Complexity Functions

A function $f(n) = O(g(n))$ if there exists a positive real number c such that $|f(n)| \leq c|g(n)|$ for sufficiently large n. More informally, we say that $f(n) = O(g(n))$ if $f(n)$ grows no faster than $g(n)$ does with increasing n. Writing $f(n) \prec g(n)$ indicates that $g(n)$ has greater order than $f(n)$ and hence grows more quickly. The hierarchy of common functions is

$$1 \prec \log(n) \prec n \prec n^k \prec c^n \prec n! \prec n^n$$

where $c, k > 1$.

Combinatorics

The number of ways of selecting k objects out of a total of n where the order of selection is important is the number of permutations:

$${}^nP_k = \frac{n!}{(n-k)!}$$

The number of ways in which k objects can be selected from n when the order of selection is not important is the number of combinations:

$${}^nC_k = \frac{n!}{(n-k)!k!}$$

where $0! = 1, n! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot \ldots \cdot (n-1) \cdot n$.

$$^nC_k = ^nC_{n-k}$$

$$^{n+1}C_k = ^nC_k + ^nC_{k-1}$$

$$^nC_0 + ^nC_1 + \ldots ^nC_{n-1} + ^nC_n = 2^n$$

$$(1+x)^n = ^nC_0 + ^nC_1 x + ^nC_2 x^2 + \ldots + ^nC_{n-1}x^{n-1} + ^nC_n x^n$$

Thus the value of ^nC_k is given by the kth entry in the nth row of Pascal's triangle:

$$\begin{array}{ccccccccccc}
 & & & & & 1 & & & & & \\
 & & & & 1 & & 1 & & & & \\
 & & & 1 & & 2 & & 1 & & & \\
 & & 1 & & 3 & & 3 & & 1 & & \\
 & 1 & & 4 & & 6 & & 4 & & 1 & \\
\vdots & & \vdots & & \vdots & & \vdots & & \vdots & & \vdots
\end{array}$$

where elements are generated as the sum of the two adjacent elements in the preceding line, the top row is designated row 0, and the left-most entry is labelled 0. For example, the 6 in the final row above is in row 4 and is entry 2, since both row and entry counting start at 0, i.e. $^4C_2 = 6$.