# **Modelling Constraints** - by Constraining the Models

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### **Talk Outline**

#### 4 Blocks:

- Purpose and Concept
- UML and OCL
- Modelling Example
- Outlook and Conclusion

### **Part 1 – Purpose and Concept**

### Goals:

(1) modelling support for constraint problems

(2) suitable reuse of public standards (UML/OCL, ...)

### Motivation

- Ittle modelling support for (re-) formulating CSPs
  - most problems still solved by experts
- constraint engineering



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- Ittle modelling support for (re-) formulating CSPs
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structure information gained in analysis

- varying levels of abstraction
- different degrees of precision

### **Classical CSP Definition**



#### CSP:

- variables  $X := \{x_1, ..., x_n\}$
- domains  $D := \{d_1, ..., d_n\}$
- constraints  $C \subseteq d_1 \times d_2 \times \ldots \times d_{n-1} \times d_n$

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$$X := \{x_1, ..., x_n\}$$

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• constraints 
$$C \subseteq d_1 \times d_2 \times \ldots \times d_{n-1} \times d_n$$

#### **Example:**

$$X = \{a, b\}; D = \{\mathbb{N}, \mathbb{N}\}\$$
$$C \equiv a < 22 \land a \le b \land b > 12$$

### **Purpose: Why use class models**

objective: complex data in terse format

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- constraint (hyper-) graph inappropriate



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- objective: complex data in terse format
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 $\Rightarrow$  no structural abstraction

## **Purpose: Modelling Support**

#### Support In Other Disciplines:

Semantic Data Models in Database Design

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- Semantic Data Models in Database Design
- Semantic Networks in Knowledge Engineering

# **Purpose: Modelling Support**

#### Support In Other Disciplines:

- Semantic Data Models in Database Design
- Semantic Networks in Knowledge Engineering
- Ubiquitous modelling in:
  - Software development
  - Hardware development
  - Engineering in general

Problem Specification

**Source Code** 

Modelling Constraints - p.9/42



**Source Code** 

Modelling Constraints - p.9/42



Source Code



### Part 2a: UML

### The 4+1 view of UML



Scalability Performance Distribution Installation System Topology

### The 4+1 view of UML



Scalability Performance Distribution Installation System Topology

# **Types of UML Diagrams**



# **Types of UML Diagrams**



## **UML Packaging**



### **UML Packaging**



#### classification



classification

▷ association



- classification
- ▷ association
- ▷ aggregation



- classification
- ▷ association
- aggregation
- ▷ attributes



- classification
- > association
- > aggregation
- > attributes

ISA
 (specialization / generalization)



## **Example IS-A Relation**

properties explained in terms of structural differences



#### inheritance of:

- attributes
- associations
- constraints

### Part 2b: OCL

# The Object Constraint Language (OCL)

textual complement to UML

first order logic with navigation

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- textual complement to UML
- first order logic with navigation

### Use of Types

- basic types: Real, Integer, Boolean
- container types: Collection, Set, Sequence, Bag
- user-defined: Tuple { }
  - e.g. Tuple{ Set(1,3,4), ``Joe Bloggs'', 1999, Sequence(Tuple(2,1}, Tuple{2,-1}) }

# The Object Constraint Language (OCL)

- textual complement to UML
- first order logic with navigation

### Use of Types

- basic types: Real, Integer, Boolean
- container types: Collection, Set, Sequence, Bag
- user-defined: Tuple{}

UML model: all classes accessible

### Status of the OCL

- fully integrated into UML 1.4
- several implementations (Boldsoft, Argo/UML, ...)

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#### **A Flavour of OCL: N-queens**



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Attribute access

context Nqueen inv: column > 0 and column <= N and row > 0 and row <= N</pre>

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#### Attribute access

context Nqueen inv: column > 0 and column <= N and row > 0 and row <= N</pre>

Navigation

context Nqueen inv:
 nQProblem.numQueens >= 3

#### **A Flavour of OCL**



#### Pseudo - attributes

## --simplifies the other expressions context Nqueen def:

attr N : Integer = nQProblem.numQueens

### **A Flavour of OCL**



#### Pseudo - attributes

--simplifies the other expressions context Nqueen def:

attr N : Integer = nQProblem.numQueens

#### Set expressions

context NQProblem inv:
 numQueens = nqueen->size()

#### **A Flavour of OCL**



#### Quantification

--no two queens attack another:

# context NQProblem inv: nqueen->forAll(q1, q2 : Nqueen | not q1.attacks(q2) )

#### **Pseudo - Operations in OCL**



context Nqueen def:
oper attacks(other: Nqueen) : Boolean =
 self.column <> other.column and
 self.row <> other.row
 and
 (self.row - other.row).abs <>
 (self.column - other.column).abs

### Part 3 – Modelling Example

### **The Bridge - Building Problem**



schedule the setup of a 5 - segment bridge
Martin Bartusch 1983, Massimo Paltrinieri 1994

#### 11 bridge components, 46 tasks, 7 resources

Number of Constraints:

11 bridge components, 46 tasks, 7 resources

#### Number of Constraints:

- Resource Constraints:
  - most tasks multiply on n = 2...6 components

• 
$$\frac{n(n-1)}{2}$$
 constraints each  $\Rightarrow$  77 constraints

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Precedences among tasks: 66 constraints

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- Precedences among tasks: 66 constraints
- Five additional requirements: 25 constraints

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- Precedences among tasks: 66 constraints
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#### Sum: 168 constraints

### Bridge - Building: Constraint Graph



Modelling Constraints - p.26/42

### Bridge - Building: Constraint Graph



### Bridge - Building: generic constraints 1/2

#### Disjunction: Unary Resources

### Bridge - Building: generic constraints 1/2

#### Disjunction: Unary Resources

	Task
0*	resource start duration
previous	part

#### context Task inv:

Task.allInstances->forAll(a,b: Task|

a <> b and

a.name() = b.name() and

a.resource = b.resource
implies (

a.start + a.duration <= b.start xor b.start + b.duration <= a.start )</pre>

### Bridge - Building: generic constraints 2/2

Precedence Constraints



#### context Task inv:

```
Task.allInstances->forAll(t: Task|
    t.previous->forAll(prev: Task|
    prev.start +
    prev.duration <=
    t.start ) )</pre>
```

### Bridge - Building: All Tasks



- generic constraints apply to all 14 subtypes
- 2 instead of 143

### requ3: Erection of temporary housing



context Erection inv:
 start + 6 <= formwork.start</pre>

#### Part 4 – Outlook and Conclusion

#### **Outlook: Application Development**

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class foo : public baz {

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Browse ...

Browse

Start

Modelling Constraints - p.32/42

#### **Outlook: Distributed CSPs**



### **Benefits of using UML & OCL**

- simplified, comprehensible knowledge acquisition
- seamless integration:
  - constraint-based reasoning
  - software development
- open to emerging developments (Semantic Web)

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#### Experiences: UML/OCL in

- Software Architecture Description Languages (ADL)
- excellent experiences for configuration KBS
- Ontology Description
- CommonKADS, Rule-Based Systems

- informal notation  $\leftrightarrow$  informal use
  - semantics in 'precise' English
  - conflicting definitions, ambiguities, imprecision
- meta-circular: defines itself by itself
- commercial standard dependent on OMG politics
  - slow adaptation (6-12 months)

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#### Conclusion

#### Useful for further work

- intuitively appealing notation
- informal notation as 'scratch pad'
- abstraction / generalisation / packaging facilities
- translation to XML

#### What to do next

- define a precise UML subset
- interfacing with eg. Localizer
- explore model transformation

#### Thank you for your attention.

### **Configuration Knowledge Base in UML**



#### **Additional Information: MOF**



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#### Meta Object Facility (MOF)

- MOF is a simple language for defining languages
- UML is a MOF-based metamodel

Level	MOF Terms	Examples
M3	meta-metamodel	MOF Model
M2	meta-metadata	UML Metamodel
M1	metadata	Class Diagram
MO	data	User Objects

### **Additional Information: XMI**

- XMI = XML Metadata Interchange (OMG)
- is a way to save UML models in XML
  - for any MOF-based metamodel
- ability to move UML models between tools

```
<XMI.header>

<XMI.documentation>

<XMI.exporter>Together</XMI.exporter>

<XMI.exporterVersion>4.0</XMI.exporterVersion>

</XMI.documentation>

<XMI.metamodel xmi.name = 'UML' xmi.version = '1.1'/>

</XMI.header>
```
## **Additional Information: What's in XMI?**

- Document Type Definition (DTD) rules for transforming MOF based models into XML DTDs
- XML Document production rules for MOF based data

UML can be regarded as:

- an XML document conforming to a DTD
- an XML DTD to which UML models must conform

*Caveat:* version differences between XMI and UML

Iimits tool-to-tool interchange