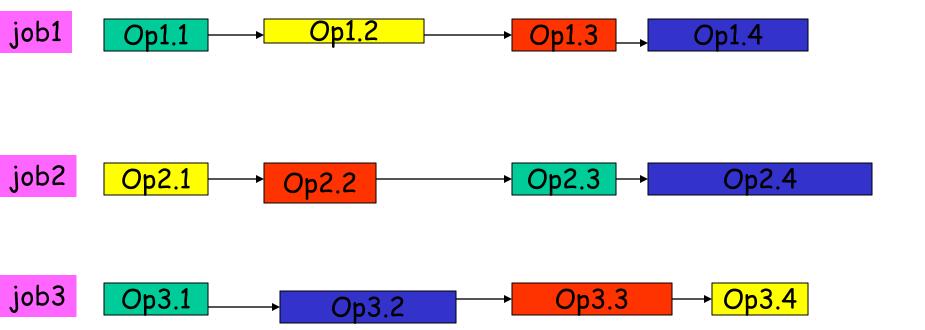
jobshop scheduling

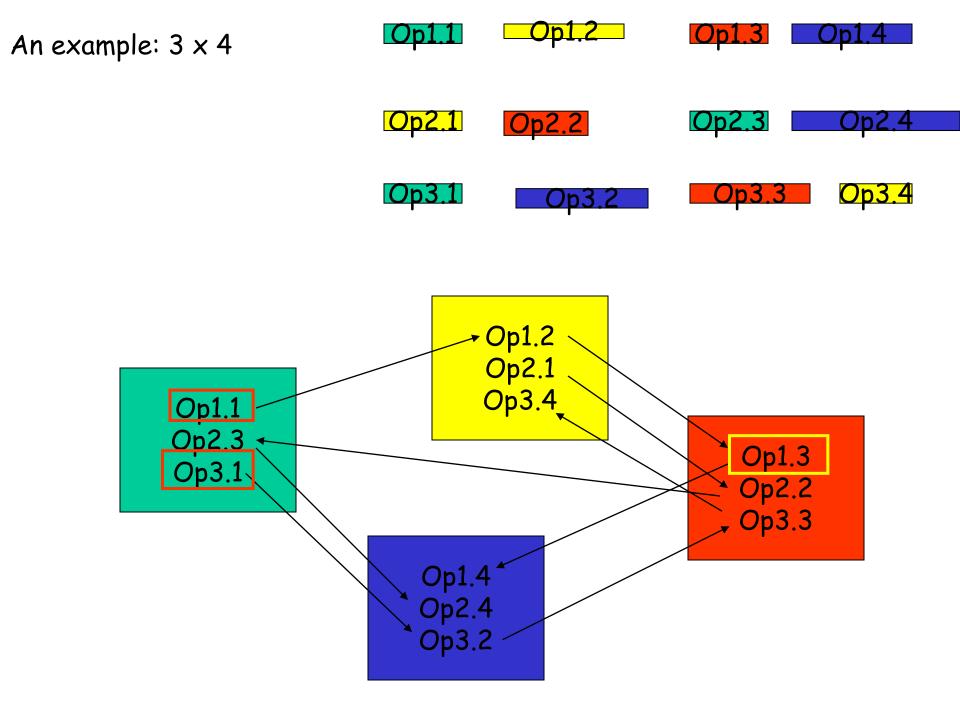
#### We have

- · a set of resources
- · a set of jobs
  - · a job is a sequence of operations/activities
- sequence the activities on the resources

An example:  $3 \times 4$ 



- · We have 4 resources: green, yellow, red and blue
- · a job is a sequence of operations (precedence constraints)
- each operation is executed on a resource (resource constraints)
- · each resource can do one operation at a time
- the duration of an operation is the length of its box
- · we have a due date, giving time windows for operations (time constraints)



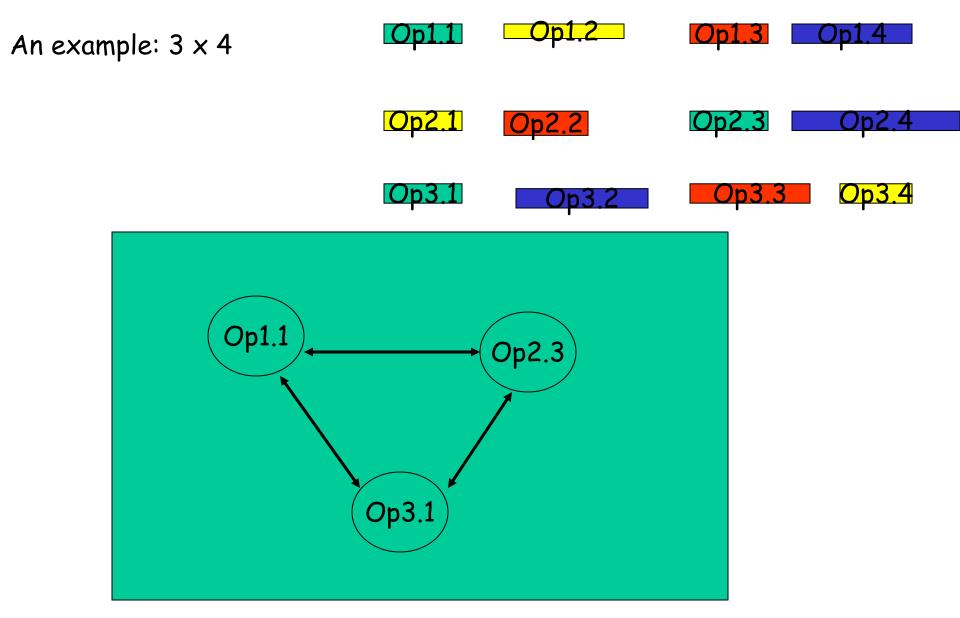
The problem

Assign a start time to each operation such that
(a) no two operations are in process on the same
machine at the same time and
(b) temporal constraints are respected

The problem

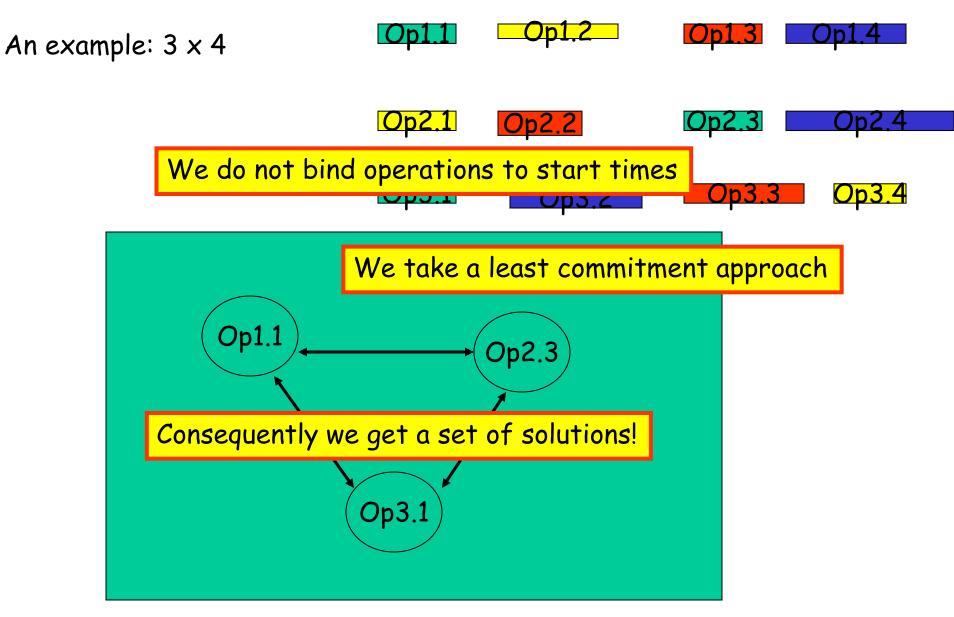
Alternatively ... sequence operations on resources

This gives a set of solutions, and might be considered a "least commitment approach"



On the "green" resource, put a direction on the arrows

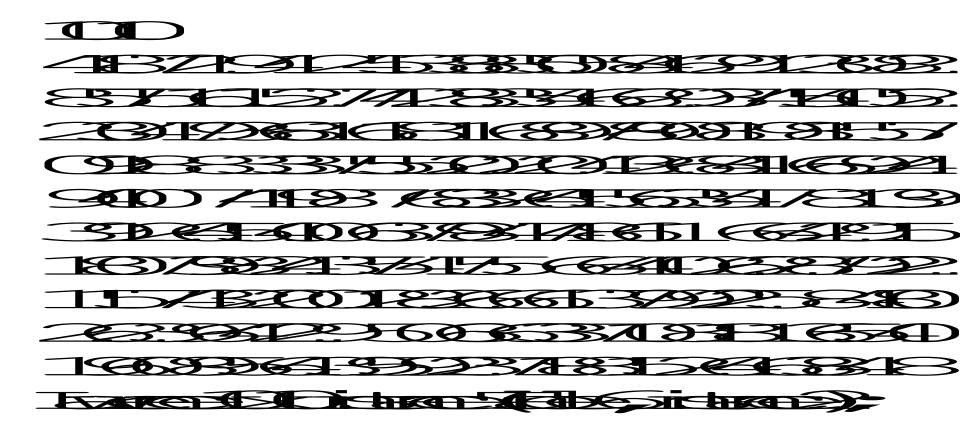
A disjunctive graph



On the "green" resource, put a direction on the arrows

A disjunctive graph

```
76
210316375346
                     What is makespan!
182540500034
253458091147
150525334859
291345540331
133359004421
244402375213
//ai/sosible
//catricov(lasse) isajo
//cetroblan (pertions (netro-detional)
// celtiple is
// —andrie Oto 5
//—additionalativetie (HoD)
//
// landomistofialla-kutesudesen
//werdermistationeriedcomplex.
//
```

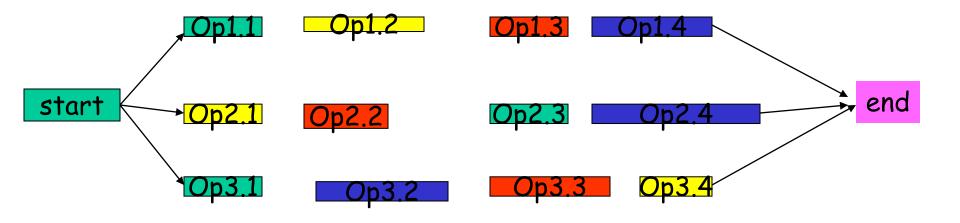


For a long time, unsolved

# Why bother?

- · Minimise makespan
  - · what is makespan?
- Maximise start
  - · JIT, minimise inventory levels
- · minimise idle time on resources
  - · maximise ROI

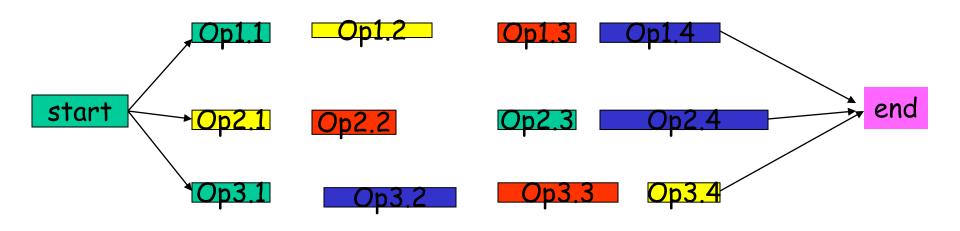
•



Find the smallest value for end minimise makespan

How can we view this as a csp?

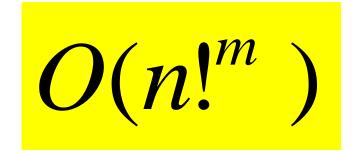
Each operation is a variable domain is set of start times there are precedence constraints between operation in a job operations on a resource have *disjunctive constraints* 



#### Complexity

What is the complexity of this problem?

- Assume we have m resources and n jobs
- · on each resource we will have n operations
- · we can order these in n! ways
- therefore we have  $O(n!^m)$  states to explore



But we want to optimise, not satisfy

How do you optimise with CP?

A sequence of decision problems

```
Is there a solution with makespan 395?
```

Yip!

•

•

Is there a solution with makespan 300?

- · Let me think about that ...
- · Yes
- Is there a solution with makespan 299?
  - · Hold on, ..., hold on
  - · NO!
- · Minimum makespan is 300.

When optimising, via a sequence of decision problems, will all decisions be equally difficult to answer?

What does branch and bound (BnB) do?

Who cares about jobshop scheduling?

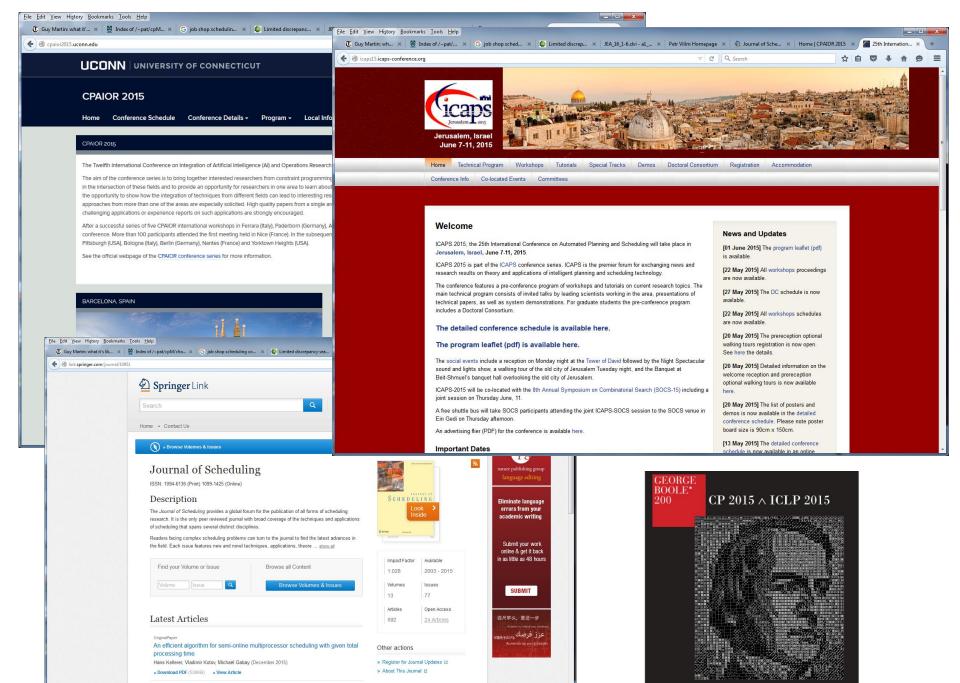
Manufacturing inc.

## Is CP any good for this class of problem?

- Main competitor is OR technology
- · Mark Fox 1980's,
  - ISIS constraint directed search(CMU)
- · OPIS
  - Steve Smith, Peng Si Ow (CMU)
- · DAS
  - Burke et al (SU)
- MicroBoss
  - Norman Sadeh (CMU)
- Edge-finding
  - a novel technique to infer constraints on a resources
    - J Carlier & E. Pinson 1994
  - · CP solves open benchmarks, beats OR
- Texture based heuristics
  - J-C Beck and Mark Fox 1990's
- · ILOG-Scheduler
  - · Claude Le Pape, Wim Nuijten, Philippe Babtiste, J-Chris Beck
  - and many others
- · 2000
  - ILOG buy CPLEX

Is CP any good for this class of problem?

- ·2000
  - · ILOG buy CPLEX
- •
- · 2009
  - · P. Vilim, edge-finding filtering
- ...



Share

Identical coupled task scheduling: polynomial complexity of the cyclic case

Cork, Ireland

# Variants of jsp

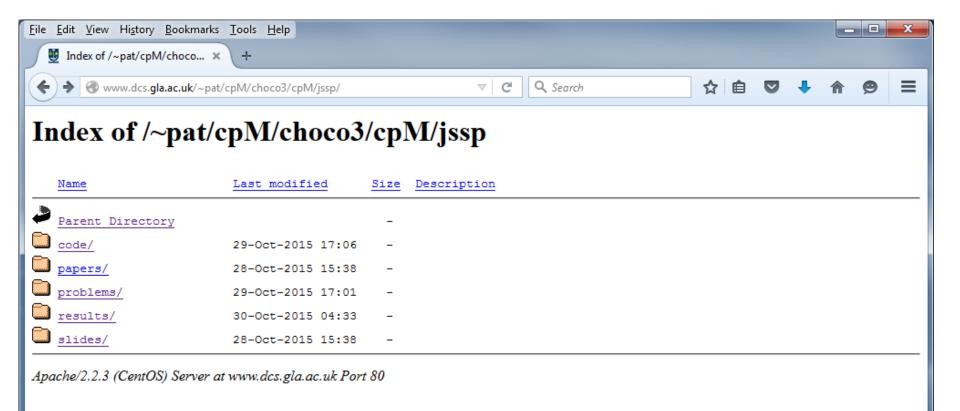
- · openness:
  - variety of resources can perform an operation
  - processing time dependant on resource used
- set up costs, between jobs (transition cost)
- consumable resources
  - such as gas, oil, etc
- pre-emption
  - can stop and restart an operation
- · resource can perform multiple operations simultaneously
  - batch processing
- secondary resources
  - people, tools, cranes, etc
- · etc

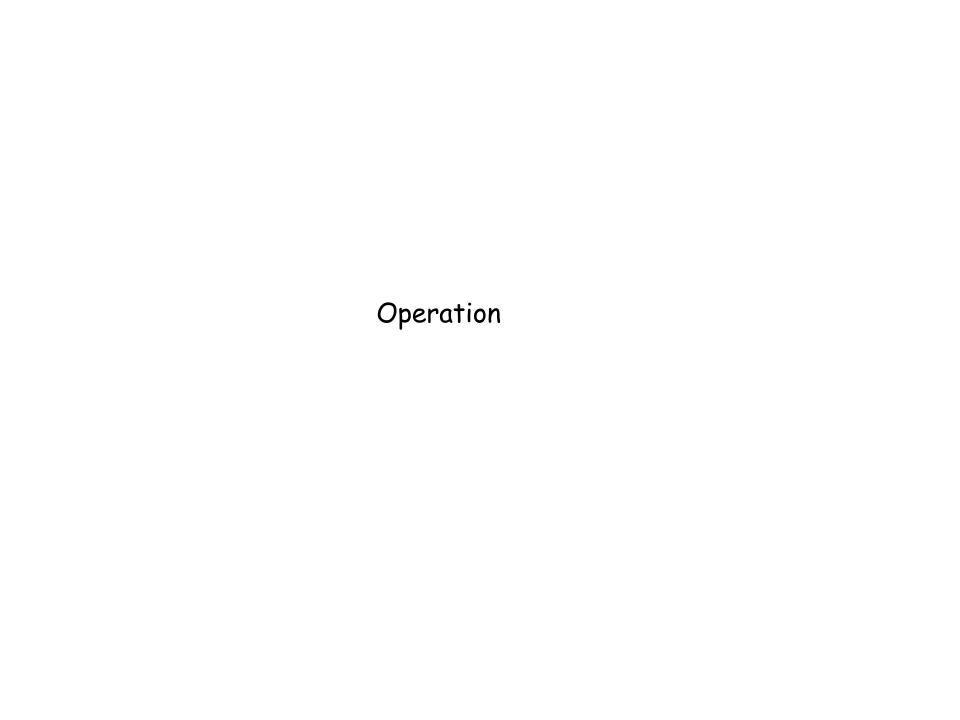
Chris Beck (2006) "The jssp has never been spotted in the wild."

# Why might CP be technology of choice for scheduling?

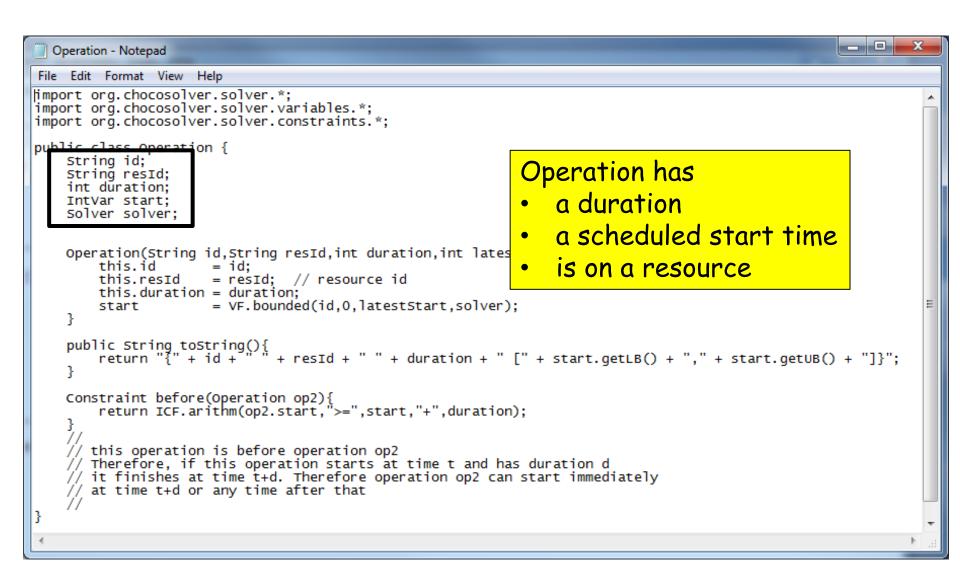
- · can model rich real-world problems
  - addition of side constraints etc
- incorporate domain knowledge
  - in the form of variable and value ordering heuristics
- · powerful reasoning/inference allied to novel search techniques

We can get a solution up and running quickly





```
Operation - Notepad
File Edit Format View Help
import org.chocosolver.solver.*;
import org.chocosolver.solver.variables.*;
import org.chocosolver.solver.constraints.*;
public class Operation {
    String id:
    String resId;
    int duration:
    IntVar start:
    Solver solver;
    Operation(String id, String resId, int duration, int latestStart, Solver solver) {
        this.id
                       = id:
        this.resId = resId; // resource id
        this.duration = duration:
        start = VF.bounded(id,0,latestStart,solver);
    public String toString(){
    return "{" + id + " " + resId + " " + duration + " [" + start.getLB() + "," + start.getUB() + "]}";
    Constraint before(Operation op2){
   return ICF.arithm(op2.start,">=",start,"+",duration);
    // this operation is before operation op2
    // Therefore, if this operation starts at time t and has duration d
    // it finishes at time t+d. Therefore operation op2 can start immediately
    // at time t+d or any time after that
```



```
Operation - Notepad
File Edit Format View Help
import org.chocosolver.solver.*;
import org.chocosolver.solver.variables.*;
import org.chocosolver.solver.constraints.*;
public class Operation {
    String id:
    String resId;
    int duration:
    IntVar start:
    Solver solver;
    Operation(String id, String resId, int duration, int latestStart, Solver solver){
        this.id
                       = id:
        this.resId = resId; // resource id
        this.duration = duration:
        start = VF.bounded(id,0,latestStart,solver);
    public String toString(){
    return "{" + id + " " + resId + " " + duration + " [" + start.getLB() + "," + start.getUB() + "]}";
    Constraint before(Operation op2){
   return ICF.arithm(op2.start,">=",start,"+",duration);
    // this operation is before operation op2
    // Therefore, if this operation starts at time t and has duration d
    // it finishes at time t+d. Therefore operation op2 can start immediately
    // at time t+d or any time after that
```

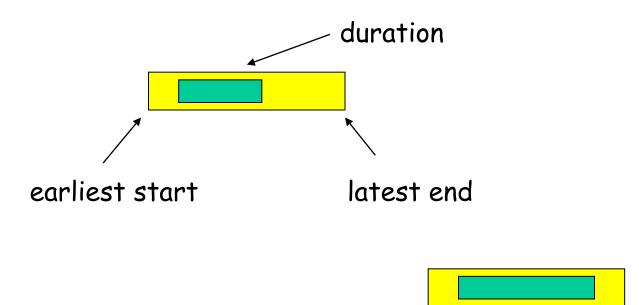
```
Operation - Notepad
File Edit Format View Help
import org.chocosolver.solver.*;
import org.chocosolver.solver.variables.*;
import org.chocosolver.solver.constraints.*;
public class Operation {
    String id:
    String resId;
    int duration:
    IntVar start:
    Solver solver;
    Operation(String id, String resId, int duration, int latestStart, Solver solver) {
        this.id
                       = id:
        this.resId = resId; // resource id
        this.duration = duration:
        start = VF.bounded(id,0,latestStart,solver);
    public String toString(){
    return "{" + id + " " + resId + " " + duration + " [" + start.getLB() + "," + start.getUB() + "]}";
   Constraint before(Operation op2){
   return ICF.arithm(op2.start,">=",start,"+",duration);
    // this operation is before operation op2
    // Therefore, if this operation starts at time t and has duration d
    // it finishes at time t+d. Therefore operation op2 can start immediately
    // at time t+d or any time after that
```

```
Constraint before(Operation op2){
    return ICF.arithm(op2.start,">=",start,"+",duration);
}
//
// this operation is before operation op2
// Therefore, if this operation starts at time t and has duration d
// it finishes at time t+d. Therefore operation op2 can start immediately
// at time t+d or any time after that
//
```

see next slides

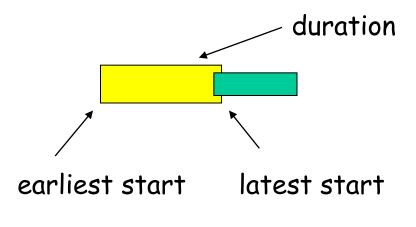
Picture of an operation

op1.before(op2)

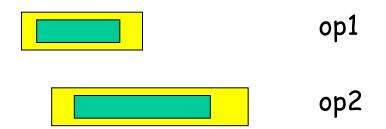


Picture of an operation

op1.before(op2)



Constrained integer variable represents start time



op1.before(op2) 
$$\longrightarrow$$
 op1.start() + op1.duration()  $\leq$  op2.start()

op1.before(op2)  $\longrightarrow$  op1.start() + op1.duration()  $\leq$  op2.start()

op1

op2

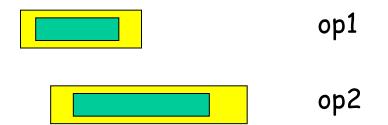
op1.before(op2)

propagate

op1.before(op2)  $\longrightarrow$  op1.start() + op1.duration()  $\leq$  op2.start()

Update latest start of operation op1

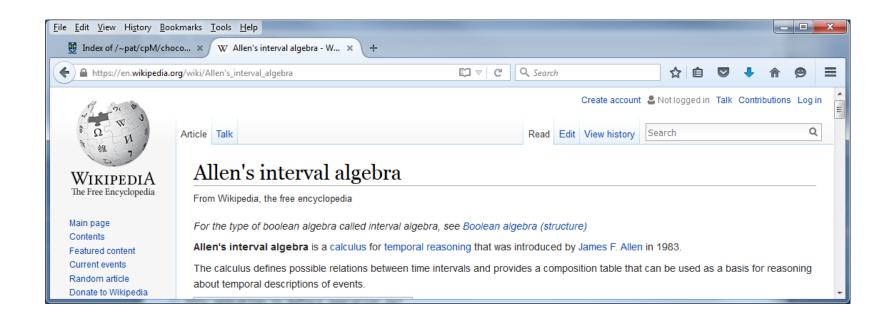
No effect on this instance

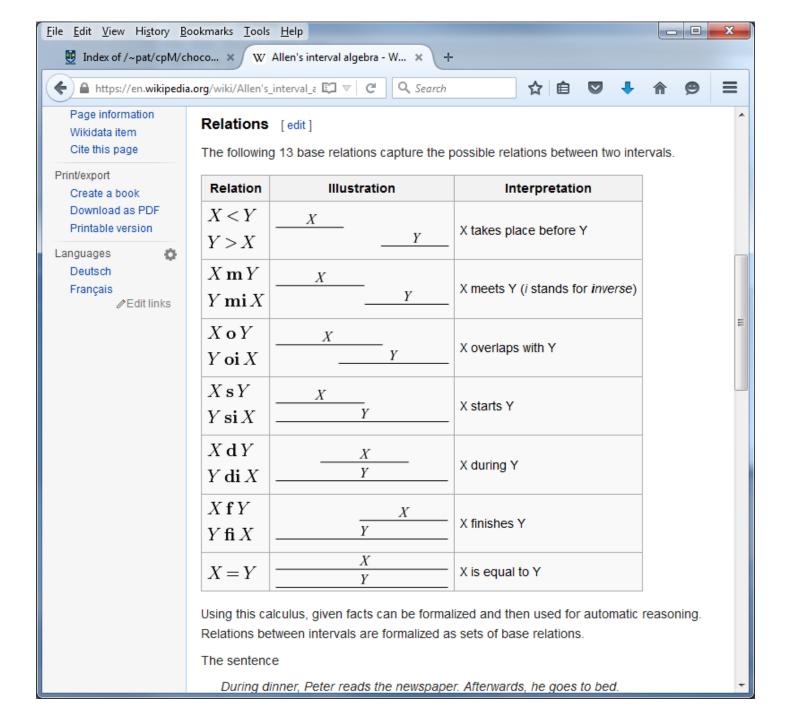


op1 and op2 cannot be in process at same time

op1.before(op2) OR op2.before(op1)

Not easy to propagate until decision made (disjunction broken)





Job

```
_ D X
Job - Notepad
File Edit Format View Help
import java.util.ArrayList;
import org.chocosolver.solver.*;
import org.chocosolver.solver.variables.*;
import org.chocosolver.solver.constraints.*;
public class Job {
    String id;
    ArrayList<Operation> operations:
    int length:
    Solver solver:
    Job(String id,Solver solver){
        this.id
                    = id:
        operations = new ArrayList<Operation>();
        length = 0;
        this.solver = solver;
    void add(Operation op){
        if (!operations.isEmpty())
            solver.post(operations.get(length-1).before(op));
        operations.add(op);
        length++;
    Operation get(int i){return operations.get(i);}
    public String toString(){
    String s = "(" + id + " ";
        for (int i=0;i<operations.size();i++) s = s + ((Operation)operations.get(i)).toString() + " ";
        return s + ")";
```

```
public class Job {
   String id;
   ArrayList<Operation> operations;
   int length;
   Solver solver;
```

```
Job(String id,Solver solver){
    this.id = id;
    operations = new ArrayList<Operation>();
    length = 0;
    this.solver = solver;
}
```

Job is a sequence of operations

```
void add(Operation op){
    if (!operations.isEmpty())
        solver.post(operations.get(length-1).before(op));
    operations.add(op);
    length++;
}
```

Creating/building a job as a sequence of operations each one *before* the other



Use a 0/1 decision variable d[i][j] as follows

$$d[i][j] = 0 \rightarrow op[i]1.before(op[j])$$
  
 $d[i][j] = 1 \rightarrow op[j]1.before(op[i])$ 

$$d[i][j] = 0 \rightarrow op[i]1.before(op[j])$$
 op1 before op2

$$d[i][j] = 0 \rightarrow op[i]1.before(op[j])$$
 op1 before op2

$$d[i][j] = 1 \rightarrow op[j]1.before(op[i])$$
 op2

op2 before op1

$$d[i][j] = 1 \rightarrow op[j]1.before(op[i])$$
 op2 before op1

```
Decision - Notepad
File Edit Format View Help
import org.chocosolver.solver.variables.*;
public class Decision {
    IntVar d;
    Operation op_i;
    Operation op_j;
    Decision(IntVar d,Operation op_i,Operation op_j){
        this.d = d;
        this.op_i = op_i;
        this.op_j = op_j;
    public String toString(){return "{" + d + " " + op_i + " " + op_j + "}";}
    IntVar getD(){return d;}
    Operation getOp_i(){return op_i;}
Operation getOp_j(){return op_j;}
   A Decision is a triple where 0/1 variable d decides
    the ordering between two operations on a resource
```

```
public class Decision {
    IntVar d;
    Operation op_i;
    Operation op_j;

    Decision(IntVar d,Operation op_i,Operation op_j){
        this.d = d;
        this.op_i = op_i;
        this.op_j = op_j;
    }
}
```

## A decision is a triple:

- a zero/one variable d
- an operation op\_i
- an operation op\_j

Value of d decides relative order of The two operations (before or after)



```
_ D X
  Resource - Notepad
File Edit Format View Help
jimport java.util.ArrayList;
import org.chocosolver.solver.*;
import org.chocosolver.solver.variables.*:
import org.chocosolver.solver.constraints.*;
public class Resource {
    String id:
    ArrayList<Operation> operations:
    ArrayList<Decision> decisions;
    Solver solver:
    Resource(String id, Solver solver){
        this.id
                   = id;
        operations = new ArrayList<Operation>();
        decisions = new ArrayList<Decision>():
        this.solver = solver:
    void add(Operation op){
        int n = operations.size():
        for (int i=0; i<n; i++){
            Operation op_i = operations.get(i);
            IntVar decision = VF.enumerated("dec_"+ i +","+ n,0,1,solver);
            decisions.add(new Decision(decision.op_i.op));
            LCF.ifThen(ICF.arithm(decision,"=",0),op_i.before(op)); // decision = 0 -> op_i before op LCF.ifThen(ICF.arithm(decision,"=",1),op.before(op_i)); // decision = 1 -> op before op_i
        operations.add(op);
   " NoDecVars: " + decisions.size();}
```

```
public class Resource {
    String id;
    ArrayList<Operation> operations;
    ArrayList<Decision> decisions;
    Solver solver;

    Resource(String id,Solver solver){
        this.id = id;
        operations = new ArrayList<Operation>();
        decisions = new ArrayList<Decision>();
        this.solver = solver;
}
```

Resource is a collection of operations and decisions that will be made on their ordering/sequencing on this resource

```
void add(Operation op){
   int n = operations.size();
   for (int i=0;i<n;i++){
        Operation op_i = operations.get(i);
        IntVar decision = VF.enumerated("dec_"+ i +","+ n,0,1,solver);
        decisions.add(new Decision(decision,op_i,op));
        LCF.ifThen(ICF.arithm(decision,"=",0),op_i.before(op)); // decision = 0
        LCF.ifThen(ICF.arithm(decision,"=",1),op.before(op_i)); // decision = 1
   }
   operations.add(op);
}</pre>
```

Add an operation to a resource and then constrain it ...

```
void add(Operation op){
   int n = operations.size();
   for (int i=0;i<n;i++){
        Operation op_i = operations.get(i);
        IntVar decision = VF.enumerated("dec_"+ i +","+ n,0,1,solver);
        decisions.add(new Decision(decision,op_i,op));
        LCF.ifThen(ICF.arithm(decision,"=",0),op_i.before(op)); // decision = 0
        LCF.ifThen(ICF.arithm(decision,"=",1),op.before(op_i)); // decision = 1
}
operations.add(op);
}</pre>
```

decision = 0 implies op\_i before op decision = 1 implies op before op\_i JSSP

```
JSSP - Notepad
File Edit Format View Help
jimport java.util.*;
import java.io.*;
import org.chocosolver.solver.*;
import org.chocosolver.solver.variables.*;
import org.chocosolver.solver.constraints.*;
public class JSSP {
    String id;
                                      // file name
                                         number of iobs
    int n:
    int m;
                                         number of resources
                                        aka makespan
    int dueDate;
                                      // jobs to complete
    ArrayList<Job> jobs;
    ArrayList<Resource> resources; // resources to use
                                      // last operation for ALL jobs!
    Operation endOp:
    Solver solver;
    JSSP(String fname, int dueDate) throws IOException {
                           = new Scanner(new File(fname));
        Scanner sc
        id
                            = fname;
                            = sc.nextInt(); // number of jobs
= sc.nextInt(); // number of resources
        n
        jobs
                            = new ArrayList<Job>();
        resources
                            = new ArrayList<Resource>();
        this.dueDate
                            = dueDate;
                            = new Solver("id");
        solver
        endop
                            = new Operation("endOp","nullRes",0,dueDate,solver);
        int totalDuration = 0;
        for (int i=0;i<m;i++) resources.add(new Resource("r_"+i,solver));</pre>
        for (int i=0; i<n; i++){
             job job = new Job("job_"+i,solver);
             for (int j=0; j < m; j++){
                 Resource resource = resources.get(sc.nextInt());
                 int duration
                                    = sc.nextInt();
                 totalDuration = totalDuration + duration;
Operation operation = new Operation("op_"+i+"_"+j,resource.id,duration,dueDate,solver);
                 resource.add(operation);
                 iob.add(operation):
             job.add(endop);
             jobs.add(job);
        solver.post(ICF.arithm(endOp.start,"<=",totalDuration));</pre>
        sc.close():
    public String toString(){
        String s = "JSSP" + n + "X" + m + "\n";
        for (int i=0; i<jobs.size(); i++){
            Job job = (Job)jobs.get(i);
s = s + job + "\n";
        return s;
    IntVar getMakeSpan(){return endOp.start;}
    Decision[] getDecisions(){
        Decision[] decisions = new Decision[((n * (n-1))/2) * m];
        for (int i=0,k=0;i<m;i++)
             for (Decision decision : resources.get(i).decisions)
                 decisions[k++] = decision;
        return decisions;
```

A jssp is a collection of jobs and resources

```
JSSP(String fname,int dueDate) throws IOException {
                       = new Scanner(new File(fname));
    Scanner sc
    id
                       = fname:
                       = sc.nextInt(); // number of jobs
    n
                       = sc.nextInt(); // number of resources
    m
    iobs
                       = new ArrayList<Job>();
                       = new ArrayList<Resource>();
    resources
    this.dueDate
                       = dueDate;
                       = new Solver("id");
    solver
    endop
                       = new Operation("endOp","nullRes",0,dueDate,solver);
    int totalDuration = 0;
    for (int i=0;i<m;i++) resources.add(new Resource("r_"+i,solver));</pre>
    for (int i=0; i<n; i++){
        Job job = new Job("job_"+i,solver);
        for (int j=0; j < m; j+\bar{+}) {
            Resource resource = resources.get(sc.nextInt());
            int duration = sc.nextInt();
            totalDuration = totalDuration + duration;
Operation operation = new Operation("op_"+i+"_"+j,resource.id,duration,dueDate,solver);
            resource.add(operation);
             job.add(operation);
         job.add(endOp);
        jobs.add(job);
    solver.post(ICF.arithm(endOp.start,"<=",totalDuration));</pre>
    sc.close();
```

```
JSSP(String fname, int dueDate) throws IOException {
                         = new Scanner(new File(fname));
    Scanner sc
    id
                         = fname;
                         = sc.nextInt(); // number of jobs
= sc.nextInt(); // number of resources
    n
    m
    jobs
                         = new ArrayList<Job>();
                         = new ArrayList<Resource>();
    resources
    this.dueDate
                         = dueDate;
                         = new Operation("endOp", "nullRes", 0, dueDate, solver);
    endop
```

```
int totalDuration = 0;
for (int i=0;i<m;i++) resources.add(new Resource("r_"+i,solver));
for (int i=0;i<n;i++){
    Job job = new Job("job_"+i,solver);
    for (int j=0;j<m;j++){
        Resource resource = resources.get(sc.nextInt());
        int duration = sc.nextInt();
        totalDuration = totalDuration + duration;
        Operation operation = new Operation("op_"+i+"_"+j,resource.id,duration,dueDate,solver);
        resource.add(operation);
        job.add(operation);
        job.add(endop);
        jobs.add(job);
}
solver.post(ICF.arithm(endop.start,"<=",totalDuration));
sc.close();</pre>
```

```
for (int i=0;i<m;i++) resources.add(new Resource("r_"+i,solver));

for (int i=0,i<m,i++);
    Job job = new Job("job_"+i,solver);
    for (int j=0;j<m;j++){
        Resource resource = resources.get(sc.nextInt());
        int duration = sc.nextInt();
        totalDuration = totalDuration + duration;
        Operation operation = new Operation("op_"+i+"_"+j,resource.id,duration,dueDate,solver);
        resource.add(operation);
        job.add(endop);
        jobs.add(job);
}

solver.post(ICF.arithm(endop.start,"<=",totalDuration));
sc.close();</pre>
```

```
int totalDuration = 0;
for (int i=0.i<m.i++) resources add(new Resource("r "+i solver)).

for (int i=0;i<n;i++) {
    Job job = new Job("job_"+i,solver);

    Resource resource = resources.get(sc.nextInt());
    int duration = sc.nextInt();
    totalDuration = totalDuration + duration;
    operation operation = new Operation("op_"+i+"_"+j,resource.id,duration,dueDate,solver);
    resource.add(operation);
    job.add(operation);
    job.add(endop);
    jobs.add(job);
}
solver.post(ICF.arithm(endop.start,"<=",totalDuration));
sc.close();</pre>
```

```
int totalDuration = 0;
for (int i=0;i<m;i++) resources.add(new Resource("r_"+i,solver));
for (int i=0;i<n;i++){
    Job job = new Job("job_"+i,solver);
    for (int j=0;j<m;j++){
        Resource resource = resources.get(sc.nextInt());
        int duration = sc.nextInt();
        totalDuration = totalDuration + duration;
        Operation operation = new Operation("op_"+i+"_"+j,resource.id,duration,dueDate,solver);
        resource.add(operation);
        job.add(endop);
        jobs.add(job);
}
solver.post(lcr.aritnm(endOp.start,"<=",totalDuration));
sc.close();</pre>
```

```
int totalDuration = 0;
for (int i=0;i<m;i++) resources.add(new Resource("r_"+i,solver));
for (int i=0;i<n;i++){
    Job job = new Job("job_"+i,solver);
    for (int j=0;j<m;j++){
        Resource resource = resources.get(sc.nextInt());
        int duration = sc.nextInt();
        totalDuration = totalDuration + duration;
        Operation operation = new Operation("op_"+i+"_"+j,resource.id,duration,dueDate,solver);
        resource.add(operation);
        job.add(operation);
        job.add(endop);
        jobs.add(job);

        solver.post(ICF.arithm(endOp.start,"<=",totalDuration));
        solver.post(ICF.arithm(endOp.start,"<=",totalDuration));
        resource.documents</pre>
```



## DecisionProblem

```
public class DecisionProblem {
  public static void main(String[] args) throws FileNotFoundException, IOException {
                         = Integer.parseInt(args[1]);
      int dueDate
                          = new JSSP(args[0],dueDate);
      JSSP jssp
      Solver solver = jssp.solver;
      Decision[] decisions = jssp.getDecisions();
                           = decisions.length;
      int n
                           = jssp.getMakeSpan();
     IntVar makeSpan
     solver.set(ISF.lexico_LB(jssp.getDecisionIntVars()));
      System.out.println("solved: " + solver.findSolution());
      System.out.println(makeSpan + " ["+ makeSpan.getLB() +","+ makeSpan.getUB() +"]");
      System.out.println("nodes: " + solver.getMeasures().getNodeCount() +
                           cpu: " + solver.getMeasures().getTimeCount());
```

## Wot!? No heuristics!?!!!

