• the dictionary ADT

• binary search

• binary search trees
The Dictionary ADT

• a dictionary is an abstract model of a database

• like a priority queue, a dictionary stores key-element pairs

• the main operation supported by a dictionary is searching by key

• simple container methods:
  - size()
  - isEmpty()

• query methods:
  - findElement(k)
  - findAllElements(k)

• update methods:
  - insertItem(k, e)
  - remove(k)
  - removeAll(k)

• special object
  - NO_SUCH_KEY, returned by an unsuccessful search
Implementing a Dictionary with a Sequence

• unordered sequence:

  - searching takes $O(n)$ time
  - inserting takes $O(1)$ time

• ordered sequence

  - searching takes $O(1)$ time
  - inserting takes $O(n)$ time

• in the ordered sequence implementation, we can search faster if the sequence is array-based ...
Binary Search

• narrow down the search range in stages
• “high-low” game
• `findElement(22)`
### Pseudo-code for Binary Search

**Algorithm** `BinarySearch(S, k, low, high)`

*if* `low > high` *then*

*return* `NO_SUCH_KEY`

*else*

- `mid ← (low+high) / 2`
- *if* `k = key(mid)` *then*
  - *return* `key(mid)`
- *else if* `k < key(mid)` *then*
  - *return* `BinarySearch(S, k, low, mid−1)`
- *else*
  - *return* `BinarySearch(S, k, mid+1, high)`
Running Time of Binary Search

- the range of candidate items to be searched is halved after comparing the key with the middle element

<table>
<thead>
<tr>
<th>comparison</th>
<th>search range</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>n</td>
</tr>
<tr>
<td>1</td>
<td>n/2</td>
</tr>
<tr>
<td>2</td>
<td>n/4</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>$2^i$</td>
<td>$n/2^i$</td>
</tr>
<tr>
<td>$\log_2 n$</td>
<td>1</td>
</tr>
</tbody>
</table>

- in the array-based implementation, access by rank takes $O(1)$ time, thus binary search runs in $O(\log n)$ time