# Probability

## Events & probabilities:

The **intersection** of two events $A$ and $B$ is $A \cap B$.

The **union** of $A$ and $B$ is $A \cup B$.

Events A and B are **mutually exclusive** if they cannot both occur, denoted $A \cap B = \emptyset$ where $\emptyset$ is called the **null event**.

For any event $A$, $0 \leq P(A) \leq 1$.

For two events $A$ and $B$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

If $A$ and $B$ are mutually exclusive then

$$P(A \cup B) = P(A) + P(B).$$

## Equally likely outcomes:

If a complete set of $n$ elementary outcomes are all equally likely to occur, then the probability of each elementary outcome is $\frac{1}{n}$. If an event $A$ consists of $m$ of these $n$ elements, then $P(A) = \frac{m}{n}$.

## Independent events:

$A$, $B$ are *independent* if and only if

$$P(A \cap B) = P(A)P(B).$$

**Conditional Probability** of $A$ given $B$:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \qquad \text{if} \quad P(B) \neq 0.$$

**Bayes ' Theorem:**

$$P(B|A) = \frac{P(A|B)P(B)}{P(A)}.$$

**Theorem of Total Probability:**

The $k$ events $B_1$, $B_2$, $\ldots B_k$ form a *partition* of the sample space $S$ if $B_1 \cup B_2 \cup B_3 \ldots \cup B_k = S$ and no two of the $B_i$'s can occur together. Then $P(A) = \sum_i P(A|B_i)P(B_i)$. In this case Bayes' Theorem generalizes to

$$P(B_i|A) = \frac{P(A|B_i)P(B_i)}{\sum_j P(A|B_j)P(B_j)} \qquad (i = 1, 2, \ldots k)$$

If $B'$ is the *complement* of the event $B$ ,

$$P(B') = 1 - P(B)$$

and

$$P(A) = P(A|B)P(B) + P(A|B')P(B')$$

This is a special case of the theorem of total probability. The complement of the event $B$ is commonly denoted $\overline{B}$.