recursion.
   3. Expressivity result for session types: not many results on types.

iii) Progress by Encoding:
   1. Gives a systematic way of understanding the notions of deadlock-freedom, livelock-freedom, progress.
   2. Encoding relates notions defined in different calculi.


In the remainder...

- Safe Dynamic Reconfiguration
- Safe Communication by Encoding
- Progress of Communication
Safe Dynamic Reconfiguration
Component-Based Calculus

- **Asynchronous Object Communication**
  - Asynchronous method calls: \( x = o!m(args) \)
  - Primitives to test and fetch the returned value.

- **Concurrent Object Groups - cog**
  - Cooperating Objects sharing the processor; only one task active at time.
  - A group’s activity consists of a set of tasks, created by asynchronous method calls on objects of the group;
  - `new cog C()` creates a new object in a new group.

- **Dynamic Reconfiguration**
  - `rebind o.p = o'` operation of ports of objects.
Client $c_1 = \text{new} \ \text{Client} \ (s)$;
Client $c_2 = \text{new cog} \ \text{Client} \ (s)$;
Ctrl $c = \text{new} \ \text{Ctrl}(c_1,c_2)\!\text{updateServer}(s_{\text{new}})$;

...
A Type System for Components

• Goal of the Type System
  1. check `rebind` performed internally to a cog.
  2. check `synchronous method call` performed internally to a cog.

• How do we do it?
  Statically track cogs identity and membership to a cog.
Component-Based Calculus in Practice:
Clients, Server and Controller

... 
Client \(c_1 = \text{new} \) Client \((s)\); ← G
Client \(c_2 = \text{new cog} \) Client \((s)\); ← G'
Ctrl \(c = \text{new} \) Ctrl\((c_1, c_2)\)!updateServer\((s_{\text{new}})\); ← G

... 
Unit updateServer\((\text{Server} \ s_{\text{new}}) \) {
  \text{rebind} \ c_1.s = s_{\text{new}}; \\
  \text{rebind} \ c_2.s = s_{\text{new}};
}

Component-Based Calculus in Practice: Clients, Server and Controller

... 
Client $c_1 = \textbf{new} \ \text{Client} \ (s); \leftarrow G$
Client $c_2 = \textbf{new cog} \ \text{Client} \ (s); \leftarrow G'$
Ctrl $c = \textbf{new} \ \text{Ctrl}(c_1,c_2) \! \text{updateServer}(s_{new}); \leftarrow G$

...

\begin{verbatim}
Unit updateServer(Server $s_{new}$) {
    rebind $c_1.s = s_{new}; \ \text{ok}$
    rebind $c_2.s = s_{new}$;
}
\end{verbatim}
Component-Based Calculus in Practice: Clients, Server and Controller

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...  
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\texttt{  rebind } \ c_1.s = s_{new}; \ \texttt{ok}  
\texttt{  rebind } \ c_2.s = s_{new}; \ \texttt{x}  
\}
Properties of the Type System for Safe Dynamic Reconfiguration

Theorem (Main Result)

Well-typed programs do not perform

i) illegal rebinding

ii) illegal synchronous method call
Safe Communication by Encoding
Session Types in Practice: Equality Test

server $\text{def} = x?(nr1).x?(nr2).x!(nr1 == nr2).0$

client $\text{def} = y!(3).y!(5).y?(eq).0$

The system is given by

$$(\nu xy) (\text{server} \mid \text{client})$$
Session Types in Practice: Equality Test

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$$(\nu x y) (\text{server} \mid \text{client})$$

Where

$x : {?}\text{Int.}?\text{Int.}!\text{Bool.end}$

and

$y : !\text{Int.}!\text{Int.}?\text{Bool.end}$
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$y : !\text{Int.}!\text{Int.}?\text{Bool}.\text{end}$
Session Types vs. Standard $\pi$-Types

- **Session types** are structured $x : \text{?Int} \cdot \text{?Int} ! \text{Bool}. \text{end}$;

- **Standard $\pi$-channel types** specify the type of the carried value: $x : \ell_i[\text{Int}]$ or $x : \ell_o[\text{Int}]$.

- **Encoding** is based on:
  1. Linearity of $\pi$-calculus channel types;
  2. Input/Output channel capabilities;
  3. Continuation-Passing principle.
Encoding Session Types

Let

\[ S = \text{?Int.?Int.!Bool.end} \]

Then

\[ [S] = \ell_i[\text{Int}, \ell_i[\text{Int}, \ell_o[\text{Bool}, \emptyset[]]]] \]
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Properties of the Encoding

Theorem (On types)

*Encoding preserves typability of programs.*
Properties of the Encoding

Theorem (On types)

*Encoding preserves typability of programs.*

Theorem (On reductions)

*Encoding preserves evaluation of programs.*
Advanced Features on Safety by Encoding

Does the encoding handle extensions? Extend the calculi with:

- Subtyping
- Polymorphism
- Higher-Order
- Recursion
Advanced Features on Safety by Encoding

Does the encoding handle extensions? Extend the calculi with:

- Subtyping
- Polymorphism
- Higher-Order
- Recursion

Theorems ‘On types’ and ‘On reductions’ still hold.
Progress of Communication
Comparing Properties of Communication

- **Deadlock-Freedom**: communications eventually succeed, *unless the whole process diverges*. (Standard $\pi$)

- **Livelock-Freedom**: communications eventually succeed even if the whole process diverges. (Standard $\pi$)

- **Progress**: each *session*, once started, is guaranteed to satisfy all the requested interactions. (Session $\pi$)
What can we say about Progress?

Theorem

*Progress is a compositional form of livelock-freedom property.*

- We use the encoding to relate progress in the session $\pi$-calculus to livelock-freedom in the standard $\pi$-calculus.
- Reusability of type system and tools for livelock-freedom.
- More accurate analysis of the progress property.
Progress in Practice: “Bad” Process

Consider

$$(\nu ab)(\nu cd)( a?(z).d!z \mid c?(w).b!w )$$

By encoding we obtain the process:

$$((\nu x)(\nu y)( x?z . y!z \mid y?w . x!w ))$$

The type system for livelock-freedom rejects it!
Progress in Practice: “Bad” Process

Consider

$$(\nu ab)(\nu cd)(a?(z).d!\langle z \rangle \mid c?(w).b!\langle w \rangle)$$

By encoding we obtain the process:

$$(\nu x)(\nu y)(x?(z).y!\langle z \rangle \mid y?(w).x!\langle w \rangle)$$

The type system for livelock-freedom rejects it!
Progress in Practice: “Good” Process

Consider the process

\[
(\nu_{ab})(b!(1) \mid (\nu_{cd})(d!(1) \mid c?(y).a?(z)))
\]
Progress in Practice: “Good” Process

Consider the process

\[(\nu_{ab})(b!\langle 1 \rangle \mid (\nu_{cd})(d!\langle 1 \rangle \mid c?(y).a?(z)))\]

By the encoding we obtain the process:

\[(\nu_k)(k!\langle 1 \rangle \mid (\nu_t)(t!\langle 1 \rangle \mid t?(y).k?(z)))\]

The type system for livelock-freedom accepts it!
Conclusions and Future Work 1/2

Problem: guaranteeing consistency and safety properties in distributed programs.

Approach: types and type systems.

i) Type system for safe dynamic reconfiguration in a concurrent object-oriented language for distributed systems.

ii) Encoding of session $\pi$-calculus into standard typed $\pi$-calculus permitting large reusability of existing theory and properties.

iii) Progress in session $\pi$-calculus as livelock-freedom in standard typed $\pi$-calculus via encoding.
Conclusions and Future Work 1/2

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• **Problem**: guaranteeing **consistency** and **safety** properties in distributed programs.

• **Approach**: types and **type systems**.
   
i) **Type system** for **safe dynamic reconfiguration** in a concurrent object-oriented language for distributed systems.

ii) **Encoding** of session $\pi$-calculus into standard typed $\pi$-calculus permitting **large reusability** of existing theory and properties.

iii) **Progress** in session $\pi$-calculus as **livelock-freedom** in standard typed $\pi$-calculus via **encoding**.
Conclusions and Future Work 2/2

- **Type System for Components** relevant in practice: designed for component-extension of ABS used in HATS and Envisage.

- **Encoding of Session Types** relevant for BETTY and ABCD.

- Extend the encoding to more general settings than dyadic session types, in particular multiparty session types.

- **Session Types in Practice (ABCD)**

- **Tool** for progress property in session types. Progress in more general settings.
Thank You!!
During my PhD I produced the following 4 papers. The last one [4] is not part of my PhD thesis, as it resulted from my Master’s work.


Component Extension of Core ABS  1/2

\[ P ::= \overline{Dl} \ \{ \ s \} \]

\[ Dl ::= D \ | \ F \ | \ I \ | \ C \]

\[ T ::= V \ | \ D[\langle T\rangle] \ | \ (I, r) \]

\[ r ::= \perp \ | \ G[f : T] \ | \ \alpha \ | \ \mu\alpha.r \]

\[ D ::= \text{data} \ D[\langle T\rangle] = \text{Co}[(\overline{T})]|\text{Co}[(\overline{T})]; \]

\[ F ::= \text{def} \ T \ \text{fun}[\langle T\rangle](\overline{T} \ x) = e; \]

\[ I ::= \text{interface} \ I \ [\text{extends} \ \overline{I}] \ \{ \ \text{port} \ \overline{T} \ x; \overline{S} \ \} \]

\[ C ::= \text{class} \ C[(\overline{T} \ x)] \ [\text{implements} \ \overline{I}] \ \{ \ \overline{Fl} \ \overline{M} \ \} \]

\[ Fl ::= [\text{port}] \ \overline{T} \ x \]

\[ S ::= [\text{critical}] \ (G, r) \ T \ m(\overline{T} \ x) \]

\[ M ::= S \ \{ \ s \} \]
Component Extension of Core ABS 1/2

\[
\begin{align*}
  s & ::= \text{skip} \mid s ; s \mid T x \mid x = z \mid \text{await } g \\
  & \quad \mid \text{if } e \text{ then } s \text{ else } s \mid \text{while } e \{ s \} \mid \text{return } e \\
  & \quad \mid \text{rebind } e.p = z \mid \text{suspend} \\
  z & ::= e \mid \text{new [cog] } C(\bar{e}) \mid e.m(\bar{e}) \mid e!m(\bar{e}) \mid \text{get}(e) \\
  e & ::= v \mid x \mid \text{fun}(\bar{e}) \mid \text{case } e \{ p \Rightarrow e_p \} \mid \text{Co}[(\bar{e})] \\
  v & ::= \text{true} \mid \text{false} \mid \text{null} \mid \text{Co}[(\bar{v})] \\
  p & ::= _ \mid x \mid \text{null} \mid \text{Co}[(\bar{p})] \\
  g & ::= e \mid e? \mid \| e \| \mid g \land g
\end{align*}
\]
Standard $\pi$-types

$$\tau ::= \emptyset[\overline{T}] \quad \text{channel with no capability}$$
$$l_i[\overline{T}] \quad \text{linear input}$$
$$l_o[\overline{T}] \quad \text{linear output}$$
$$l_\#[\overline{T}] \quad \text{linear connection}$$

$$T ::= \tau \quad \text{linear channel type}$$
$$\langle l_i - T_i \rangle_{i \in I} \quad \text{variant type}$$
$$\# T \quad \text{standard channel type}$$
$$\text{Bool} \quad \text{boolean type}$$
$$\ldots \quad \text{other constructs}$$
Session Types

\[ q ::= \text{lin} \mid \text{un} \quad \text{qualifiers} \]

\[ p ::= !T.U \quad \text{send} \]
\[ ?T.U \quad \text{receive} \]
\[ \oplus \{l_i : T_i\}_{i \in I} \quad \text{select} \]
\[ \&\{l_i : T_i\}_{i \in I} \quad \text{branch} \]

\[ T ::= q \ p \quad \text{qualified pretype} \]
\[ \text{end} \quad \text{termination} \]
\[ \text{Bool} \quad \text{boolean type} \]
Encoding of session types

\[
\begin{align*}
[\text{end}] & \overset{\text{def}}{=} \emptyset \quad \text{(E-END)} \\
[! T \cdot U] & \overset{\text{def}}{=} \ell_o[[T], [U]] \quad \text{(E-OUT)} \\
[? T \cdot U] & \overset{\text{def}}{=} \ell_i[[T], [U]] \quad \text{(E-INP)} \\
[\oplus\{l_i : T_i\}_{i \in I}] & \overset{\text{def}}{=} \ell_o[\langle l_i - [T_i]\rangle_{i \in I}] \quad \text{(E-SELECT)} \\
[\&\{l_i : T_i\}_{i \in I}] & \overset{\text{def}}{=} \ell_i[\langle l_i - [T_i]\rangle_{i \in I}] \quad \text{(E-BRANCH)}
\end{align*}
\]
Encoding of session processes

\[
\begin{align*}
[0]_f & \overset{\text{def}}{=} 0 \\
[x!(v).P]_f & \overset{\text{def}}{=} (\nu c)f_x!(v, c).[P]_f,\{x\mapsto c\} \\
[x?(y).P]_f & \overset{\text{def}}{=} f_x?(y, c).[P]_f,\{x\mapsto c\} \\
[x \triangleright l_j.P]_f & \overset{\text{def}}{=} (\nu c)f_x!(l_j-c).[P]_f,\{x\mapsto c\} \\
[x \triangleright \{l_i : P_i\}_{i \in I}]_f & \overset{\text{def}}{=} f_x?(y). \text{ case } y \text{ of } \{l_i-c \triangleright [P_i]_f,\{x\mapsto c\}\}_{i \in I} \\
[\text{if } v \text{ then } P \text{ else } Q]_f & \overset{\text{def}}{=} \text{ if } f_v \text{ then } [P]_f \text{ else } [Q]_f \\
[P \mid Q]_f & \overset{\text{def}}{=} [P]_f \mid [Q]_f \\
[(\nu xy)P]_f & \overset{\text{def}}{=} (\nu c)[P]_f,\{x,y\mapsto c\}
\end{align*}
\]
Subtyping in standard $\pi$-calculus

\[
\begin{align*}
T &\leq T \quad (S_{\pi-} \text{REFL}) \\
T \leq T' &\quad T' \leq T'' \quad (S_{\pi-} \text{TRANS}) \\
\tilde{T} \leq \tilde{T'} &\quad \ell_i[\tilde{T}] \leq \ell_i[\tilde{T}'] \quad (S_{\pi-} \text{ii}) \\
\tilde{T}' \leq \tilde{T} &\quad \ell_o[\tilde{T}] \leq \ell_o[\tilde{T}'] \quad (S_{\pi-} \text{oo}) \\
I \subseteq J &\quad T_i \leq T'_j \quad \forall i \in I \\
\langle l_i - T_i \rangle_{i \in I} &\leq \langle l_j - T'_j \rangle_{j \in J} \quad (S_{\pi-} \text{VARIANT})
\end{align*}
\]
Polymorphism

Example of polymorphism in the $\pi$-calculus with/without sessions:

$$x : !\langle X; D \rangle . \text{end}, \ y : ?\langle X; D \rangle . \text{end}$$

$$\vdash x!\langle \text{Int}; 5 \rangle | y?(z). \ \text{open} \ z \ \text{as} \ (X; w) \ \text{in} \ nj!\langle w \rangle$$

$$\rightarrow \ \text{open} \ \langle \text{Int}; 5 \rangle \ \text{as} \ (X; w) \ \text{in} \ nj!\langle w \rangle$$

$$\rightarrow \ nj!\langle 5 \rangle$$
Semantics of Bounded Polymorphism

\[(\nu xy)(x \triangleleft l_j(B).P \mid y \triangleright \{ l_i(X_i \triangleleft B_i) : P_i \}_{i \in I} \mid R) \rightarrow (\nu xy)(P \mid P_j[B/X_j] \mid R) \quad j \in I\]

\textbf{case} l_j(B) \_ v \textbf{ of } \{ l_i(X_i \leq B_i) \_ x_i \triangleright P \}_{i \in I} \rightarrow P_j[B/X_j][v/x_j] \quad j \in I
Higher-order constructs

\[\sigma ::= T\]  \hspace{1cm} \text{general type}

\[\Diamond\]  \hspace{1cm} \text{process type}

\[T ::= \text{Unit}\]  \hspace{1cm} \text{unit type}

\[T \rightarrow \sigma\]  \hspace{1cm} \text{functional type}

\[T \rightarrow^1 \sigma\]  \hspace{1cm} \text{linear functional type}

\[P ::= PQ\]  \hspace{1cm} \text{application}

\[v\]  \hspace{1cm} \text{values}

\[v ::= \lambda x : T.P\]  \hspace{1cm} \text{abstraction}

\[\star\]  \hspace{1cm} \text{unit value}
Encoding Higher-Order

\[
\begin{align*}
\llbracket T \rightarrow^1 \sigma \rrbracket & \overset{\text{def}}{=} \llbracket T \rrbracket \rightarrow^1 \sigma \\
\llbracket T \rightarrow \sigma \rrbracket & \overset{\text{def}}{=} \llbracket T \rrbracket \rightarrow \sigma \\
\llbracket \lambda x : T . P \rrbracket_f & \overset{\text{def}}{=} \lambda x : \llbracket T \rrbracket . \llbracket P \rrbracket_f \\
\llbracket P Q \rrbracket_f & \overset{\text{def}}{=} \llbracket P \rrbracket_f \llbracket Q \rrbracket_f 
\end{align*}
\]

Where \( \sigma ::= T \mid \diamond \)
On progress for sessions

Definition (Progress)

A process $P$ has progress if for all $C[\cdot]$ such that $C[P]$ is well-typed, $C[P] \rightarrow^* E[R]$ (where $R$ is an input or an output) implies that there exist $C'[\cdot]$, $E'[\cdot][\cdot]$ and $R'$ such that $C'[E[R]] \rightarrow^* E'[R][R']$ and $R \Join_{\{x,y\}} R'$ for some $x$ and $y$ such that $(\nu xy)$ is a restriction in $C'[E[R]]$. 
Results for Progress

Theorem (Progress ⇔ Lock-freedom)
Let $P$ be a well-typed closed process. Then $P$ is livelock-free if and only if $P$ has progress.

Theorem (Progress ⇔ Closed Lock-Free)
If $P$ is well-typed then $P$ has progress if and only if $\text{close}(P)$ is livelock-free.
1: procedure PROGRESS(Γ, P)
2:     Check Γ ⊢ P
3:     Build close(P) from Γ
4:     Encode [close(P)]_f = P'
5:     return TyPiCal(P')
6: end procedure