Matrices and Determinants

The 2×2 matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ has determinant

$$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

The 3×3 matrix $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$ has determinant

$$|A| = a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

The inverse of a 2×2 matrix

If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ then $A^{-1} = \dfrac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$
provided that $ad - bc \neq 0$.

Matrix multiplication: for 2×2 matrices

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} \alpha & \gamma \\ \beta & \delta \end{pmatrix} = \begin{pmatrix} a\alpha + b\beta & a\gamma + b\delta \\ c\alpha + d\beta & c\gamma + d\delta \end{pmatrix}$$

Remember that $AB \neq BA$ except in special cases.

Binary Relations

A **binary relation**, R, from set A to set B is a subset of the Cartesian Product, $A \times B$. If $(a, b) \in R$ we write aRb.
A binary relation on a set A is a subset of $A \times A$.

For a relation R on a set A:
R is **reflexive** when $aRa \; \forall a \in A$.
R is **antireflexive** when $aRb \implies a \neq b$, $a, b \in A$.
R is **symmetric** when $aRb \implies bRa,\ a, b \in A$.
R is **antisymmetric** when aRb and $bRa \implies a = b, a, b \in A$.
R is **transitive** when aRb and $bRc \implies aRc,\ a, b, c \in A$.

An **equivalence relation** is reflexive, symmetric and transitive.

A **partial order** is reflexive, antisymmetric and transitive.

Functions

A binary relation, f, on $A \times B$ is a **function** from A to B, written $f : A \to B$, if for every $a \in A$ there is one and only one $b \in B$ such that $(a, b) \in f$. We write $b = f(a)$.
We call A the **domain** of f and B the **codomain** of f.
The **range** of f is denoted by $f(A)$ where $f(A) = \{f(a) : a \in A\}$.
A function $f : A \to B$ is **one-to–one** or **injective** if $f(a_1) = f(a_2) \implies a_1 = a_2$.
A function $f : A \to B$ is **onto** or **surjective** if for every $b \in B$ there exists an $a \in A$ so that $b = f(a)$.
A function is **bijective** if it is both injective and surjective.