

Magic square

An example of how to represent a problem

An idea from Chris Beck

- put a number in each square
- each number is different
- a number is in the range 1 to 16
- the sum of a column is
 - the same as a sum of a row
 - the same as the sum of a main diagonal



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Lo Shu Square

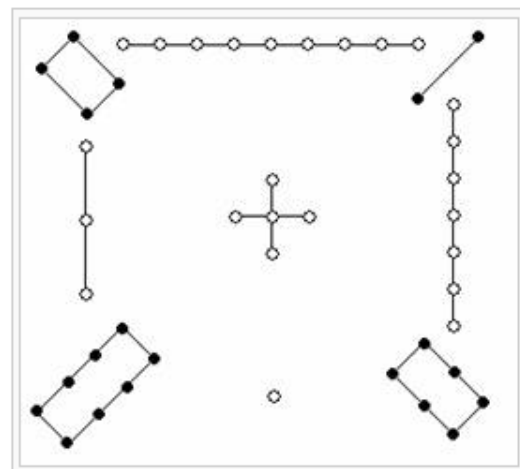
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Lo Shu Square (**Simplified Chinese**: 洛书; **Traditional Chinese**: 洛書; **pinyin**: luò shū; also written 雒書; literally: Luo (River) Book/Scroll) or the **Nine Halls Diagram** (**Simplified Chinese**: 九宫图; **Traditional Chinese**: 九宮圖; **pinyin**: jiǔ gōng tú), is the unique normal **magic square** of order three. Lo Shu is part of the legacy of the most ancient Chinese mathematical and divinatory (*Yi Jing* 易经) traditions, and is an important emblem in **Feng Shui** (風水), the art of geomancy concerned with the placement of objects in relation to the flow of *qi* (氣) 'natural energy'.

Chinese legends concerning the pre-historic Emperor Yu (夏禹) tell of the Lo Shu, often in connection with the **Ho Tu** (河圖) figure and **8 trigrams**. In ancient China there was a huge **deluge**: the people offered sacrifices to the god of one of the flooding rivers, the Lo river (洛水), to try to calm his anger. A magical **turtle** emerged from the water with the curious and decidedly unnatural (for a turtle shell) Lo Shu pattern on its shell: circular dots giving unitary (base 1) representations (**figurate numbers**) of the integers **one** through **nine** are arranged in a three-by-three grid.

The odd and even numbers alternate in the periphery of the Lo Shu pattern, the 4 even numbers are at the four corners, and the 5 odd numbers (outnumbering the even numbers by one) form a cross in the center of the square. The sums in each of the 3 rows, in each of the 3 columns, and in both diagonals, are all 15 (**fifteen** is the number of days in each of the 24 cycles of the **Chinese solar year**). Since the center cell is 5, the sum of any two non-central cells in the same row, column, or diagonal, is 10 (e.g., opposite corners add up to 10, the number of the **Ho Tu** (河圖)).

The Lo Shu is sometimes connected numerologically with the **8 trigrams**, which can be arranged in the 8 outer cells, reminiscent of circular trigram diagrams. The numbers 1 (001, the beginning of all things, bottom of the central column) and 9 (completion, 1001, top of the central column) are considered most auspicious, while the number 5 (101) at the very center is the perfectly balanced number (also at the heart of the **Ho Tu**). Like the **Ho Tu** (河圖), the Lo Shu square, in conjunction with the **8 trigrams**, is sometimes used as a cosmological representation important in **Feng Shui** (風水).



The 洛書 luòshū.

4	9	2
3	5	7
8	1	6

Modern representation of the *Lo Shu* square as a magic square.

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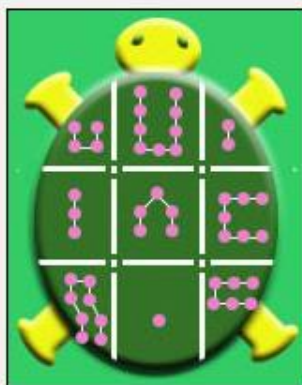
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LO SHU MAGIC SQUARE

One of most ancient tool of Feng Shui and widely used by the practitioner is the Lo Shu magic square. 6000 years ago; this form was found on the back of the Turtle, one of the celestial animals, as the Turtle appeared on the river Lo, in the following pattern



The reason it is called magic square is because of its unique placement and numbers. The total in any direction will add upto figure 15. Feng Shui formulas of astrology, flying star and I-Ching are all based on this Lo Shu magic square.

4	9	2
3	5	7

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Recreational Mathematics > Magic Figures > Magic Squares

Lo Shu



8	1	6
3	5	7
4	9	2

The unique [magic square](#) of order three. The Lo Shu is an [associative magic square](#), but not a [panmagic square](#).

SEE ALSO: [Associative Magic Square](#), [Magic Square](#), [Panmagic Square](#).
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REFERENCES:

Gardner, M. *The Sixth Book of Mathematical Games from Scientific American*. Chicago, IL: University of Chicago Press, pp. 19 and 24, 1984.

Hunter, J. A. H. and Madachy, J. S. *Mathematical Diversions*. New York: Dover, pp. 23-24, 1975.

Kraitchik, M. *Mathematical Recreations*. New York: W. W. Norton, pp. 146-147, 1942.

Wells, D. *The Penguin Dictionary of Curious and Interesting Numbers*. Middlesex, England: Penguin Books, pp. 75-76, 1986.

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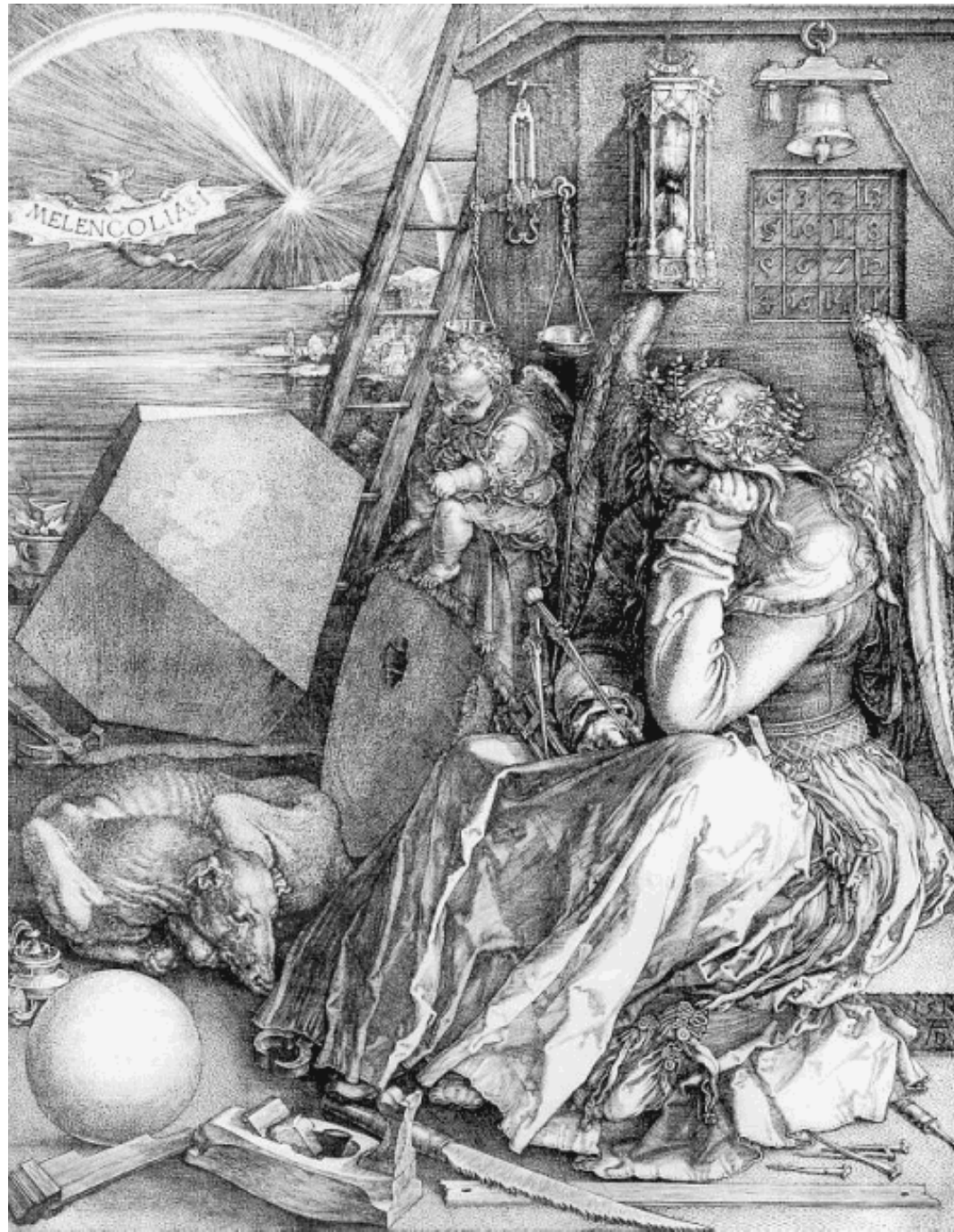
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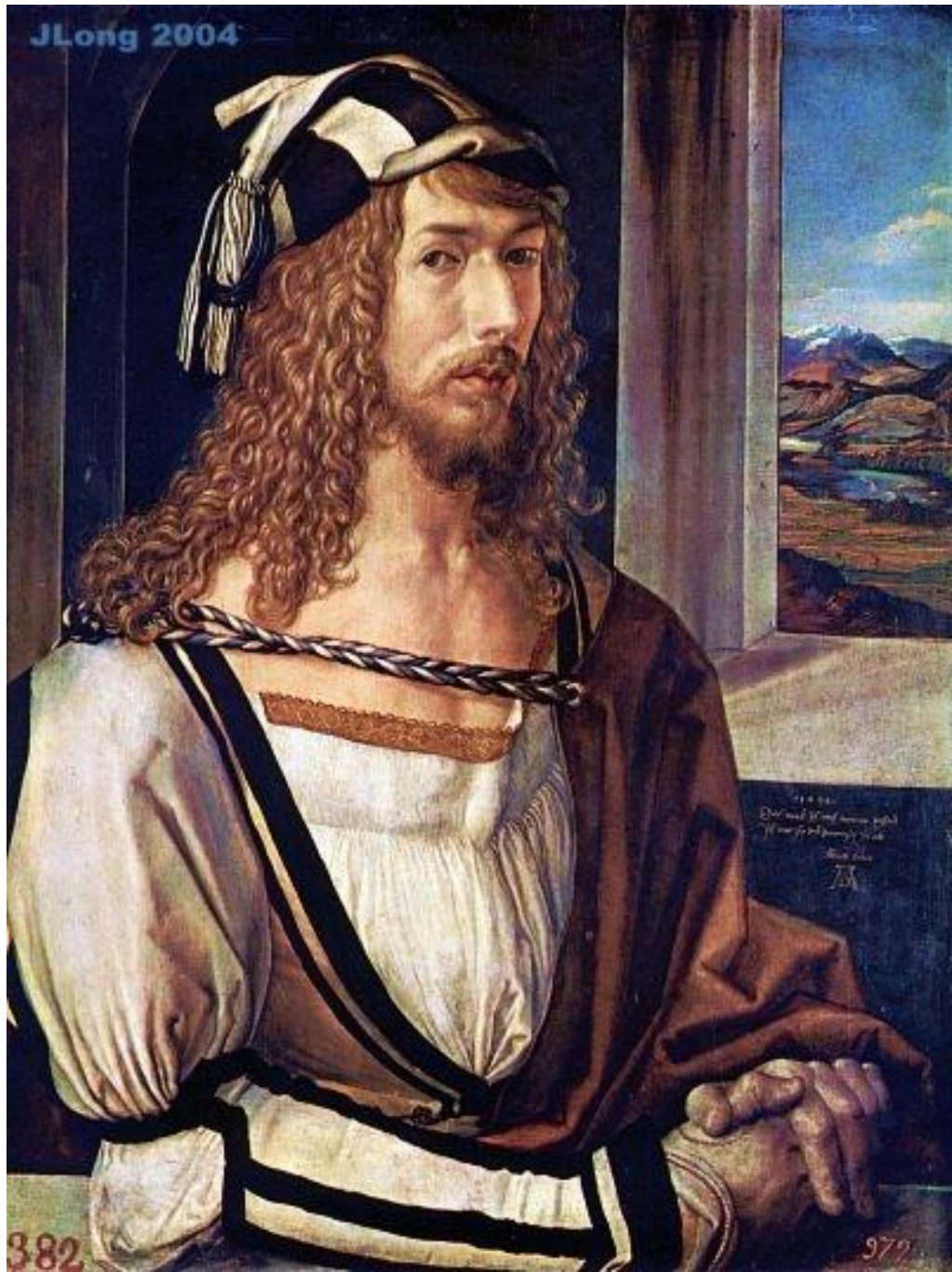




1500
AD

Albertus Durerus Noricus
episcopus ac proprie lectissimus
gubernatoris quatuordecim
anno. M. L.

JLong 2004





16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

Dürer's magic square is a [magic square](#) with [magic constant](#) 34 used in an engraving entitled *Melancholia I* by Albrecht Dürer (The British Museum, Burton 1989, Gellert *et al.* 1989). The engraving shows a disorganized jumble of scientific equipment lying unused while an intellectual sits absorbed in thought. Dürer's magic square is located in the upper right-hand corner of the engraving. The numbers 15 and 14 appear in the middle of the bottom row, indicating the date of the engraving, 1514.

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

Dürer's magic square has the additional property that the sums in any of the four quadrants, as well as the sum of the middle four numbers, are all 34 (Hunter and Madachy 1975, p. 24). It is thus a [gnomon magic square](#). In addition, any pair of numbers symmetrically placed about the center of the square sums to 17, a property making the square even more magical.

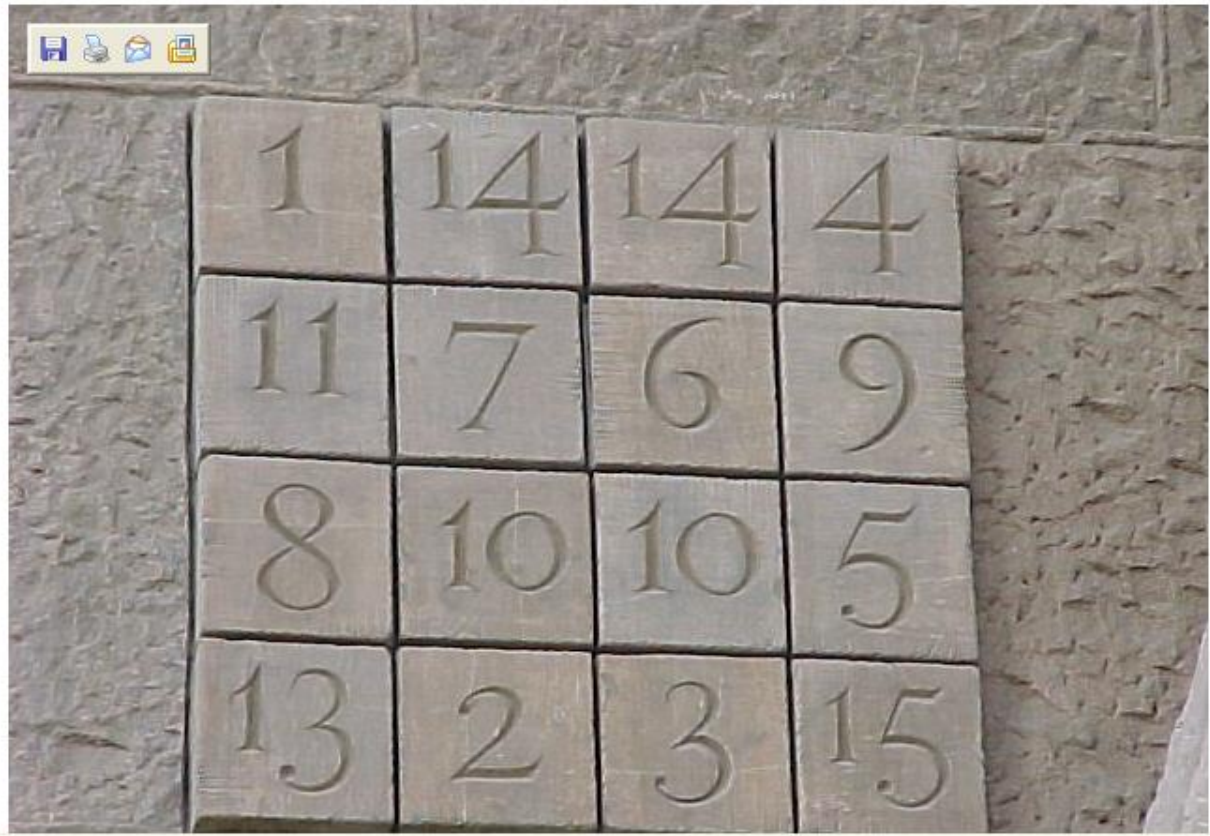
SEE ALSO: [Dürer's Solid](#), [Gnomon Magic Square](#), [Magic Square](#). [[Pages Linking Here](#)]

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sum of 34 in 86 different ways. For example, the four numbers in the four corners, any quadrant of the square, or the four nearest the center all add up to 34.

4	5	16	9
14	11	2	7
1	8	13	12
15	10	3	6

The photo below shows a different kind of magic square. This one is on the Passion facade of the Sagrada Familia, the unfinished cathedral in Barcelona designed by Antoni Gaudi. Each row and column add up to 33, the supposed age of Christ at his death. In fact there are supposed to be exactly 33 different four number groupings that add up to 33; can you find them all?





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Chapter 11

First Example: Magic square

A simple magic square of order 3 can be seen as the “Hello world!” program in Choco. First of all, we need to agree on the definition of a magic square of order 3. [Wikipedia](#) tells us that :

A **magic square** of order n is an arrangement of n^2 numbers, usually distinct integers, in a square, such that the n numbers in all rows, all columns, and both diagonals sum to the same constant. A normal magic square contains the integers from 1 to n^2 .

So we are going to solve a problem where unknowns are cells value, knowing that each cell can take its value between 1 and n^2 , is different from the others and columns, diagonals and rows are equal to the same constant M (which is equal to $n * (n^2 + 1)/2$).

We have the definition, let see how to add some Choco in it.

11.1 First, the model

To define our problem, we need to create a Model object. As we want to solve our problem with constraint programming (of course, we do), we need to create a CPMModel

Done

Unknown Zone

start

http://www.dcs.gla.a... magic

Microsoft PowerPoint ...

14:28

1st stab

- put a number in each square
- each number is different
- a number is in the range 1 to 16
- the sum of a column is
 - the same as a sum of a row
 - the same as the sum of a main diagonal

$$D_i \in [1..16]$$

$$\sum row_i = \sum row_{i+1}$$

$$\sum col_j = \sum col_{j+1}$$

$$\sum diag_l = \sum diag_{l+1}$$

$$m_{i,j} \neq m_{k,l}$$

Use $sum(x) = sum(y)$

where x and y are

- different rows
- different columns
- different diagonals

Every element of the array is different

- represent as a clique of not equals

How does it go?
For propagation and search?

2nd stab

- put a number in each square
- each number is different
- a number is in the range 1 to 16
- the sum of a column is
 - the same as a sum of a row
 - the same as the sum of a main diagonal

Use allDiff

$$D_i \in [1..16]$$

$$\sum row_i = k$$

$$\sum col_j = k$$

$$\sum diag_l = k$$

But what is k?

How does model perform?

```

import org.chocosolver.util.tools.ArrayUtils;
import org.chocosolver.solver.Model;
import org.chocosolver.solver.Solver;
import org.chocosolver.solver.variables.IntVar;

public class Magic {

    public static void main(String[] args) throws Exception {
        int n = Integer.parseInt(args[0]);
        int k = n*(n*n + 1)/2;

        Model model = new Model("magic");
        Solver solver = model.getSolver();
        IntVar S[][] = model.intVarMatrix("S",n,n,1,n*n);

        model.allDifferent(ArrayUtils.flatten(S)).post();

        for (int i=0;i<n;i++) model.sum(S[i], "=", k).post();
        for (int j=0;j<n;j++) model.sum(ArrayUtils.getColumn(S,j), "=", k).post();

        IntVar diagL[] = new IntVar[n];
        for (int i=0;i<n;i++) diagL[i] = S[i][i];
        model.sum(diagL, "=", k).post();

        IntVar diagR[] = new IntVar[n];
        for (int i=0;i<n;i++) diagR[i] = S[i][n-i-1];
        model.sum(diagR, "=", k).post();

        solver.solve();

        for (int i=0;i<n;i++){
            for (int j=0;j<n;j++){
                System.out.print(S[i][j].getValue() + " ");
                System.out.println();
            }
        }

        System.out.println(solver.getMeasures());
    }
}

```


Command Prompt

```
C:\Documents and Settings\pat>cd ..
C:\Documents and Settings>cd ..
C:\>cd pat
C:\pat>cd cpm
C:\pat\cpM>cd JChoco
C:\pat\cpM\JChoco>cd magic
C:\pat\cpM\JChoco\magic>javac Magic.java
C:\pat\cpM\JChoco\magic>java Magic 3
20[+0] millis.
4[+0] nodes
feasible: true
k = 15
4 3 8
9 5 1
2 7 6
C:\pat\cpM\JChoco\magic>
```

8	1	6
3	5	7
4	9	2



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Magic square on the web

<http://mathworld.wolfram.com/MagicSquare.html>

<http://mathforum.org/alejandre/magic.square.html>

Is there an order 3 magic square with the number 5 in one of the outer corners?

Thanks to Chris Beck



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I joined the Department of Mechanical and Industrial Engineering, University of Toronto in August, 2004. I am an Assistant Professor in the area of Information Engineering but also do a lot of what might be considered Operations Research.

Before joining U of T I spent 2 years as a Staff Scientist at the Cork Constraint Computation Centre in Cork, Ireland. I spent 3 years as Senior Scientist and Software Engineer on the Scheduler R&D team at ILOG in Paris, France.

I do research in the areas of optimization, heuristic search, constraint programming, constraint-directed scheduling, hybrid algorithms, dynamic and uncertain problems, and problem modeling. Broadly, I am interested in when and why a particular problem solving technique works: what characteristics make a problem suitable (or unsuitable) for a search technique.

Contact Information