

Algorithmic Foundations 2

Assessed Exercise 1

Notes for guidance:

1. There are two assessed exercises. Each is worth 10% of your final grade for this module. Your answers must be the result of your own individual efforts.
2. Hand in a paper copy of your answers - these need not be word-processed; however handwritten answers must be legible. **Important:** you **must** write your name, matriculation number and lab group at the top of your script. Without this information you risk gaining no credit for your answers.
3. The **deadline** for completing this assessed exercise is **Thursday 11th February at 4.30 pm** - note that this is a strict deadline.
4. Completed exercises must be posted in the appropriate pigeonholes and you must attach a completed "*Declaration of Originality*" form to your submission.

Questions:

1. Using the laws of logical equivalence, decide whether the proposition $(q \wedge (p \rightarrow \neg q)) \rightarrow \neg p$ is a tautology.
2. Define the logical functions not, and, or, xor, implies, and ifOnlyIf below, where true is equivalent to the integer 1 and false is equivalent to the integer 0. In your definitions you may only use the arithmetic operators +, -, *, Math.min, and Math.max. You are not allowed to use if statements or tests for equality.

```
public class Logic {

    public static int not(int p){return -999;}
    public static int and(int p, int q){return -999;}
    public static int or(int p, int q){return -999;}
    public static int xor(int p, int q){return -999;}
    public static int implies(int p, int q){return -999;}
    public static int ifOnlyIf(int p, int q){return -999;}

    public static void truthTable(){
        System.out.println("P Q & | xor -> <->");
        for (int p=0;p<2;p++)
            for (int q=0;q<2;q++)
                System.out.println(p + " " + q + " " +
                    and(p,q) + " " +
                    or(p,q) + " " +
                    xor(p,q) + " " +
                    implies(p,q) + " " +
                    ifOnlyIf(p,q));
    }

    public static void main(String[] args) {
        truthTable();
    }
}
```

3. Suppose the variable x represents people, the variable y represents movies, and $S(x,y)$, $L(x,y)$, $A(y)$ and $C(y)$ are statements as follows:

$S(x,y)$: x saw y ;
 $L(x,y)$: x liked y ;

$A(y)$: y won an award;
 $C(y)$: y is a comedy.

Write each of the following English statements using the above predicates and any needed quantifiers:

- a) No comedy won an award. c) Lois saw Casablanca, but didn't like it.
 b) Some people have seen every comedy. d) No one liked every movie he has seen.
4. Given the definitions of Question 3, write each of the following in good English. Do not use variables in your answers.

- a) $\neg\forall y S(\text{Margaret}, y)$ c) $\forall x \exists y L(x,y)$
 b) $\exists y \forall x L(x,y)$ d) $\forall y ((C(y) \wedge A(y)) \rightarrow \exists x (S(x,y) \wedge L(x,y)))$

5. Determine whether each of the following sets is the power set of some set A . In each case, if the answer is yes, give the set A .

- a) \emptyset c) $\{\emptyset, \{a\}, \{\emptyset, a\}\}$
 b) $\{\emptyset, \{a\}\}$ d) $\{\emptyset, \{a\}, \{b\}, \{a,b\}\}$

6. Using a membership table, decide whether $A \oplus B = (A - B) \cup (B - A)$.

7. Which of the functions below are one-to-one and which functions below are onto?

```
public class Functions {
    public static int f1(int n){return n-1;}
    public static int f2(int n){return n*n+1;}
    public static int f3(int n){return n*n*n;}
    public static int f4(int n){return n/2;}
    public static int f5(int n){return f4(f2(n));}

    public static void main(String[] args) {
        int n = Integer.parseInt(args[0]);
        System.out.println(f1(n) + " " + f2(n) + " " +
                           f3(n) + " " + f4(n) + " " + f5(n));
    }
}
```

8. Suppose $g: A \rightarrow B$ and $f: B \rightarrow C$ where $A=B=C=\{1, 2, 3, 4\}$, $g = \{(1,4),(2,1),(3,1),(4,2)\}$ and $f = \{(1,3),(2,2),(3,4),(4,2)\}$.

- a) Find $f \circ g$ c) Find $g \circ g$
 b) Find $g \circ f$ d) Find $g \circ (g \circ g)$

9. Let $P(x,y)$ be a propositional function. Is the expression $\exists x \forall y P(x,y) \rightarrow \forall y \exists x P(x,y)$ a tautology? Explain your answer.