

## Algorithms

Suppose we have two positive integers  $m$ ,  $n$ , with  $m$  greater than  $n$ . When  $m$  is divided by  $n$ , the result is a whole number part plus a remainder. For example given 16 and 5, then  $\frac{16}{5} = 3$ , remainder 1. The number 3 is called the **quotient**, 1 is called the **remainder**, and 5 is called the **divisor**.

### Algorithm to convert decimal to binary

**Step 1:** Divide the number by 2. Retain the quotient and record the remainder.

**Step 2:** If the quotient in Step 1 is 0 then stop.

**Step 3:** If the quotient in Step 1 is not 0 go to Step 1 using the quotient as the number which is divided by 2.

The binary representation of the initial decimal number is given by the remainders in the reverse order to that in which they were obtained.

### Euclid's Algorithm for the Greatest Common Divisor of two positive integers $a$ and $b$ , $\text{GCD}(a, b)$

**Step 1:** Divide the larger of the two integers by the smaller.

**Step 2:** If the remainder is zero then stop, the  $\text{GCD}(a, b)$  is the divisor.

**Step 3:** If the remainder is not zero then divide the divisor by the remainder and go to Step 2.

## Prim's Algorithm for the minimum spanning tree in a network of $n$ vertices.

**Step 1:** Choose any vertex. Choose the edge of shortest length incident on this vertex. Call this graph  $P$ .

**Step 2:** Choose the edge  $(i, j)$  with the shortest length amongst all the edges  $(i, k)$  where  $i$  is in  $P$  and  $k$  is not in  $P$ . Add this edge to  $P$ . (If there are multiple edges of the same shortest length then choose one of them arbitrarily.)

**Step 3:** If  $P$  has  $n - 1$  edges then stop - it is a minimal spanning tree, otherwise go to Step 2.

## Binary Search Algorithm to find an element $x$ in an ordered list $L$ made up of $n$ elements $a_1 < a_2 < \dots < a_n$ .

**Step 1:** Check if  $x$  is greater than the middle element of the list  $L$ . If this is true then set this upper half of the list to be the new search list  $L$ . If false set the lower half of the list to be the new search list.

**Step 2:** If there is only one element  $a_L$  remaining in the list then stop. If  $x = a_L$  the element is found. If  $x \neq a_L$  the element is not in the list.

**Step 3:** If there is more than one element in the list then go to step 1.

## Bubble Sort Algorithm to arrange an unordered list of $n$ numbers $a_1, a_2, \dots, a_n$ in ascending order.

**Step 1:** Set counter  $j = 2$ .

**Step 2:** From  $i = n$  to  $j$ , if  $a_i < a_{i-1}$  swap  $a_i$  and  $a_{i-1}$ .

**Step 3:** Increase counter value  $j$  by 1.

**Step 4:** If  $j = n$  stop, the list is sorted, otherwise go to step 2.