(2,4) TREES

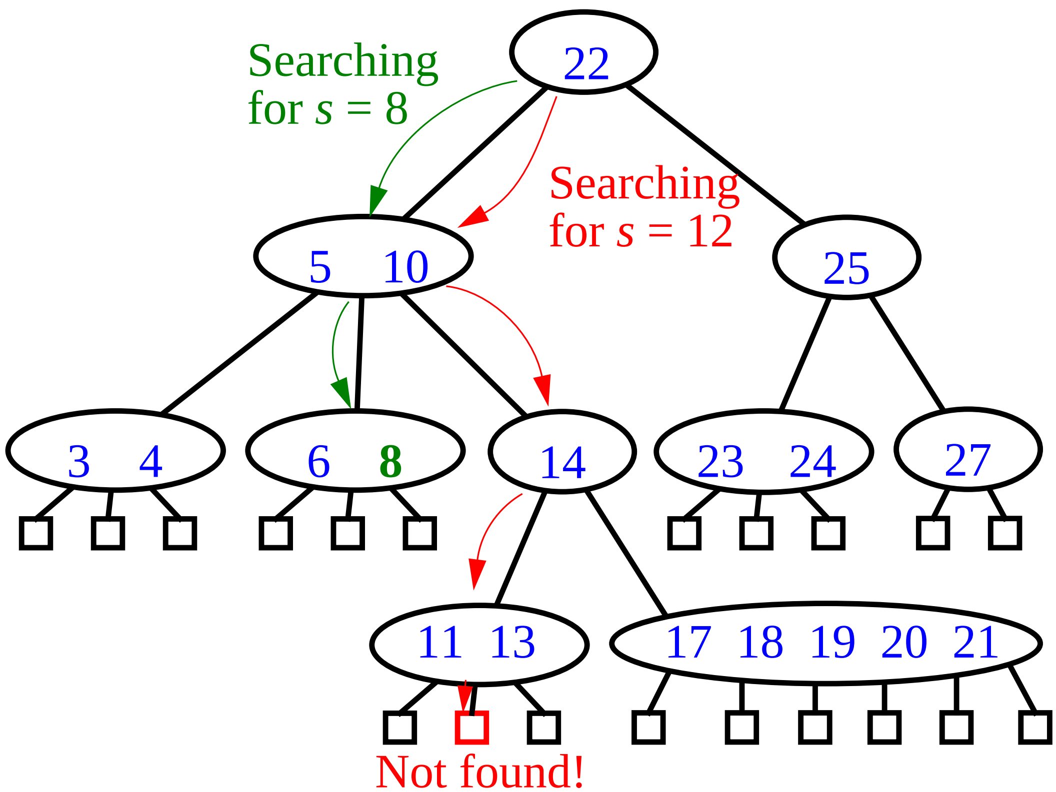
- Search Trees (but not binary)
- also known as 2-4, 2-3-4 trees
- very important as basis for Red-Black trees (so pay attention!)

Multi-way Search Trees

- Each internal node of a multi-way search tree T:
  - has at least two children
  - stores a collection of items of the form (k, x), where k is a key and x is an element
  - contains d - 1 items, where d is the number of children
  - “contains” 2 pseudo-items: $k_0 = -\infty$, $k_d = \infty$
- Children of each internal node are “between” items
  - all keys in the subtree rooted at the child fall between keys of those items
- External nodes are just placeholders

Multi-way Searching

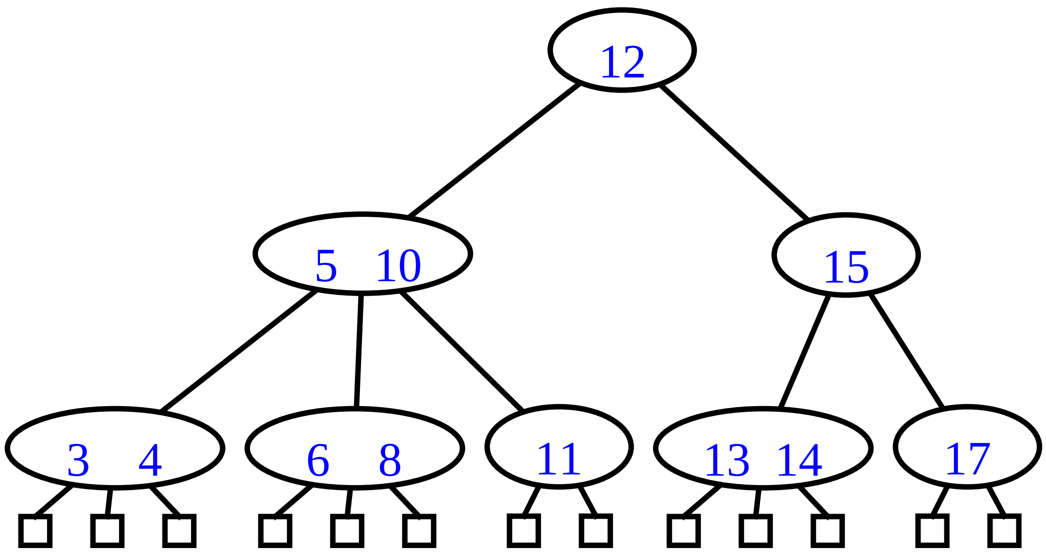
- Similar to binary searching
- If search key $s < k_1$, search the leftmost child
- If $s > k_{d-1}$, search the rightmost child
- That's it in a binary tree; what about if $d > 2$?
- Find two keys k_{i-1} and k_i between which s falls, and search the child v_i.



- What would an in-order traversal look like?

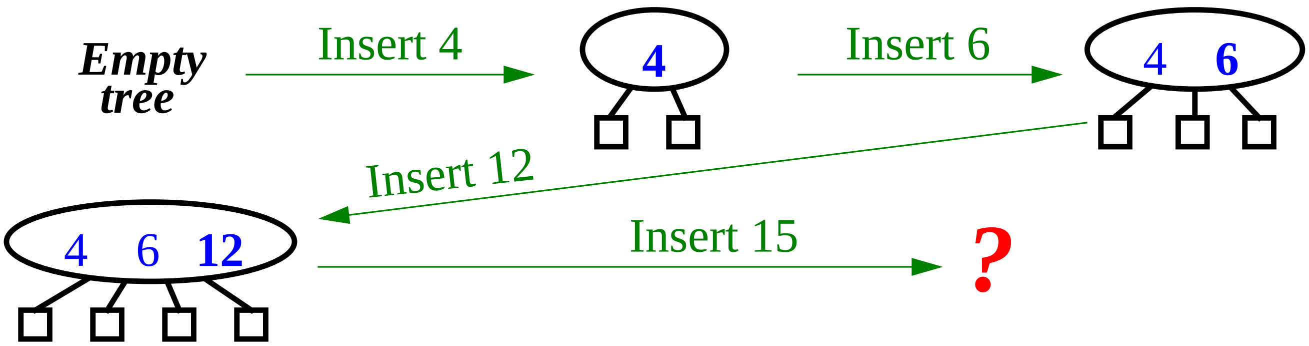
(2,4) Trees

- At most 4 children
- All external nodes have same depth
- Height h of (2,4) tree is $O(\log n)$.
- How is this fact useful in searching?



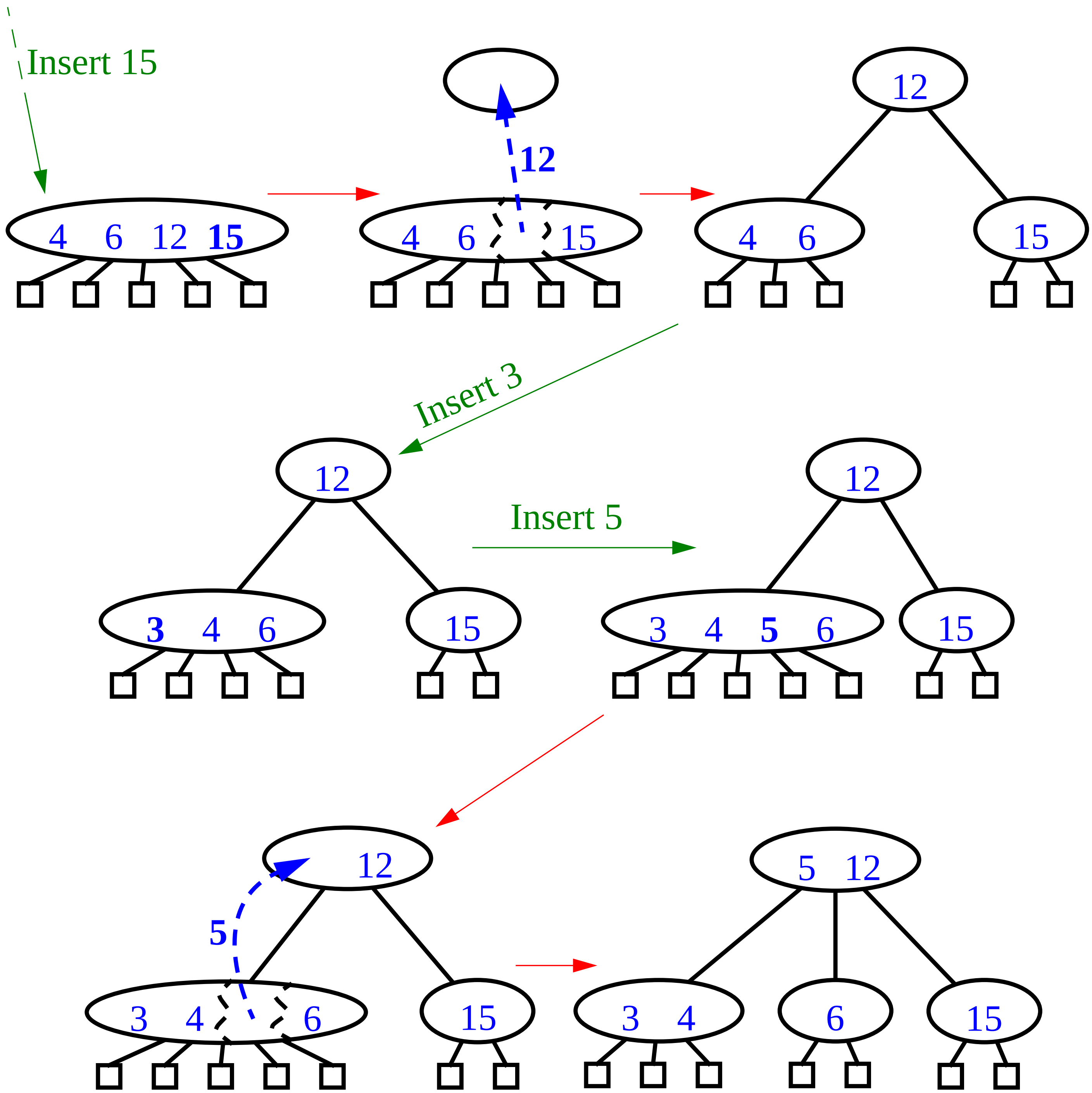
(2,4) Insertion

- Always maintain depth condition
- Add elements only to existing nodes



- What if that makes a node too big?
  - *overflow*
- Must perform a *split* operation
  - replace node *v* with two nodes *v*' and *v*''
  - *v*' gets the first two keys
  - *v*'' gets the last key
  - send the other key up the tree
    - if *v* is root, create new root with third key
    - otherwise just add third key to parent
- Much clearer with a few pictures...

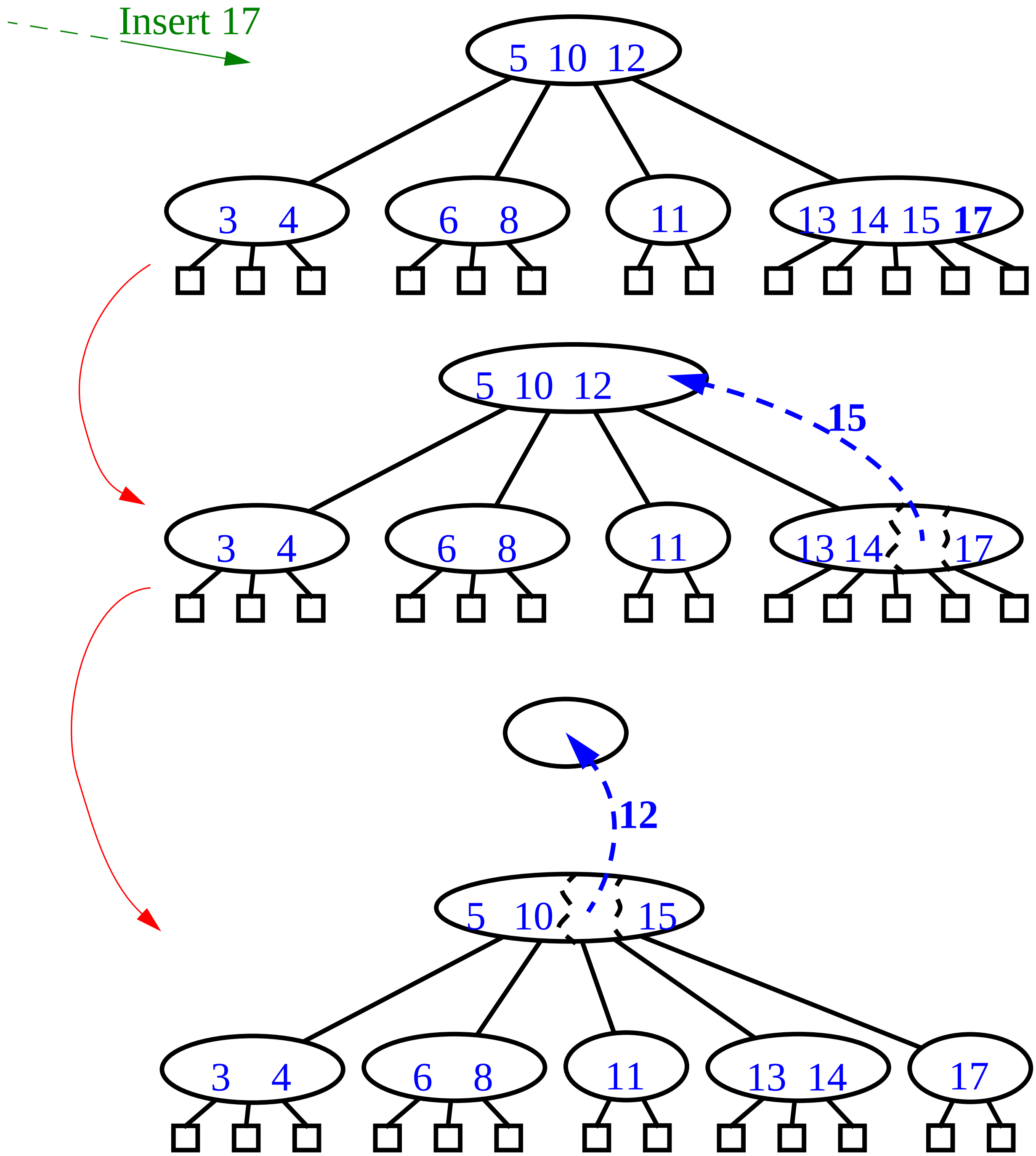
(2,4) Insertion (cont.)



- Tree always grows from the top, maintaining balance
- What if parent is full?

(2,4) Insertion (cont.)

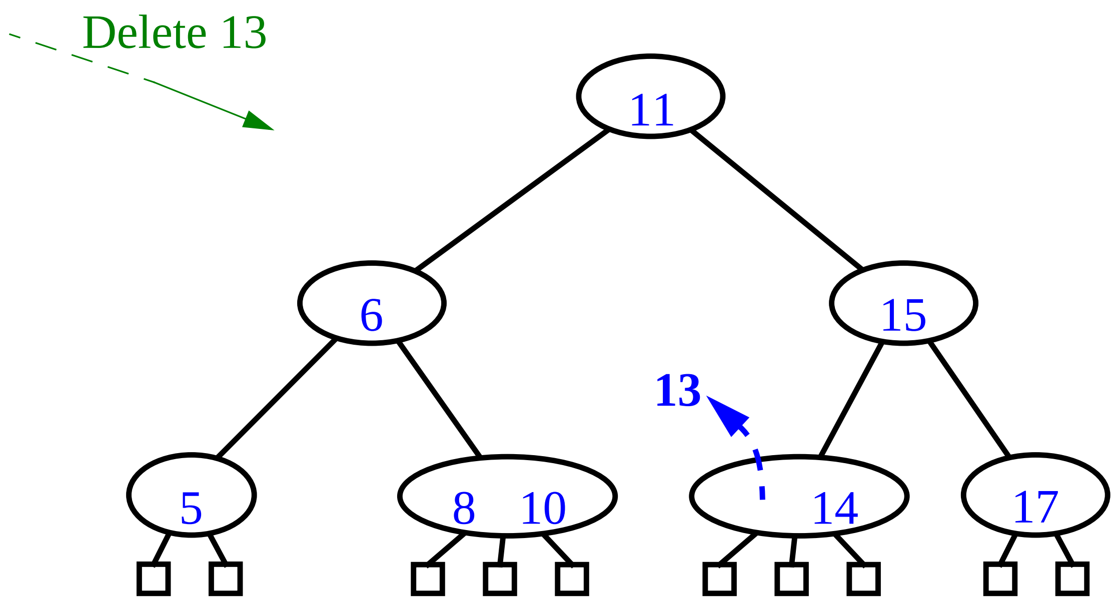
- Do the same thing:



- Overflow cascade all the way up to the root
  - still at most $O(\log n)$

(2,4) Deletion

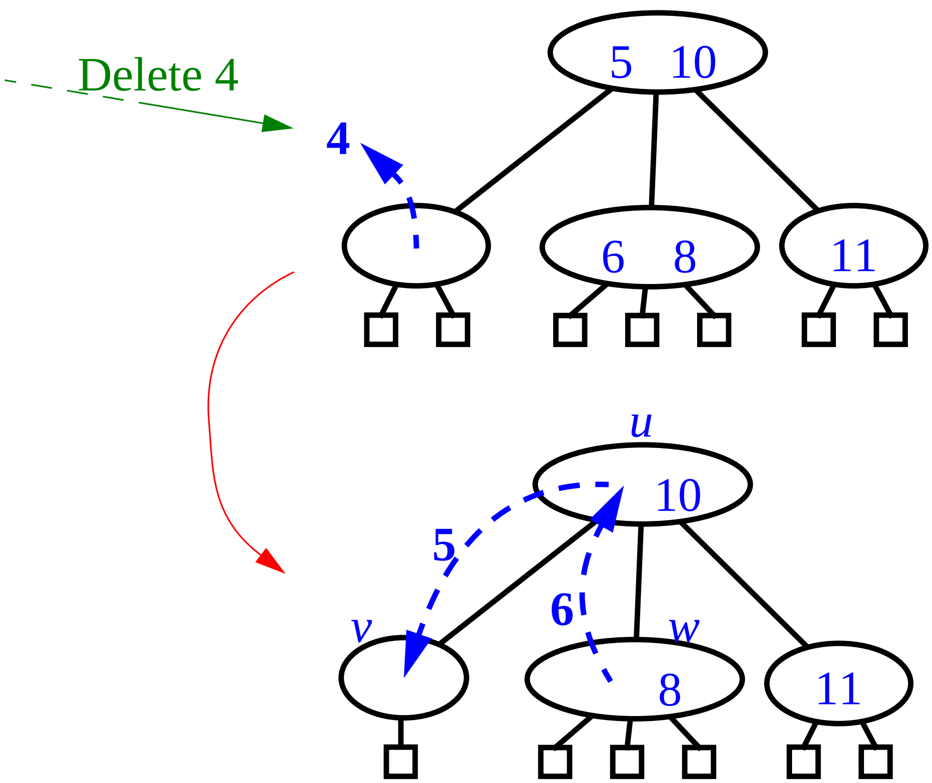
- A little trickier
- First of all, find the key
  - simple multi-way search
- Then, reduce to the case where deletable item is at the bottom of the tree
  - Find item which precedes it in in-order traversal
  - Swap them
- Remove the item



- Easy, right?
- ...but what about removing from 2-nodes?

(2,4) Deletion (cont.)

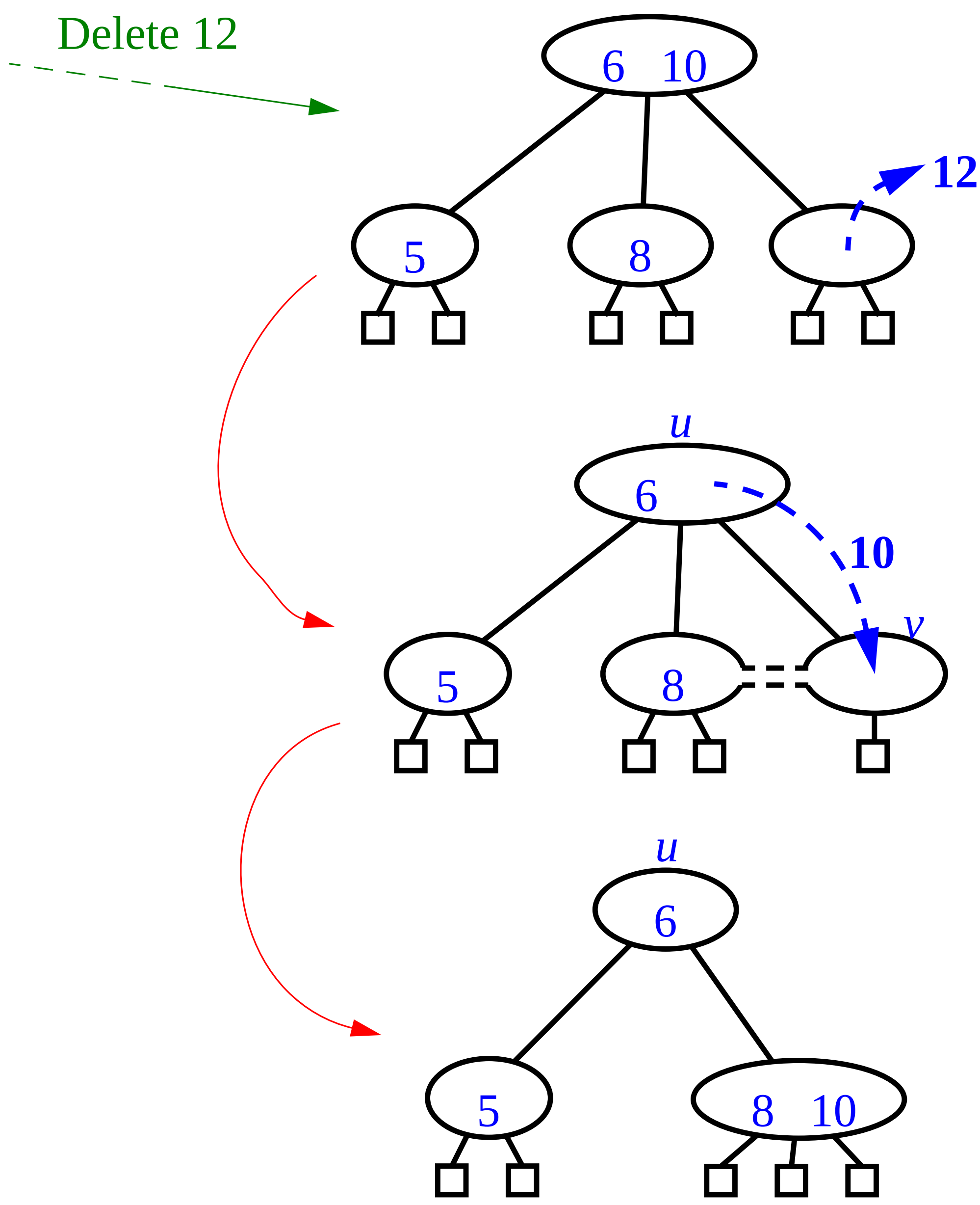
- Not enough items in the node
  - *underflow*
- Pull an item from the parent, replace it with an item from a sibling
  - called *transfer*



- Still not good enough! What happens if siblings are 2-nodes?
- Could we just pull one item from the parent?
  - too many children
- But maybe...

(2,4) Deletion (cont.)

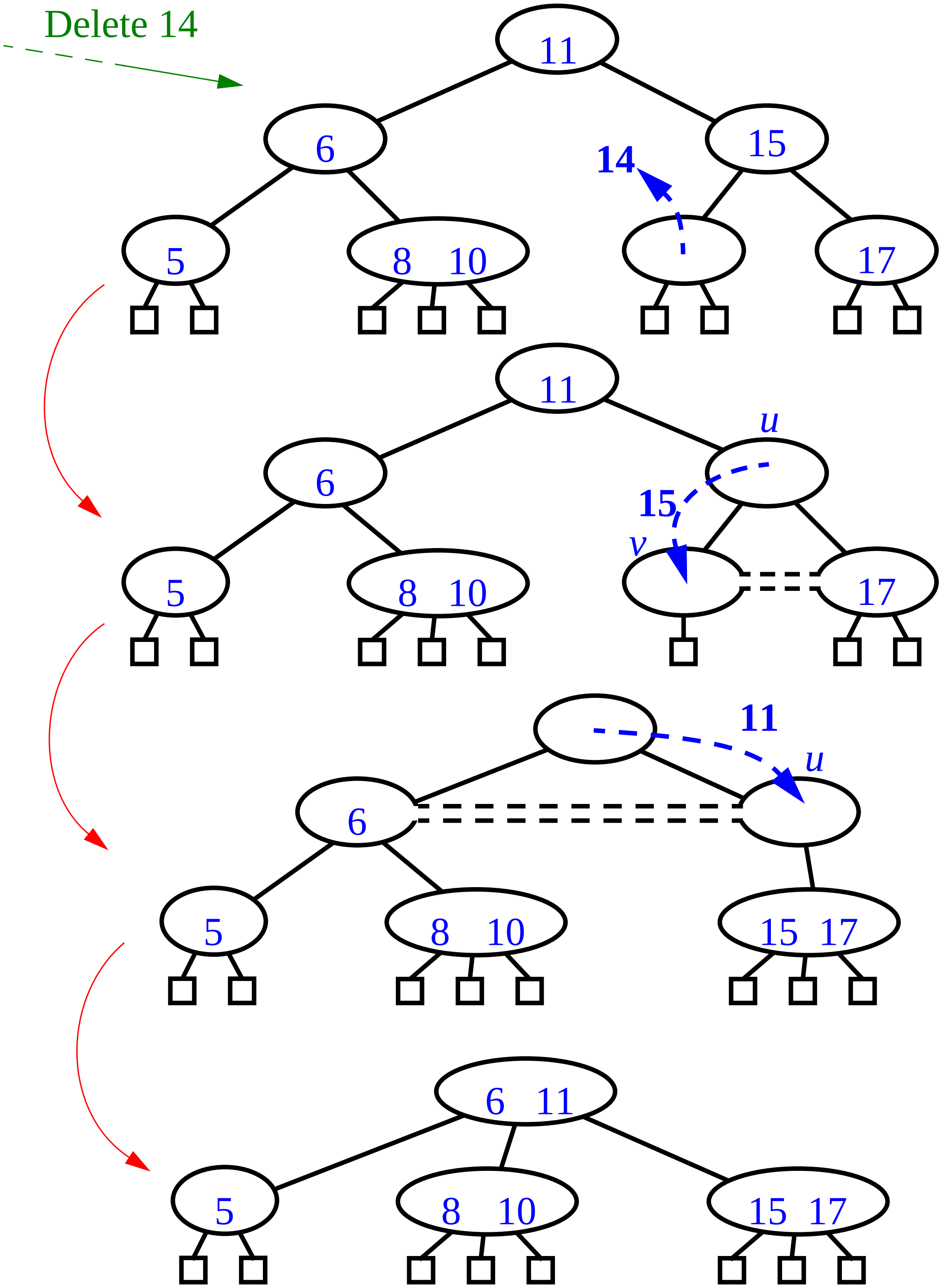
- We know that the node's sibling is just a 2-node
- So we *fuse* them into one
  - after stealing an item from the parent, of course



- Last special case, I promise: what if the parent was a 2-node?

(2,4) Deletion (cont.)

- Underflow can cascade up the tree, too.



(2,4) Conclusion

- The height of a (2,4) tree is $O(\log n)$.
- Split, transfer, and fusion each take $O(1)$.
- Search, insertion and deletion each take $O(\log n)$.
- Why are we doing this?
  - (2,4) trees are fun! Why else would we do it?
  - Well, there's another reason, too.
  - They're pretty fundamental to the idea of Red-Black trees as well.
  - And you're covering Red-Black trees on Monday.
  - Perhaps more importantly, your next project is a Red-Black tree.
- Have a nice weekend!