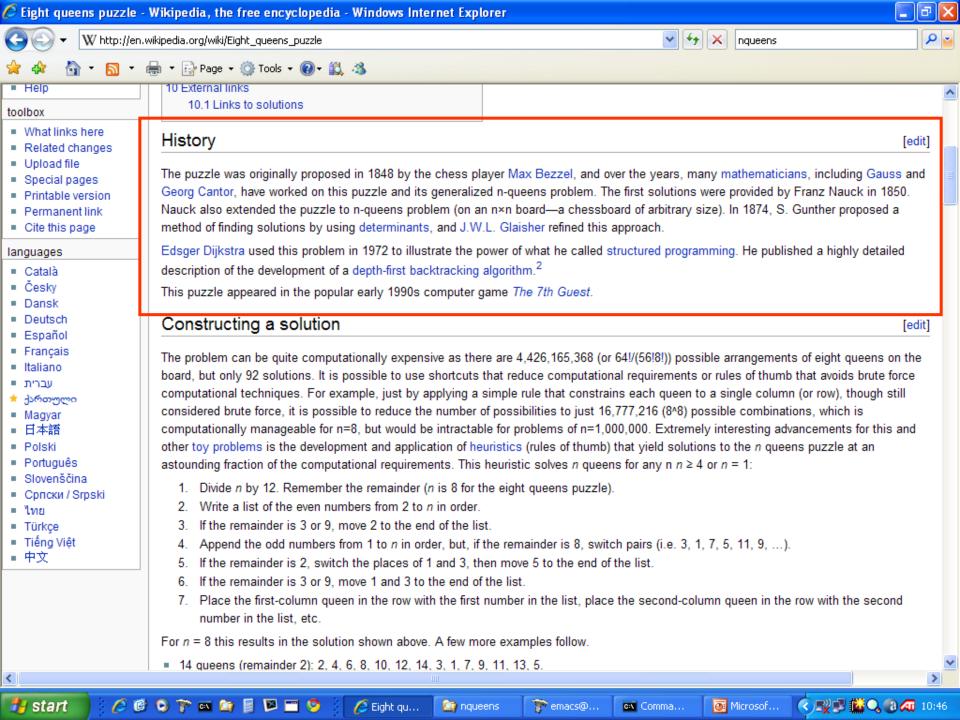
nqueens

(CP: "hello world")





## m-queens

The famous n-queens problem is actually a simplification of the much more important m-queens problem, which is originally attributed to Adardhir I the founder of the Sassanid Empire. His mother Rodhagh challenged him to cover his empire with the minimum number of castles so that no place was not protected by cavalry within 1 week riding. Adardhir simplified this problem into the m-queens problem, based on the game of chess, which was becoming popular at the time, under the name shatranj. His son Shapur I is attributed with the first optimal solution on a standard 8x8 chessboard. The m-queens problem, generalizing Adardhir's problem, is to cover an n x n chess board with as few queens as possible so that no queen can take another and no more queens can be placed on the board without being taken.

For example given n = 5 a solution might be q = [2, 0, 5, 3, 1]; representing the solution

- . Q . . .
- . . . . .
- . . Q . .
- Q . . . .

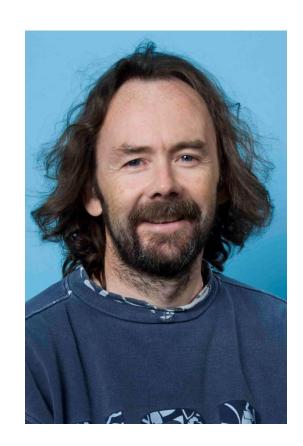
where 0 means no queen on that row. Note that above a 5th queen cannot be placed on the board witjout being under attack.

and a minimal solution might be q = [5,2,0,3,0]; representing the solution

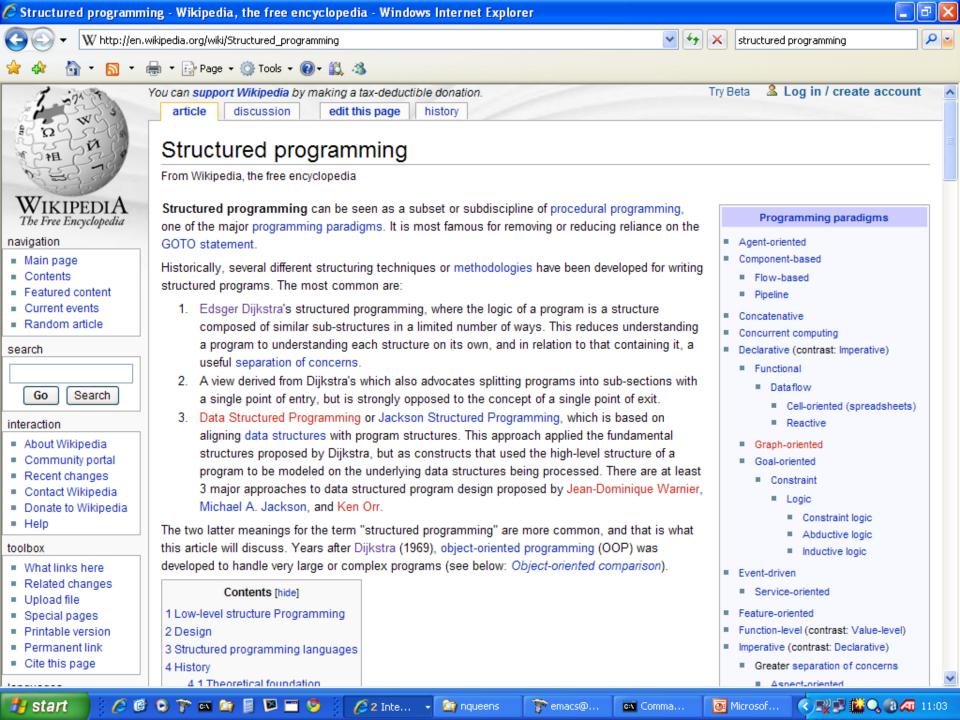
- . . . . Q
- . Q . . .
- . . . . .
- . . Q . .
- . . . . .

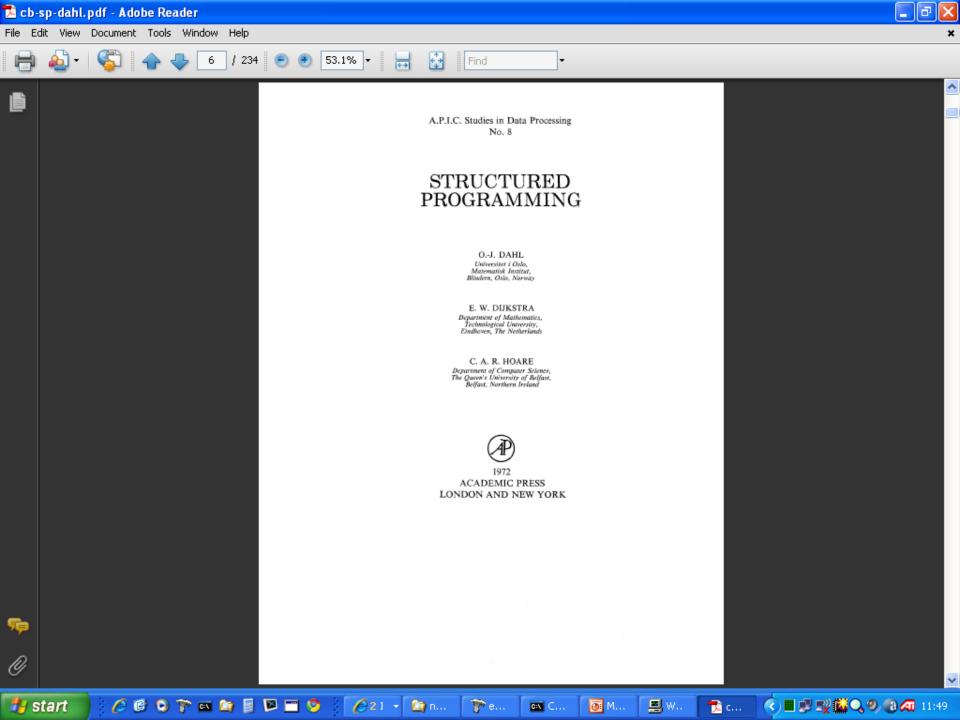
That is, above we see that by placing 3 queens on the 5x5 board the queens do not attack each other and all other vacant positions are under attack. The above solution is minimal

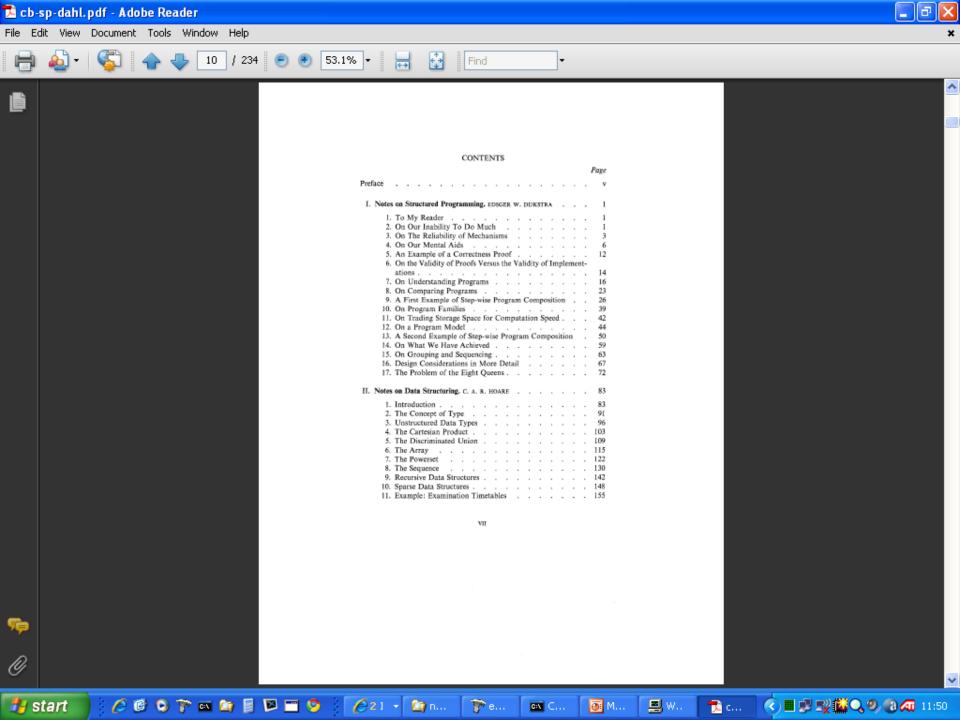
## Peter Stuckey myth?

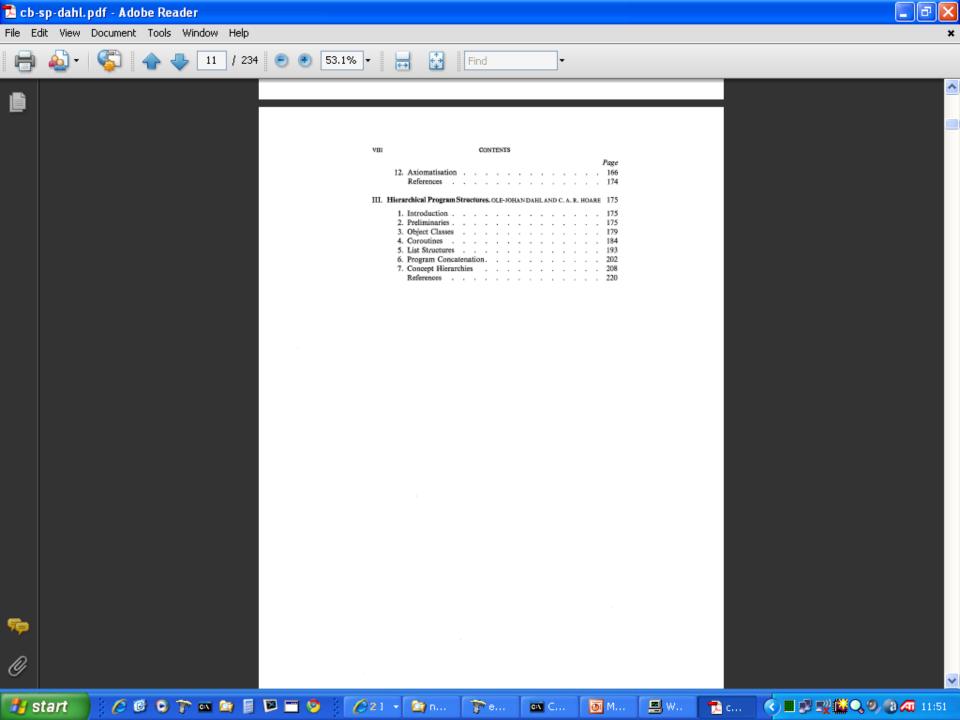


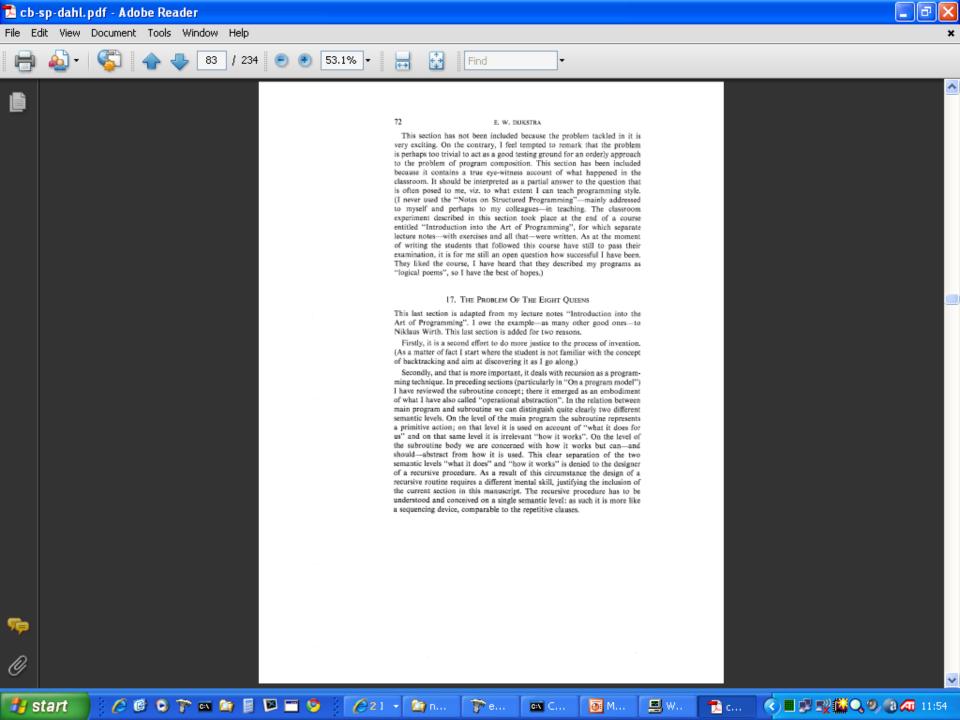


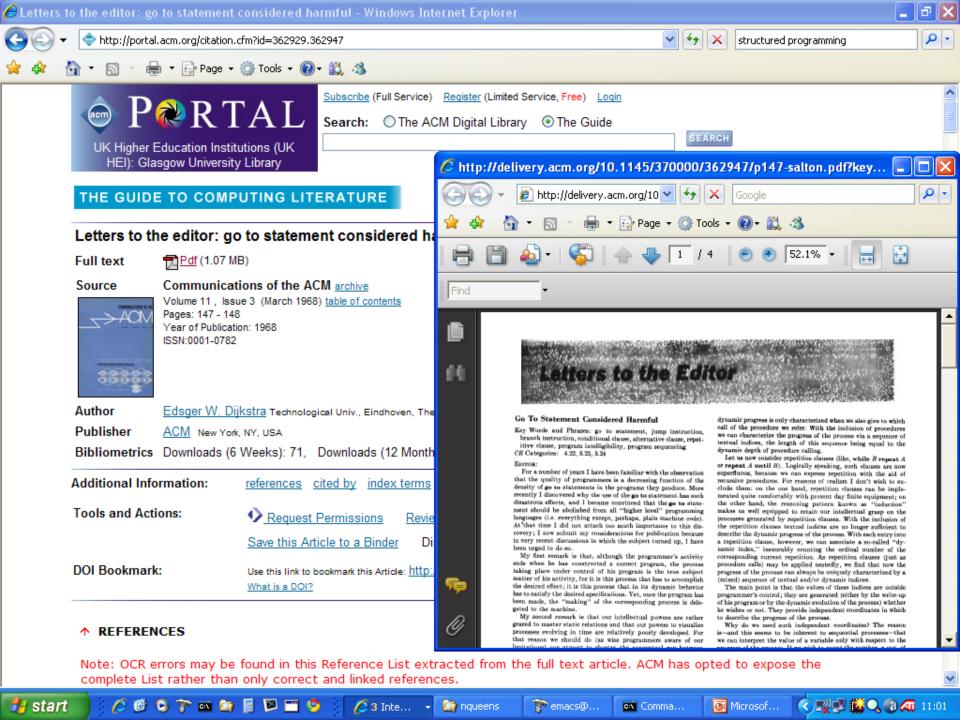


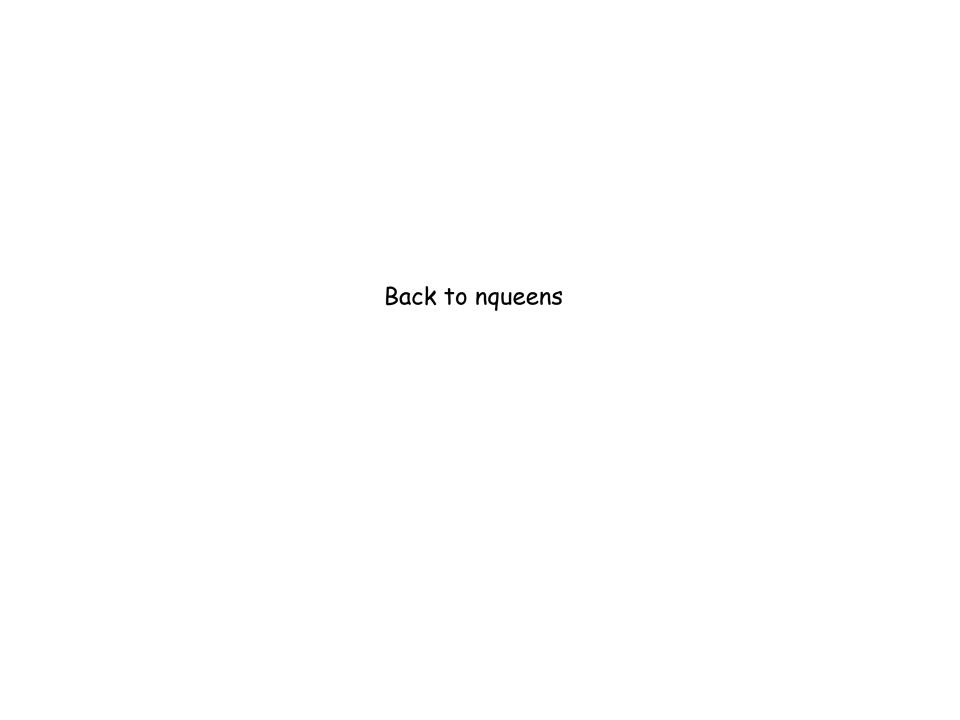


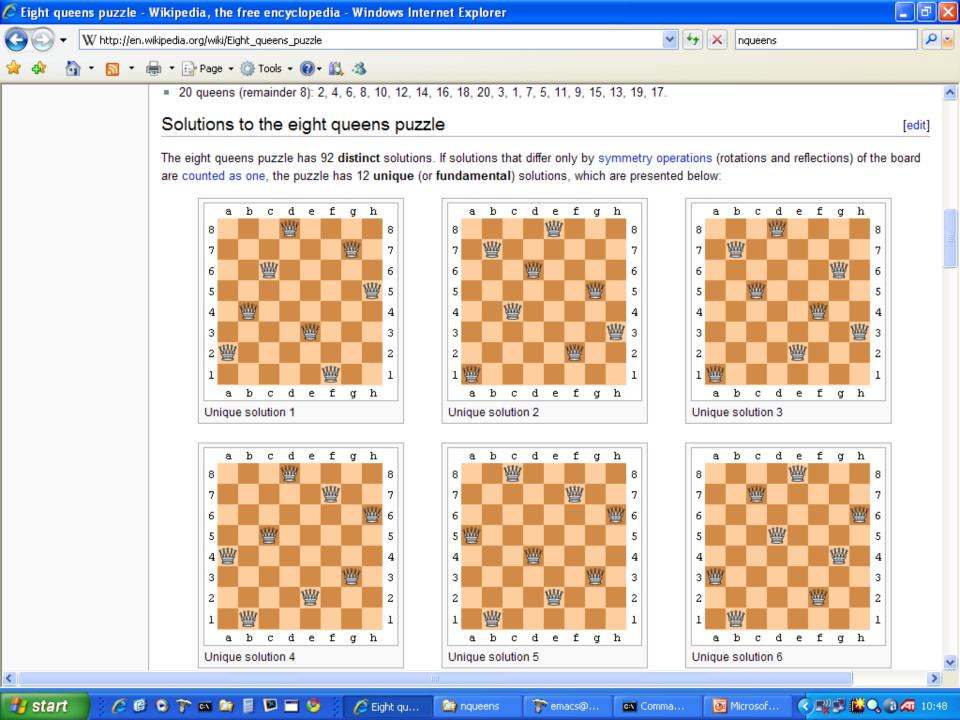




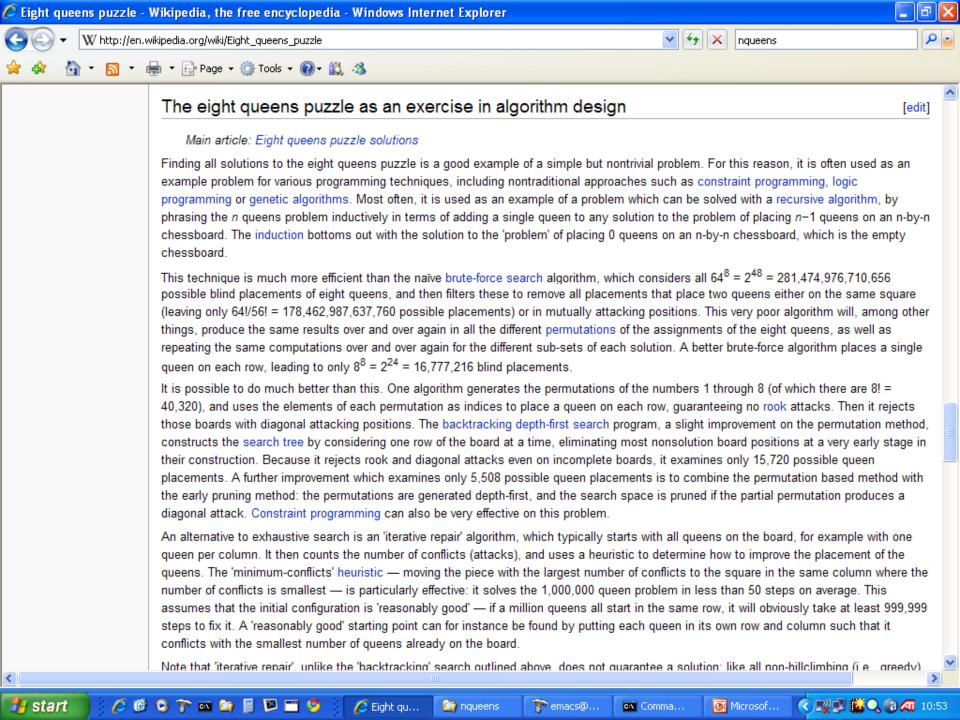


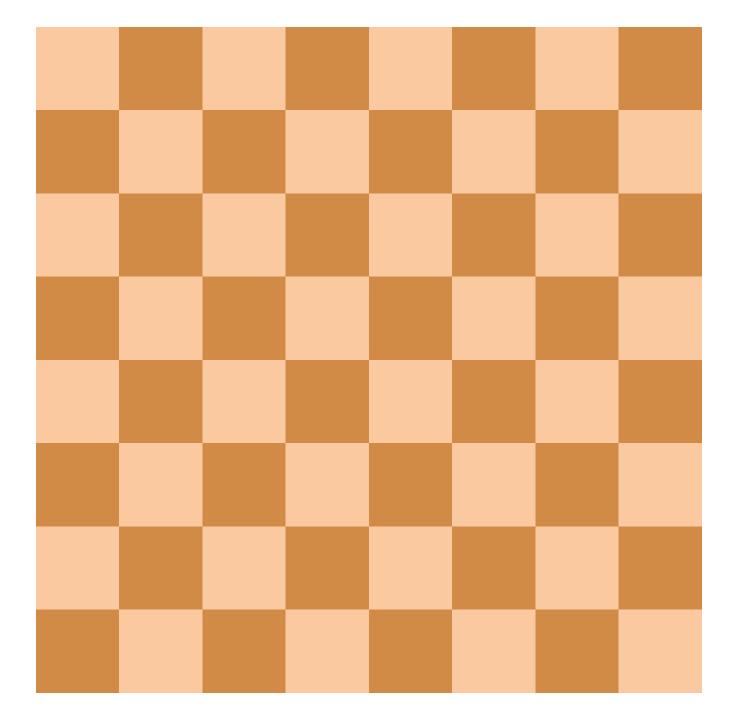




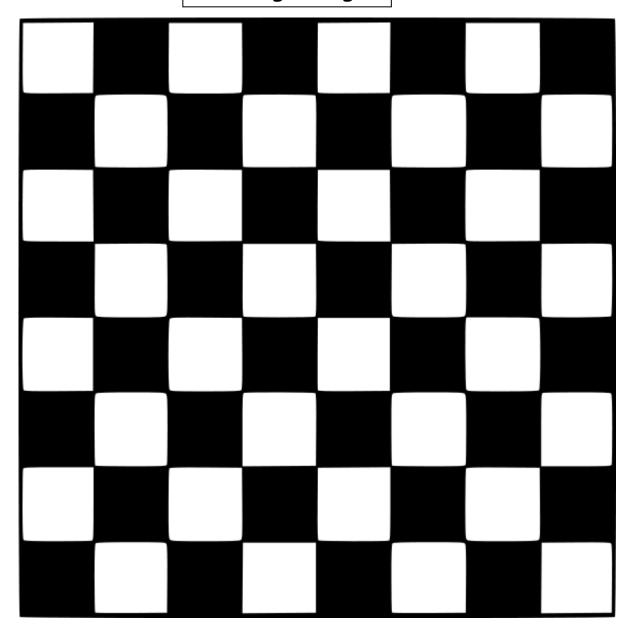


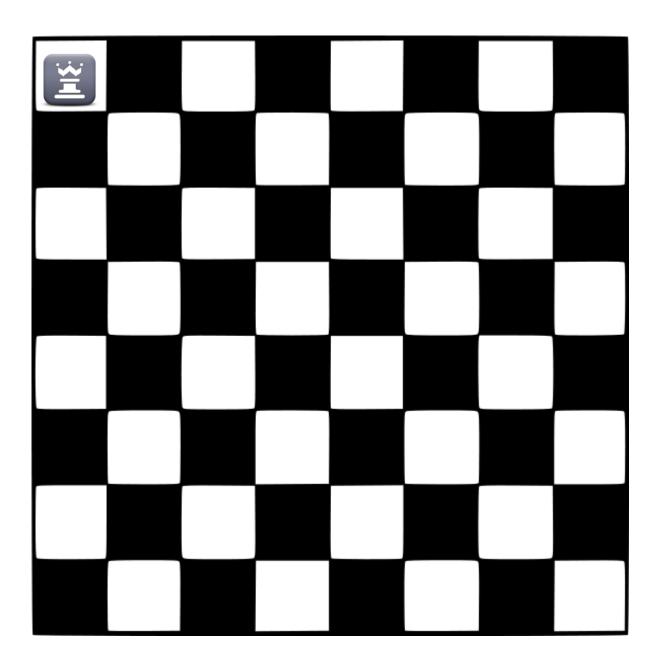


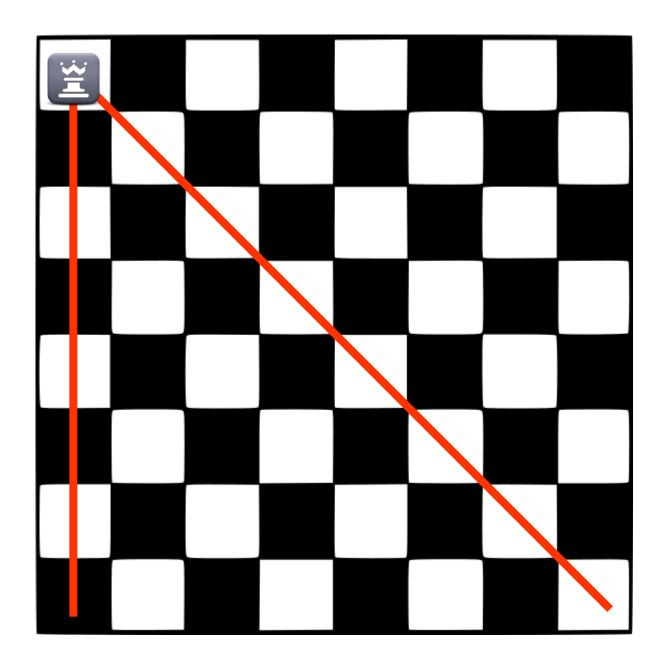


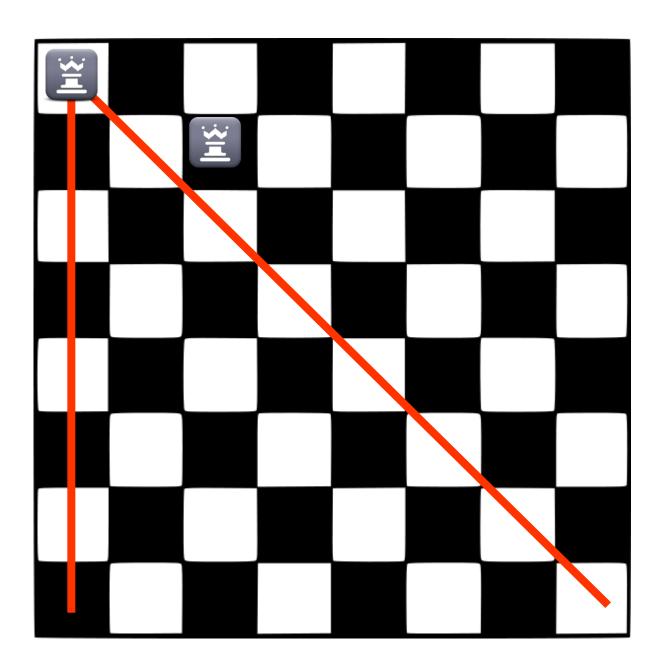


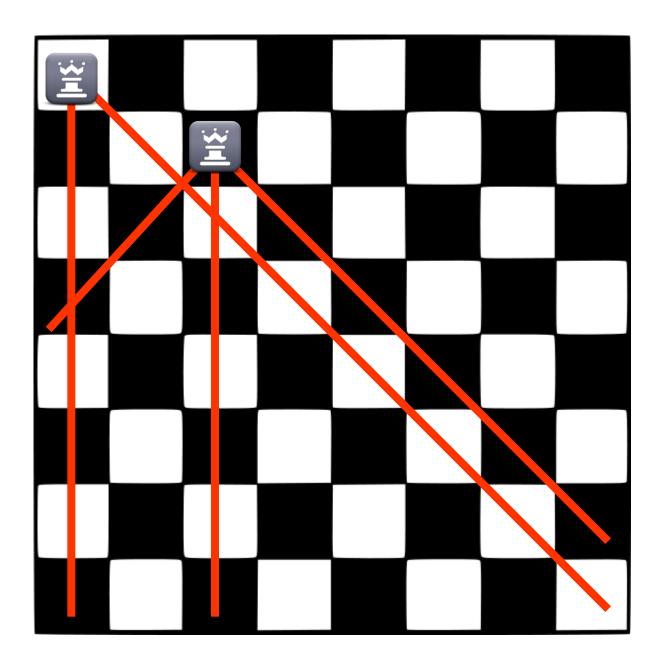
How might it go?

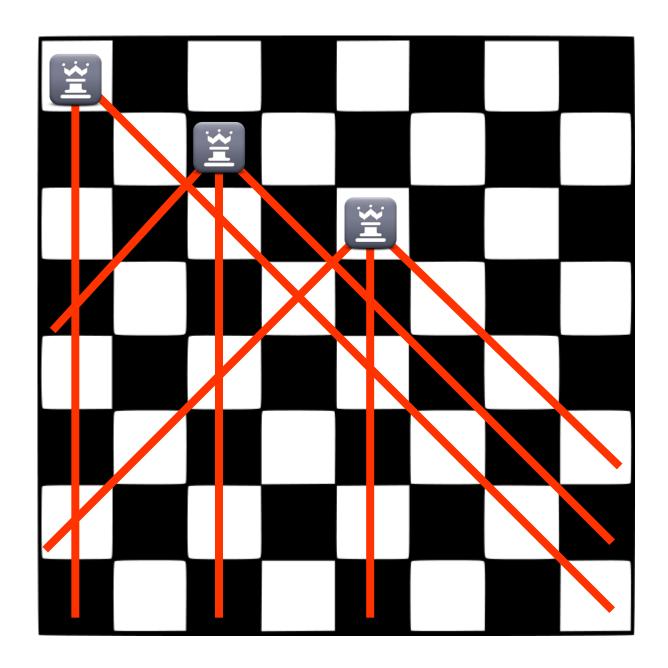


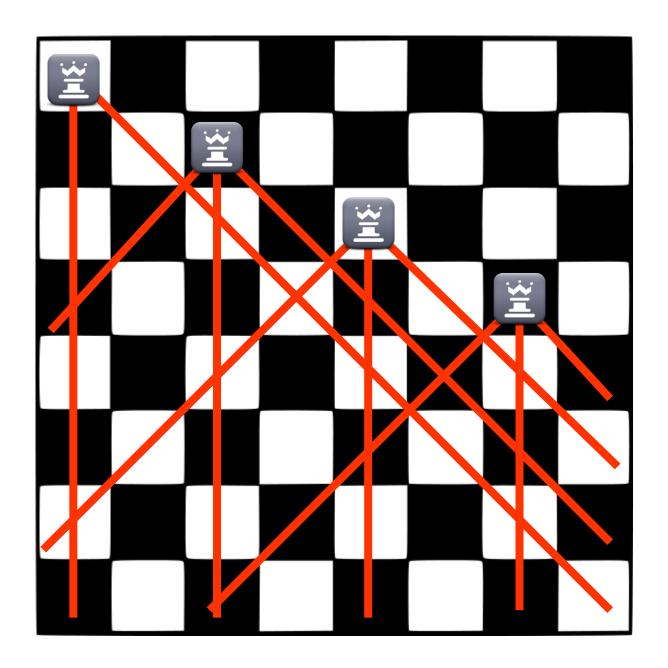


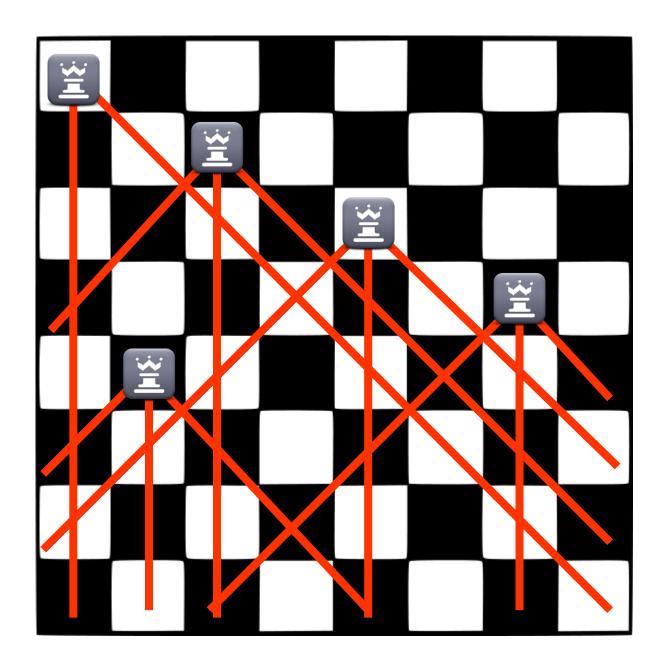


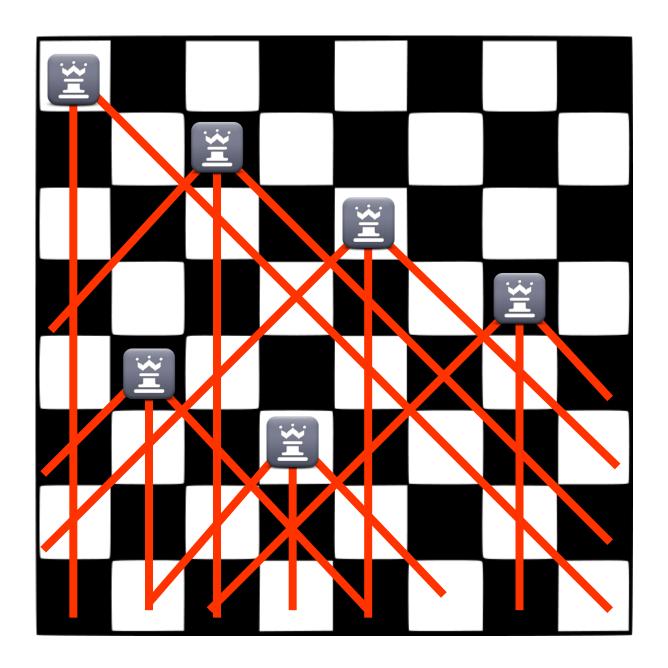


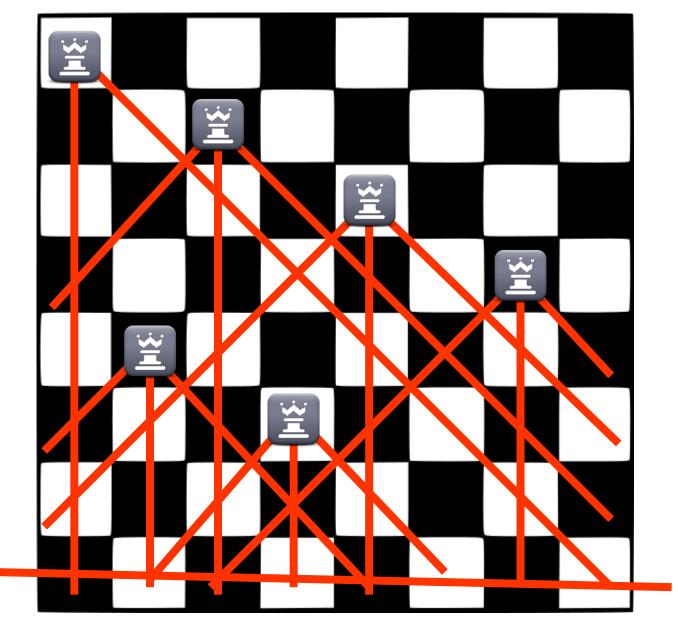


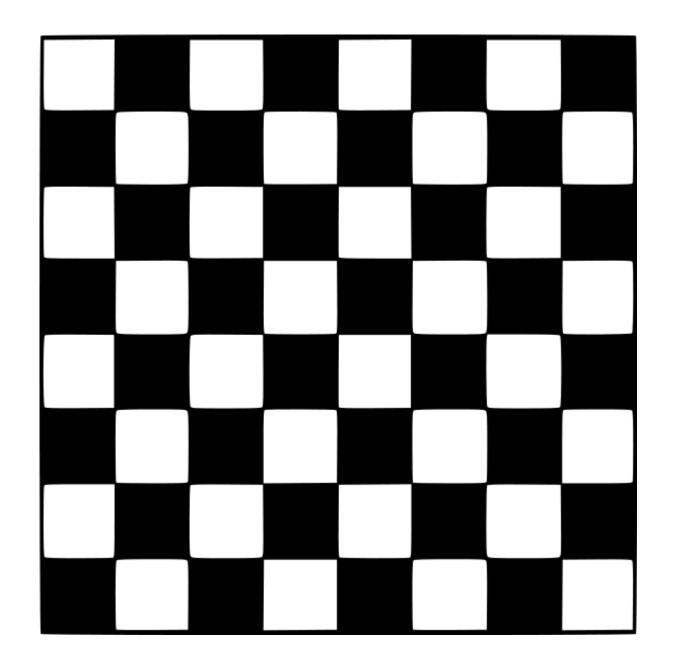


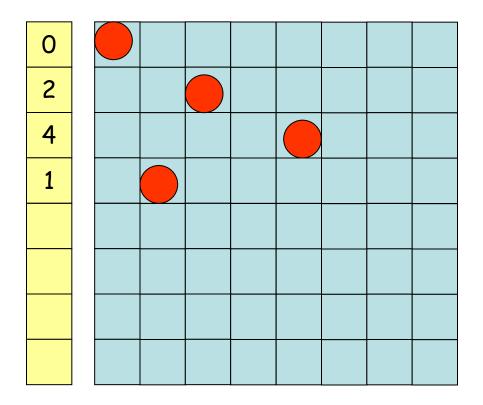




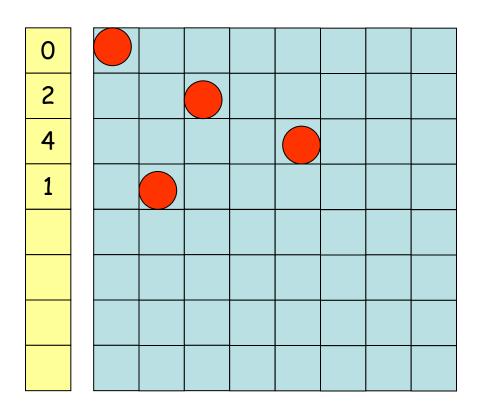








Representation/model 8 constrained integer variables Domains [0..7] A variable represents a row  $v[i] = j \leftrightarrow queen on row i, column j$ 



Representation/model
Constraints
no column attacks
v[i] ≠ v[j] for all i ≠ j
no diagonal attacks
|v[i] - v[j]| ≠ |i - j|

```
import java.io.*;
import java.util.*;
import org.chocosolver.solver.Model;
import org.chocosolver.solver.Solver;
import org.chocosolver.solver.variables.IntVar;
import org.chocosolver.solver.constraints.IIntConstraintFactory.*;
```

```
int n = Integer.parseInt(args[0]);
Model model = new Model("nqueens");
Solver solver = model.getSolver();
IntVar[] q = model.intVarArray("queen",n,0,n-1);
```

```
//
// columns constraint
//
for (int i=0;i<n-1;i++)
    for (int j=i+1;j<n;j++)
        model.arithm(q[i],"!=",q[j]).post();</pre>
```

```
//
// diagonal constraint
//
for (int i=0;i<n-1;i++)
    for (int j=i+1;j<n;j++)
        model.distance(q[i],q[j],"!=",Math.abs(i-j)).post();</pre>
```

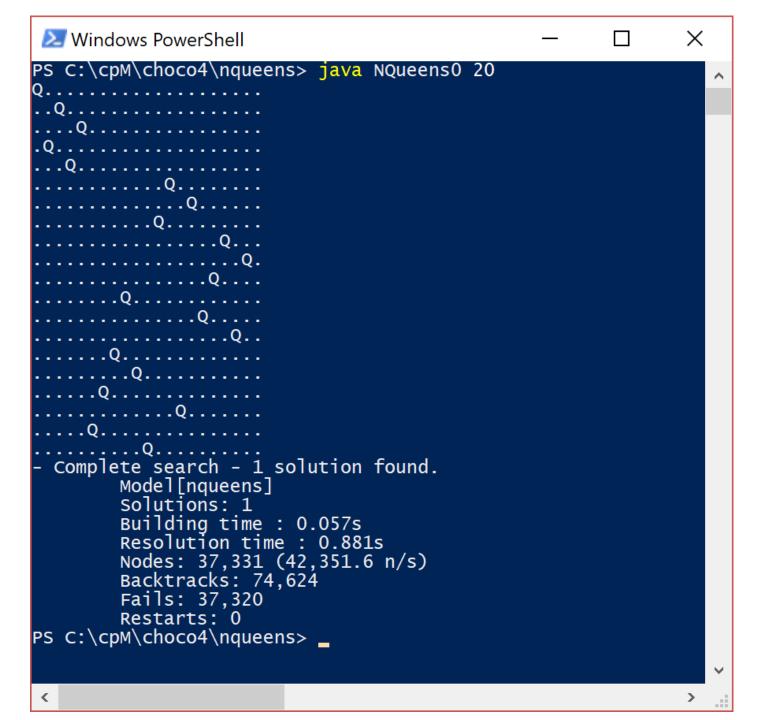


```
solver.setSearch(Search.inputOrderLBSearch(q)); // simplest
boolean solved = solver.solve();
```

```
if (solved) {
    for (int i=0;i<n;i++) {
        for (int j=0;j<q[i].getValue();j++) System.out.print(".");
        System.out.print("Q");
        for (int j=q[i].getValue();j<n;j++) System.out.print(".");
        System.out.println();
    }
}</pre>
System.out.println(solver.getMeasures());
```

```
💹 Windows PowerShell
PS C:\cpM\choco4\nqueens> java NQueens0 4
.Q...
..Q..
 Complete search - 1 solution found.
         Model[nqueens]
         solutions: 1
         Building time : 0.050s
Resolution time : 0.030s
         Nodes: 4 (131.3 n/s)
         Backtracks: 3
         Fails: 2
         Restarts: 0
PS C:\cpM\choco4\nqueens> _
<
```

```
Windows PowerShell
PS C:\cpM\choco4\nqueens> java NQueens0 8
. . . . Q . . . .
. . . . . Q. . .
. . . . . . Q. .
. . . Q . . . . .
 Complete search - 1 solution found.
         Model[nqueens]
          Solutions: 1
         Building time : 0.051s
Resolution time : 0.032s
          Nodes: 27 (848.3 n/s)
          Backtracks: 43
          Fails: 24
          Restarts: 0
PS C:\cpM\choco4\nqueens>
```





```
solver.setSearch(Search.minDomLBSearch(q)); // fail-first
boolean solved = solver.solve();
```

<

```
int solutions = 0;
while (solver.solve()) solutions++;
System.out.println("solutions: "+ solutions);
System.out.println(solver.getMeasures());
```

```
Windows PowerShell
                                                                           X
PS C:\cpM\choco4\nqueens> java NQueens3 8
                                                                               \wedge
solutions: 92
 Complete search - 92 solution(s) found.
          Model[nqueens]
Solutions: 92
          Building time : 0.057s
Resolution time : 0.089s
          Nodes: 455 (5,096.8 n/s)
          Backtracks: 727
          Fails: 272
          Restarts: 0
PS C:\cpM\choco4\nqueens> java NQueens3 12 solutions: 14200

    Complete search - 14,200 solution(s) found.

          Model[nqueens]
          Solutions: 14,200
          Building time : 0.058s
Resolution time : 2.017s
Nodes: 116,973 (58,003.8 n/s)
          Backtracks: 205,547
          Fails: 88,574
          Restarts: 0
PS C:\cpM\choco4\nqueens> t
 <
```

Version 3.2 (2-core)							
<	N-Queens Solut	ions>	< time>				
N:	Total	Unique	days hh:mm:ss.—				
5:	10	2	0.00				
6:	4	1	0.00				
7:	40	6	0.00				
8:	92	12	0.00				
9:	352	46	0.00				
10:	724	92	0.00				
11:	2680	341	0.00				
12:	14200	1787	0.00				
13:	73712	9233	0.02				
14:	365596	45752	0.05				
15:	2279184	285053	0. 22				
16:	14772512	1846955	1 47				
17:	95815104	11977939	9.42				
18:	666090624	83263591	1:11 21				
19:	4968057848	621012754	8:32.54				
20:	39029188884	4878666808	1:10:55.48				
	314666222712	39333324973	9:24:40.50				
21.	011000222712	30000Z4070	0.21.40.00				

AMD Athlon(tm) Dual Core Processor 5050e 2.60 GHz Microsoft Visual C++ 2008 Express Edition with SP1 Windows SDK for Windows Server 2008 and .NET

```
//
// columns constraint ... allDiff!
//
model.allDifferent(q).post();
```

```
//
// columns constraint ... allDiff!
//
model.allDifferent(q).post();
```

<b>Windows</b> Pov	verShell	_		×		
		Q				
				Q		
- Complete search - 1 solution found.  Model[nqueens]  Solutions: 1						
Building time : 0.174s Resolution time : 56.920s Nodes: 147,014 (2,582.8 n/s) Backtracks: 293,662 Fails: 146,838 Restarts: 0 PS C:\cpM\choco4\nqueens>						
				<b>~</b>		
<				>		

```
Windows PowerShell
  Complete search - 1 solution found.
         Model[nqueens]
         Solutions: 1
         Building time : 0.139s
Resolution time : 2.742s
         Nodes: 306 (111.6 n/s)
         Backtracks: 250
         Fails: 128
         Restarts: 0
PS C:\cpM\choco4\nqueens> _
```

Harder questions

Could we have a different representation?

Could we solve it a different way (other than systematic search)?

Does the problem get harder or easier as it gets larger?

What happens if we start with some queens already on the board?



Could we view nqueens differently, maybe as a permutation problem?

