Sequences and Series

Arithmetic progression:

$$a, a+d, a+2d, \ldots$$

$a =$ first term, $d =$ common difference,
kth term $= a + (k-1)d$.
Sum of n terms,

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

Sum of the first n integers:

$1 + 2 + 3 + \ldots + n =$

$$\sum_{k=1}^{n} k = \frac{1}{2}n(n+1)$$

Sum of the squares of the first n integers:

$1^2 + 2^2 + 3^2 + \ldots + n^2 =$

$$\sum_{k=1}^{n} k^2 = \frac{1}{6}n(n+1)(2n+1)$$

Geometric progression:

$$a, ar, ar^2, \ldots$$

$a =$ first term, $r =$ common ratio,
kth term $= ar^{k-1}$
Sum of n terms,

$$S_n = \frac{a(1-r^n)}{1-r}, \text{ provided } r \neq 1$$

Sum of an infinite geometric series:

$$S_\infty = \frac{a}{1-r}, \quad -1 < r < 1$$

Proof by Induction

Let $P(n)$ be a statement defined for all integers $n \geq 1$. Then if

1. P(1) is true, and
2. for all $k \geq 1$, $P(k) \Rightarrow P(k+1)$ is true,

then $P(n)$ is true for all $n \geq 1$.

This can be compactly written in symbolic form as

$$P(1) \wedge (\forall k(P(k) \Rightarrow P(k+1))) \Rightarrow \forall n \; P(n).$$