



University  
of Glasgow

XXXday May XX, 2011  
XX.XX am/pm – XX.XX am/pm  
(Duration: X hour XX minutes)

DEGREES OF MSci, MEng, BEng, BSc, MA and MA (Social Sciences)

## Algorithms and Data Structures 2

(Answer all questions.)

This examination paper is worth a total of 50 marks

You must not leave the examination room within the first hour or the last half-hour of the examination. **(for exams of 2 hours duration)**

**or**

You must not leave the examination room within the first half hour or the last fifteen minutes of the examination. **(for exams of less than 2 hours duration)**

1. A binary search tree is a binary tree  $T$  such that each node of  $T$  stores an item  $e$ . Items stored in the left subtree rooted at a node  $v$  are less than the item in node  $v$ , and items stored in the right subtree rooted at a node  $v$  are greater than the item in node  $v$ . Below is java code for the `BNode` class, and below that code for the `BSTree` class.

```
public class BNode {
    private int    item;
    private BNode left;
    private BNode right;

    public BNode(int e, BNode left,BNode right){
        this.item = e;
        this.left = left;
        this.right = right;
    }

    public int    getItem(){return item;}
    public BNode getLeft(){return left;}
    public BNode getRight(){return right;}
    public void  setLeft(BNode nd){left = nd;}
    public void  setRight(BNode nd){right = nd;}
}

public class BSTree {
    private BNode root;

    public BSTree(){root = null;}

    public BNode root(){return root;}
    public boolean isEmpty(){return root == null;}

    public void insert(int e){
        if (isEmpty()) {root = new BNode(e,null,null);}
        else insert(e,root);
    }

    private static void insert(int e,BNode nd){...}

    public boolean isPresent(int e){return root != null && isPresent(e,root);}

    private static boolean isPresent(int e,BNode nd){...}
}

```

- (a) Write java code for the method `insert(int e,BNode nd)` in class `BSTree`, where the method inserts the integer  $e$  into the tree if and only if  $e$  is not already present in the tree. [5]
- (b) Write java code for the method `isPresent(int e,Bnode nd)`, where the method delivers true if and only if  $e$  is in the tree. [5]
- (c) Assume that the following items are inserted into an empty `BSTree` in the following order: 30, 40, 24, 58, 48, 26, 11, 13, 36.
- Draw the tree.
  - What is the height of the tree?
  - Write out the preorder, inorder and postorder traversals of the tree. [5]
- (d) Draw the tree after the node with item 30 has been deleted and outline the algorithm you used for the deletion (you do not need to write Java code). [3]

2. A stack is a container of objects that are inserted and removed according to the last-in first-out (LIFO) principle. In the class definition below for Stack, the stack is implemented as an array of Objects.

```
public class Stack {

    private Object[] S;
    int tos = -1; // top of stack pointer
    int capacity;

    public Stack(int capacity){
        this.capacity = capacity;
        S = new Object[capacity];
    }

    public int size(){...}
    public boolean isEmpty(){...}
    public boolean isFull(){...}

    public void push(Object o) throws StackException {...}
    public Object top() throws StackException {...}
    public Object pop() throws StackException {...}
}

public class StackException extends RuntimeException {
    public StackException(){...}
    public StackException(String msg) {super(msg);}
}
```

- (a) Give a Java implementation for the undefined methods above (**size, isEmpty, isFull, push, top, pop**) taking care to address stack overflow and stack underflow. [8]
- (b) The stack could have been implemented using a singly linked list (rather than a one dimensional array as above). What would be the relative advantages and disadvantages of each of those implementations? [4]

3. The dictionary abstract data type stores key-element pairs (k,v), which we call entries, where k is the key and v is the value. A dictionary allows for multiple entries with the same key, much like an English dictionary, where we can have multiple definitions of the same word. The primary use of a dictionary is to store values so that they can be located quickly using keys. We might realize a dictionary using an unordered linked list or a sorted array. Given a dictionary D, what would be the complexity of the following operations when D is realized using an unsorted linked list and when D is realized using a sorted array?

D.insert(k,v)	D.find(k)
D.size()	D.remove(k)

Explain each one of your answers, and state any assumptions that you make.

**NOTE:** D.find(k) returns an arbitrary entry (k,v) whose key is equal to k, or null if no such entry exists [8]

4. A map abstract data structure allows us to store key-value pairs  $(k,v)$  , but is different from the dictionary abstract data type (above) because each key is unique. Consequently the association of keys to values defines a mapping. We might implement the map abstract data type using a hash table.
- (a) What is a hash table and why would we want to use one? [2]
  - (b) What is a hash function and what properties would we like it to have? [2]
  - (c) What is a “collision”? [1]
  - (d) Describe the “separate chaining” method for hash collision resolution. [2]
  - (e) Describe the “linear probing” method for hash collision resolution, taking into consideration the methods insert, find, and remove [5]

