The Basis of Counting

Section 4
page 301 onwards

The Sum Rule

Imagine you go out for lunch, and as a 1st course you can have soup or pate (but not both)

- you can have
  - chicken soup or
  - carrot soup or
  - minestrone

- you can have
  - duck pate or
  - liver pate

How many different 1st courses could you have?

The Sum Rule

- you can have
  - chicken soup or
  - carrot soup or
  - minestrone
- you can have
  - duck pate or
  - liver pate

Chicken soup, or carrot soup, or minestrone, or duck pate, or liver pate

The Sum Rule

The sum rule:

If a first task can be done in M ways and
a second task can be done in N ways, and you can either do the first or the second task (but not both) then there are N + M ways to choose

Product Rule

For a main course you can have

- grilled salmon or
- steak or
- lasagne

With the main course you can have

- baked potato or
- salad or
- chips or
- assorted vegetables

How many different choices do you have for the main course?

Product Rule

For a main course you can have

- grilled salmon or
- steak or
- lasagne

With the main course you can have

- baked potato or
- salad or
- chips or
- assorted vegetables

Grilled salmon and baked potato, or
grilled salmon and salad, or
grilled salmon and chips, or
grilled salmon and assorted vegetables, or
steak and baked potato, or
steak and salad, or
steak and chips, or
steak and assorted vegetables, or
lasagne and baked potato, or
lasagne and salad, or
lasagne and chips, or
lasagne and assorted vegetables

Product Rule

The Product Rule:

If there are M ways to do a 1st task and N ways to do the second and you must do both, then there are M.N ways of doing them

How many passwords are there, where the passwords must have

- 6 alpha numeric characters
- first character must be a capital letter

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- 6 alpha numeric characters
- first character must be a capital letter

- There are 26 choices for the 1st position
- 26 + 26 + 10 for the 2nd, 3d, 4th, 5th and 6th
    - 62 for each of the other 5 positions

$$26 \times 62 \times 62 \times 62 \times 62 \times 62 = 26.62^5$$

23,819,453,632

Think of it as 26 times a 5 digit number to the base 62

Inclusion-Exclusion Principle

How many bit strings of length 8 either start with a 1 or end with 00?

10101100
01010100
11111111

- (a) There are 2^8 bit strings of length 8 (just so you know)
- (b) There are 2^7 bit strings of length 8 that start with a 1
    - note, this also includes strings that start with a 1 and end with a 00
- (c) There are 2^6 bit strings of length 8 that end with 00
    - note, this also includes strings that start with a 1
- (d) Using sum rule
    - there are $2^7 + 2^6$ bit strings that start with 1 or end with 00
        - but we have over-counted,
        - adding in twice the strings that start with 1 and end with 00
- (e) There are 2^5 bit strings of length 8 that start with a 1 and end with 00
- (f) therefore we should have $2^7 + 2^6 - 2^5$ bit strings

Inclusion-Exclusion Principle

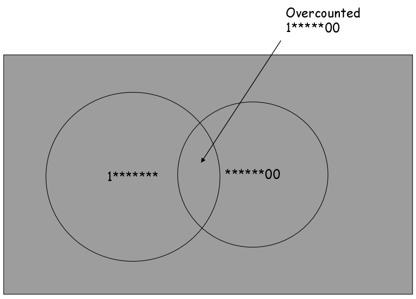
The Inclusion-Exclusion Principle:
If there are M ways to do task A and N ways to do task B and both can be done at the same time, then if we use the sum rule alone (i.e. M+N) we over-count by the amount P, where P is when we do A and B simultaneously

This is just like over-counting the cardinality of the union of 2 sets

See Rosen page 308

Inclusion-Exclusion Principle

How many bit strings of length 8 either start with a 1 or end with 00?



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