PROB038: STEEL MILL SLAB DESIGN

1. Problem description

Steel is produced by casting molten iron into slabs. A steel mill produces a bounded number of slab sizes. Steel orders are distinguished by two properties, the weight and a colour to denote a route through the steel mill. Given j input orders, the problem is to allocate slabs to orders, determine the number and size of required slabs, and to minimise the total weight of produced steel. This assignment is subject to two further constraints:

- *Capacity constraints:* the total weight of orders must not be greater than the slab size.
- *Colour constraints:* each slab can be assigned orders of at most p different colours.

2. Z- Model

Instance data. The above can thus be described by a set of input orders and a set of available slab sizes:

$$Orders, Slabs : \mathbb{F}\,\mathbb{N}$$

Thus, $\sigma = \#(Slabs)$. There is further a set of colours to denote possible routes:

$[Colours]$

Problem constraints. A further parameter is p, the attributes of *Orders* are introduced as functions in the following schema:

Slab_Design

$$\begin{array}{l} p, objective : \mathbb{N} \\ weight : Orders \longrightarrow \mathbb{N} \\ colour : Orders \longrightarrow Colours \\ Allocate : Orders \longrightarrow Slabs \end{array}$$

$$\begin{array}{l} \forall\, s : \operatorname{ran} Allocate;\ rel : Orders \longrightarrow Slabs \mid rel = Allocate \rhd \{s\} \bullet \\ \qquad \sum(weight \parallel \operatorname{dom}(rel) \parallel) \leq s \wedge \\ \qquad \#(colour (\!| \operatorname{dom}(rel) |\!)) \leq p \end{array}$$

As main problem variable, we use the function *Allocate*. Using range restriction on this variable, the two problem constraints are straightforward. The first sums up all order weights related to any allocated slab size s and constrains this value to be smaller than or equal to s. The second collects all different colours of orders related to a slab s into a set, whose cardinality is to be less than or equal to p.

Optimisation part. Since the objective in this problem is to minimise the sum of produced steel, we sum over the range of *Allocate* in the definition of the *objective* function.

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Optimisation_Part

$solution : Slab\_Design$
$objective : Slab\_Design \longrightarrow \mathbb{N}$

$\forall\, s : Slab\_Design \bullet$
$\quad objective(s) = \sum(\mathrm{ranbag}(\mathrm{s.Allocate}))$
$objective(solution) = \min(objective(\!| Slab\_Design |\!))$

For this purpose, we use the functions Σ for summing and ranbag to create the bag corresponding to the range of a function, both are introduced in the Appendix. Finally, we specify that the *solution* to the problem must exhibit a minimal value of the *objective* function.

3. Literature references

The problem was originally introduced in [1]; the most successful of three presented basic models was developed further in [2], where the addition of symmetry-breaking and implied constraints (such as additional variables and unary constraints) lead to a performance improvement. An UML/OCL model was developed for the problem in [4]. Recently, ILP, CP and hybrid ILP/CP models (created by combining the ILP and CP parts via channeling constraints) for the problem were compared in [3], where the hybrid models exhibited the best performance.

References

[1] Alan M. Frisch, Ian Miguel, and Toby Walsh. Modelling a Steel Mill Slab Design Problem. In Christian Bessière, editor, *Proceedings of the IJCAI-01 Workshop on Modelling and Solving Problems with Constraints*, pages 39–45, 2001.

[2] Alan M. Frisch, Ian Miguel, and Toby Walsh. Symmetry and Implied Constraints in the Steel Mill Slab Design Problem. In *Proceedings of the CP'01 Workshop on Modelling and Problem Formulation*, pages 8–15, 2001.

[3] Brahim Hnich, Zeynep Kiziltan, Ian Miguel, and Toby Walsh. Hybrid Modelling for Robust Solving. *Annals of Operations Research*, 130(1-4):19–39, August 2004.

[4] Gerrit Renker, Hatem Ahriz, and Inés Arana. CSP - There is more than one way to model it. In Max Bramer, Alun Preece, and Frans Coenen, editors, *Research and Development in Intelligent Systems XIX - Proceedings ES 2002, the Twenty-second SGAI International Conference on Knowledge Based Systems and Applied Artificial Intelligence*, pages 395–408. Springer, 2002.