# Allocating employees to cost centres 

A case study

- We are given a number of employees ( $n$ )
- where each employee has a name and a salary
- We are given a number of cost centres ( $m$ )
- where each cost centre has a budget
- all cost centres have the same budget
- Allocate the employees to cost centres, where
- the sum of salaries in a cost centre is within budget

```
import java.util.*;
public class Person implements Comparable<Person> {
String name;
int salary;
public Person(String name,int salary){
        this.name = name;
        this.salary = salary;
}
public int compareTo(Person p){
    return p.salary - salary;
}
public String toString(){
    return "("+ name +","+ salary +")";
}
}
```

```
import java.util.*;
public class Person implements Com
String name;
int salary;
public Person(String name,int
        this.name = name;
        this.salary = salary;
}
    public int compareTo(Person p)
        return p.salary - salary;
    }
public String toString(){
        return "("+ name +","+ sal
    }
}
```

                        Veronica 99
    Seth 72
    4 Cameron 43
    5 Constance 94
    6 Jenny 25
    7 Katharine 94
    ```
public class Allocate {
Person person[]; // the employees to be allocated to cost centres
int salary[]; // salary[i] of ith person
Model model; // the model
Solver solver; // the solver object
IntVar inCentre[][]; // inCentre[i][j] = 1 iff jth person works in ith cost centre
IntVar centreSalary[]; // centreSalaries[i] is sum of the salaries in the ith cost centre
int budget; // the budget for each cost centre
int n; // number of employees
int m; // number of cost centres
String id; // an identification for the problem
```

public Allocate(String fname, int numberOfPeople,int numberOfCostCentres,int budget) throws Exception
$\mathrm{n} \quad=$ numberOfPeople;
$\mathrm{m} \quad=$ numberOfCostCentres;
this.budget = budget;
id
= fname;
person $\quad=$ new Person[n];
salary $\quad=$ new int[n];
model $\quad=$ new Model(id);
solver = model.getSolver();
inCentre = model.intVarMatrix("inCentre",m,n,0,1);
centreSalary = model.intVarArray("centre salaries",m, 0, budget);

Scanner sc = new Scanner(new File(fname)); for (int $i=0 ; i<n ; i++$ ) person[i] = new Person(sc.next(),sc.nextInt()); sc.close();
for (int $i=0 ; i<n ; i++$ ) salary[i] = person[i].salary;

```
for (int i=0;i<m;i++)
        model.scalar(inCentre[i],salary,"=",centreSalary[i]).post();
//
// constrain centreSalary[i] to be the scalar product of inCentre[i] and salary[i]
//
for (int i=0;i<n;i++)
        model.sum(ArrayUtils.getColumn(inCentre,i),"=",1).post();
//
// constrain a person such that he/she can only be in one cost centre at a time!
// i.e. the sum of a column of the array inCentre must be equal to 1,
// such that a person is in exactly one cost centre
//
```

```
public static void main(String[] args) throws FileNotFoundException, IOException, Exception {
    if (args.length == 0){
        System.out.println("java Allocate fname budget #employees #centres");
        return;
    }
    String fname = args[0];
    int budget = Integer.parseInt(args[1]);
    int numberOfEmployees = Integer.parseInt(args[2]);
    int numberOfCentres = Integer.parseInt(args[3]);
    Allocate alloc = new Allocate(fname,numberOfEmployees,numberOfCentres,budget);
    boolean solved = alloc.solve();
    System.out.println(solved);
    if (solved) System.out.println(alloc);
    System.out.println("nodes: " + alloc.solver.getMeasures().getNodeCount() +
    " cpu: " + alloc.solver.getMeasures().getTimeCount());
}
```

C:\cpM\choco4\teamsWithBudgets>java Allocate
java Allocate fname budget \#employees \#centres
C:\cpM\choco4\teamsWithBudgets>java Allocate 0.txt 150 10 10
true
0.txt \#employees: 10 \#centres: 10 budget: 150
centre[0] 0 0 1 0 0 1 0 0 0 0 97
centre[1] 0 0 0 0 0 0 0 0 0 1 26
centre[2] 0 0 0 1 0 0 0 1 0 0 126
centre[3] 0 0 0 0 0 0 0 0 0 0 0
centre[4] 1 0 0 0 0 0 0 0 0 0 97
centre[5] 0 0 0 0 0 0 0 0 1 0 75
centre[6] 0 0 0 0 0 0 0 0 0 0 0
centre[7] 0 0 0 0 0 0 1 0 0 0 94
centre[8] 0 1 0 0 0 0 0 0 0 0 99
centre[9] 0 0 0 0 1 0 0 0 0 0 94
centre-0: (Seth,72) (Jenny,25) ... cost: 97
centre-1: (Edwin, 26) ... cost: 26
centre-2: (Cameron,43) (Sheryl,83) ... cost: 126
centre-3: ... cost: 0
centre-4: (Robyn,97) ... cost: 97
centre-5: (Tim,75) ... cost: 75
centre-6: ... cost: 0
centre-7: (Katharine,94) ... cost: 94
centre-8: (Veronica,99) ... cost: 99
centre-9: (Constance,94) ... cost: 94
nodes: 79 cpu: 0.038137063
C:\cpM\choco4\teamsWithBudgets>java Allocate 0.txt 150 10 7
true
0.txt \#employees: 10 \#centres: 7 budget: 150
centre[0] 0 0 0 0 0 1 1 0 0 0 119
centre[1] 0 0 0 1 0 0 0 0 1 0 118
centre[2] 0 1 0 0 0 0 0 0 0 0 99
centre[3] 0 0 1 0 0 0 0 0 0 0 72
centre[4] 0 0 0 0 1 0 0 0 0 1 120
centre[5] 1 0 0 0 0 0 0 0 0 0 97
centre[6] 0 0 0 0 0 0 0 1 0 0 83
centre-0: (Jenny, 25) (Katharine,94) ... cost: 119
centre-1: (Cameron,43) (Tim,75) ... cost: 118
centre-2: (Veronica,99) ... cost: 99
centre-3: (Seth,72) ... cost: 72
centre-4: (Constance,94) (Edwin,26) ... cost: 120
centre-5: (Robyn,97) ... cost: 97
centre-6: (Sheryl,83) ... cost: 83
nodes: 50 cpu: 0.02624389
C:\cpM\choco4\teamsWithBudgets>

```
C:\cpM\choco4\teamsWithBudgets>java Allocate 0.txt 150 10 6
true
0.txt #employees: 10 #centres: 6 budget: 150
centre[0] 0 0 0 1 0 0 1 0 0 0 137
centre[1] 0 1 0 0 0 1 0 0 0 0 124
centre[2] 0 0 0 0 0 0 0 1 0 1 109
centre[3] 0 0 1 0 0 0 0 0 1 0 147
centre[4] 0 0 0 0 1 0 0 0 0 0 94
centre[5] 1 0 0 0 0 0 0 0 0 0 97
centre-0: (Cameron,43) (Katharine,94) ... cost: 137
centre-1: (Veronica,99) (Jenny,25) ... cost: 124
centre-2: (Sheryl,83) (Edwin,26) ... cost: 109
centre-3: (Seth,72) (Tim,75) ... cost: 147
centre-4: (Constance,94) ... cost: 94
centre-5: (Robyn,97) ... cost: 97
nodes: 53 cpu: 0.030337468
C:\cpM\choco4\teamsWithBudgets>_
```

C: \cpM\choco4\teamsWithBudgets>java Allocate 0.txt 150105 false
nodes: 769 cpu: 0.14290735
C: \cpM\choco4\teamsWithBudgets>_

```
    Command Prompt
C:\cpM\choco4\teamsWithBudgets>java Allocate 0.txt 150 20 12
true
0.txt #employees: 20 #centres: 12 budget: 150
centre[0] 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 1 117
centre[1] 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 83
centre[2] 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 98
centre[3] 0 0 0 0 0 0 1 0 0 0 1 0 0 0 0 0 0 0 0 0 142
centre[4] 0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 142
centre[5] 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 124
centre[6] 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 1 0 0 0 0 109
centre[7] 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 1 0 0 117
centre[8] 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 00 0 0 97
centre[9] 0 0 1 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 00 0 147
centre[10] 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 39
centre[11] 0 0 0 0 0 1 0 0 0 0 0 1 0 0 0 0 0 0 0 0 110
centre-0: (Alvin,45) (Ernest,72) ... cost: 117
centre-1: (Sheryl,83) ... cost: }8
centre-2: (Julie,98) ... cost: 98
centre-3: (Katharine,94) (Hilda,48) ... cost: 142
centre-4: (Veronica,99) (Cameron,43) ... cost: 142
centre-5: (Constance,94) (Caroline,30) ... cost: 124
centre-6: (Edwin,26) (Nicole,83) ... cost: 109
centre-7: (Michael,77) (Stanley,40) ... cost: 117
centre-8: (Robyn, 97) ... cost: 97
centre-9: (Seth,72) (Tim,75) ... cost: 147
centre-10: (Marsha,39) ... cost: 39
centre-11: (Jenny,25) (Ronnie,85) ... cost: 110
nodes: 199
cpu: 0.04195229
C: \cpM\choco4\teamsWithBudgets>
```

true
0.txt \#employees: 20 \#centres: 10 budget: 150
centre[0] 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 1 149
centre[1] 0 0 0 0 0 1 0 1 0 0 0 0 0 0 0 0 0 1 0 0 148
centre[2] 0 0 0 0 0 0 0 0 0 1 0 0 1 0 0 1 0 0 0 0 148
centre[3] 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 94
centre[4] 0 0 1 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 147
centre[5] 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 00 0 0 0 0 94
centre[6] 1 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 142
centre[7] 0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 142
centre[8] 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 00 0 1 0 1 0 128
centre[9] 0 0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 133
centre-0: (Michael,77) (Ernest,72) ... cost: 149
centre-1: (Jenny,25) (Sheryl,83) (Stanley,40) ... cost: 148
centre-2: (Edwin,26) (Marsha,39) (Nicole,83) ... cost: 148
centre-3: (Constance,94) ... cost: 94
centre-4: (Seth,72) (Tim,75) ... cost: 147
centre-5: (Katharine,94) ... cost: 94
centre-6: (Robyn,97) (Alvin,45) ... cost: 142
centre-7: (Veronica,99) (Cameron,43) ... cost: 142
centre-8: (Caroline,30) (Julie,98) ... cost: 128
centre-9: (Hilda,48) (Ronnie,85) ... cost: 133
nodes: 634 cpu: 0.08598319
C: \cpM\choco4\teamsWithBudgets>_

```
\(n=20\)
C:\cpM\choco4\teamsWithBudgets>java Allocate 0.txt 150209 false
nodes: 9301860 cpu: 63.349964
C: \cpM\choco4\teamsWithBudgets>
variable \& value ordering

What are decision variables and what order are values picked?
// solve using value ordering over decision variables boolean solve()\{
//solver.setSearch(Search.minDomLBSearch(ArrayUtils.flatten(inCentre))); //solver.setSearch(Search.minDomUBSearch(AlrrayUtils.flatten(inCentre))); return solver.solve();

This is a classic problem ...


\section*{[SR1] BIN PACKING}

INSTANCE: Finite set \(U\) of items, a size \(s(u)\) in \(Z^{+}\)for each \(u\) in \(U\), a positive integer bin capacity \(B\), and a positive integer \(K\).

QUESTION: Is there a partition of \(U\) into disjoint sets \(U_{1}, U_{2}, \ldots, U_{k}\) such that the sum of the sizes of the items in each \(U_{i}\) is \(B\) or less?

Garey \& Johnson
"Computers and Intractability: A guide to the theory of NP-Completeness"

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Is there a heuristic we might use?

\section*{Bin Packing} First fit decreasing algorithm


With the first fit decreasing algorithm we sort the blocks into descending order first.
4


\section*{Bin Packing}

\section*{First fit decreasing algorithm}


Now we use the first fit algorithm


\section*{Bin Packing} First fit decreasing algorithm


Now we use the first fit algorithm



Now we use the first fit algorithm



Now we use the first fit algorithm


\section*{Bin Packing}


Now we use the first fit algorithm


\section*{Bin Packing}


Now we use the first fit algorithm



Now we use the first fit algorithm


\section*{Bin Packing}


Now we use the first fit algorithm

\section*{Bin Packing}


Now we use the first fit algorithm

\section*{Bin Packing} First fit decreasing algorithm


We have packed them into 5 bins.



\section*{Welcome to:}


National Association for Numeracy and Mathematics in Colleges

\section*{Improving Learning in Mathematics} approaches that encourage active learning including group work, discussion and open questions.
\[
\text { Friday } 23^{\text {rd }} \text { October } 2009 .
\]

University of Wales, Caerleon campus, Newport

News:
```

Scanner sc = new Scanner(new File(fname));
for (int i=0;i<n;i++) person[i] = new Person(sc.next(),sc.nextInt());
sc.close();
// EDIT
// Arrays.sort(person);
//
// first fit decreasing
//
for (int i=0;i<n;i++) salary[i] = person[i].salary;

```

Try \(1^{\text {st }}\) fit decreasing (see Person)

Slow proving optimality

Don't have a test that sum of numbers over capacity is less than or equal to the number of bins available!

\section*{Symmetries?}

Are there any symmetries that are slowing down search?
Can we remove those symmetries?

What are the symmetries in this problem?
```

// EDIT
//for (int i=0;i<m-1;i++)
// model.arithm(centreSalary[i],">=",centreSalary[i+1]).post();
//
// symmetry breaking consistent with first fit decreasing
// costliest cost centres have low index
//
// EDIT
//for (int centre=0;centre<m-1;centre++)
// model.lexLessEq(inCentre[centre+1],inCentre[centre]).post();
//
// symmetry breaking such that inCentre[i] lex>= inCentre[i+1]
//

```

Is there another model?

\(\leftarrow \rightarrow\) C ir (i) www.choco-solver.org/apidocs/index.html
… \(\boldsymbol{\sim}\) Q Search

All Classes

\section*{Packages}
org.chocosolver.memory
org.chocosolver.memory.structure
org.chocosolver.memory.trailing
org.chocosolver.memory.trailing.trail
org.chocosolver.memory.trailing.trail.chunc \(\checkmark\)
<

LongWorld
MathUtils
MaxDelta
MaxRegret
MD
MDRk
Measures
MeasuresRecorder
Member
MinDelta
MinusView
Model
MonotonicRestartStrategy
Move
MoveBinaryDDS
MoveBinaryDFS
MoveBinaryHBFS
MoveBinaryLDS
MoveLearnBinaryTDR
MoveLNS
MoveRestart
MoveSeq

\section*{binPacking}
default Constraint binPacking(IntVar[] itemBin,
\[
\begin{aligned}
& \text { int[] itemSize, } \\
& \text { IntVar[] binLoad, } \\
& \text { int offset) }
\end{aligned}
\]

Creates a BinPacking constraint. Bin Packing formulation: forall bin [o,binLoad.length-1], binLoad[b]=sum(itemSize[i] | i in [o,itemSize.length-1], itemBin[i] = b+offset forall i in [o,itemSize.length-1], itemBin is in [offset,binLoad.length-1+offset],

\section*{Parameters:}
itemBin - IntVar representing the bin of each item
itemSize - int representing the size of each item
binLoad - IntVar representing the load of each bin (i.e. the sum of the size of the items in it)
offset - 0 by default but typically 1 if used within MiniZinc (which counts from 1 to \(n\) instead of from 0 to n-1)

\section*{boolsIntChanneling}
default Constraint boolsIntChanneling(BoolVar[] bVars,
\[
\begin{aligned}
& \text { IntVar var, } \\
& \text { int offset) }
\end{aligned}
\]

Creates an channeling constraint between an integer variable and a set of boolean variables. Maps the boolean assignments variables bVars with the standard assignment variable var.
var \(=\mathrm{i}\) <-> bVars[i-offset] = 1

```

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IIntConstraintFactory (Choco-4. X
\leftarrow
->C狊
(i) https://link.springer.com/chapter/10.
2 A Constraint for Bin Packing|S }
\&s.//ink.springer.com/chapter/10 \cdots \& Q Search
Buy options

```

\section*{Abstract}
```

We introduce a constraint for one-dimensional bin packing. This constraint uses propagation rules incorporating knapsack-based reasoning, as well as a lower bound on the number of bins needed. We show that this constraint can significantly reduce search on bin packing problems. We also demonstrate that when coupled with a standard bin packing search strategy, our constraint can be a competitive alternative to established operations research bin packing algorithms.

```
lII 1

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\section*{Thanks me ol' mucka!}





File Edit Options Buffers Tools Java Help

public AllocateBP(String fname, int numberOfPeople, int numberOfCostCentres, int budget) throws Exception \{
    \(\mathrm{n} \quad=\) numberOfPeople;
    \(\mathrm{m} \quad=\) numberOfCostCentres;
    this.budget \(=\) budget;
    id \(=\) fname;
    person \(\quad=\) new Person[n];
    salary \(\quad=\) new int \([\mathrm{n}]\);
    model \(\quad\) new Model (id);
    solver \(\quad=\) model.getSolver ();
    employee \(\quad\) model.intVarArray ("employee", \(\mathrm{n}, \mathrm{0}, \mathrm{m}-1\) );
    costCentre = model.intVarArray("cost centre", m, 0, budget);
    Scanner \(s c=\) new Scanner(new File(fname));
    for (int \(i=0 ; i<n ; i++\) ) person[i] = new Person(sc.next(),sc.nextInt());
    sc.close () :
    for (int \(i=0 ; i<n ; i++\) ) salary[i] = person[i].salary;
    model.binPacking (employee, salary, costCentre, 0) . post ():
\}
// solve using value ordering over decision variables
boolean solve() \{
    solver.setSearch (Search.minDomUBSearch (employee));
    return solver. solve();
\}
AllocateBP.java \(30 \%\) L47 (Java/l Abbrev)

What have we learned?
1. Identify the decision variables
2. What is value ordering doing to the search?
3. Can we use any heuristics?
4. Are there symmetries that we can break?
5. Are there any simple/redundant tests/constraints overlooked?
6. Is there an alternative model?
- How would we modify our model to address
- Two people must be in same cost centre
- Two people must not be in same cost centre
- Cost centres have a limit on
- Sum of salaries and ...
- Number of employees in cost centre```

