Variable & Value Ordering Heuristics

Heuristics for backtracking algorithms

- Variable ordering
  - what variable to branch on next
- Value ordering
  - given a choice of variable, what order to try values
- Constraint ordering
  - what order to propagate constraints
  - most likely to fail or cheapest propagated first

Variable ordering

- Domain dependent heuristics
- Domain independent heuristics
- Static variable ordering
  - fixed before search starts
- Dynamic variable ordering
  - chosen during search

Basic idea

- Assign a heuristic value to a variable that estimates how difficult/easy it is to find a satisfying value for that variable

SVO

Static variable orderings

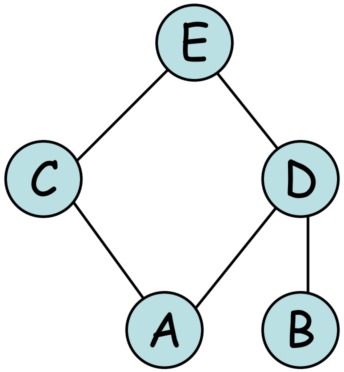
- based on constraint graph topology
  - minimum width
  - minimum induced width
  - max degree ordering
  - minimum bandwidth ordering
- based on something else

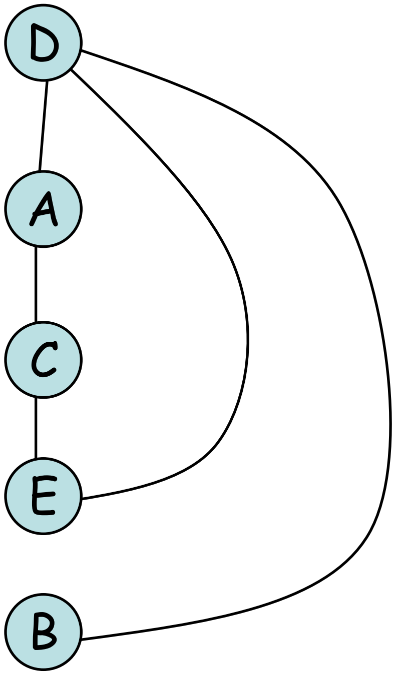
Usually for backward checking algorithms

- why?

Static variable orderings

"order" the constraint graph in a certain way





Minimum width ordering
- width of a node is number of adjacent predecessors
- width of an ordering is maximum width of the nodes
- width of a graph is minimal width of all orderings

Max degree ordering (shown)
- in non-decreasing degree sequence

Why should this work?
Is there anything bad bout it?

Minimum width aka degeneracy ordering

Minimum width aka degeneracy ordering

1. Select vertex v of maximum degree
2. Remove v from graph
   - reduce degree of vertices adjacent to v
3. If vertices remain, go to 1

Minimum Bandwidth Ordering (MBO)

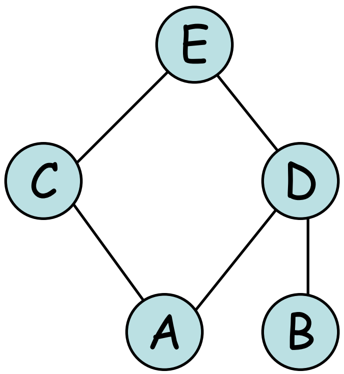
What is that?

What's its complexity?

Do we need it if we can jump?

- Bandwidth of a variable is the "distance" between variables in the ordered constraint graph
- Bandwidth of ordering is max bandwidth of varaibles/vertices

Minimum Bandwidth Ordering (MBO)



Measuring backwards

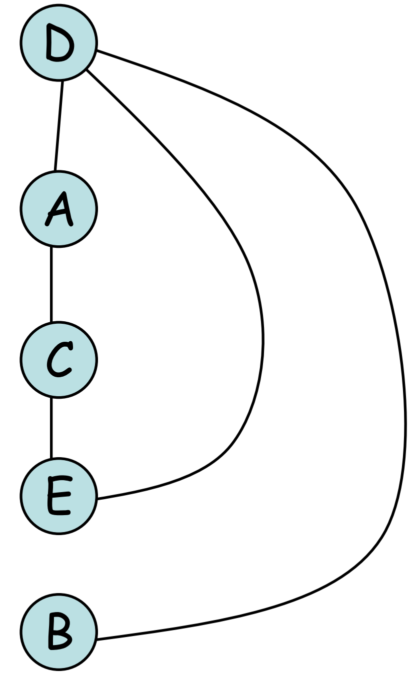
bw(D) = 0

bw(A) = 1

bw(C) = 1

bw(E) = 3

bw(B) = 4



Bandwidth of ordering is 4

MBO is minimum of all orderings
NP-hard to find ☹

Bandwidth is the “distance” between variables in the ordered constraint graph

DVO

Dynamic variable ordering (dvo)

Domain Indepentent

- Mainly based on the FF principle
- Mainly used by MAC and FC (why?)
    - smallest domain first
    - brelaz
    - dom/deg

Regret
For each variable measure it's regret as (best value – next best value)
Chose variable with maximum regret

Fail First Principle: "*To succeed, try first where you are most likely to fail*" Haralick & Elliott 1980

3.2 Search Strategies

The search space induced by variable domains is equal to $S = |d_1| * |d_2| * ... * |d_n|$ where d_i is the domain of the i^{th} variable. Most of the time (not to say always), constraint propagation is not sufficient to build a solution, that

is, to remove all values but one from variable domains. Thus, the search space needs to be explored using one or more *search strategies*. A search strategy defines how to explore the search space by computing *decisions*. A decision involves a variables, a value and an operator, e.g. $x = 5$, and triggers new constraint propagation. Decisions are computed and applied until all the variables are instantiated, that is, a solution has been found, or a failure has been detected (backtrack occurs). Choco 4.0.5 builds a binary search tree: each decision can be refuted (if $x = 5$ leads to no solution, then $x! = 5$ is applied). The classical search is based on Depth First Search.

Note: There are many ways to explore the search space and this steps should not be overlooked. Search strategies or heuristics have a strong impact on resolution performances. Thus, it is strongly recommended to adapt the search space exploration to the problem treated.

3.2.1 Default search strategy

File Edit View History Bookmarks Tools Help

Formulating a Constraint Satisfactio × | Search (Choco-4.0.6: an Open-S ×

www.choco-solver.org/apidocs/index.ht | Search

All Classes

Packages

org.chocosolver.memory
org.chocosolver.memory.st
org.chocosolver.memory.tr
org.chocosolver.memory.tr

Rules
RuleStore
S64BitSet
SafeIntProcedure
SatConstraint
ScaleView
Search
SearchMonitorList
SearchState
SequenceNeighborhood
Set_BitSet
Set_CstInterval
Set_FixedArray
Set_LinkedList
Set_ReadOnly
Set_Std_BitSet
Set_Std_Swap
Set_Std_Swap2
Set_Swap
Set_Swap2
SetDecision

OVERVIEW PACKAGE CLASS USE TREE DEPRECATED INDEX HELP

PREV CLASS NEXT CLASS FRAMES NO FRAMES

SUMMARY: NESTED | FIELD | CONSTR | METHOD DETAIL: FIELD | CONSTR | METHOD

org.chocosolver.solver.search.strategy

Class Search

java.lang.Object
org.chocosolver.solver.search.strategy.Search

```
public class Search
extends Object
```

Constructor Summary

Constructors

Constructor and Description
`Search()`

Method Summary

```
solver.setSearch(Search.minDomLBSearch(q)); // fail-first
```

Dom over weighted degree (example)

When propagation of a constraint results in a dwo (domain wipe out)
Increment the weight of that constraint

For a variable v, sum up the weight of the constraints it is involved in

h(v) = card(dom(v))/weightedDegree(v)

Select variable with minimum h(v)

Some more recent dvo's

- Conflict ordering search [cp2015]
- Reasoning from last conflict(s) [AIJ 173, 2009]
- Boosting systematic search by weighting constraints [ECAI2004]

Cutset decomposition

If constraint graph is a tree then *AC* is a decision procedure
(result due to E.C. Freuder (Gene))

Select a variable that cuts the constraint graph

Value Ordering

Value ordering

- All solutions
  - value ordering not important
  - why?
- One solution
  - if a solution exists, there exists a *perfect* value ordering
- Insoluble instance
  - like all solutions
  - why?

Value ordering: Intuition (*promise*)

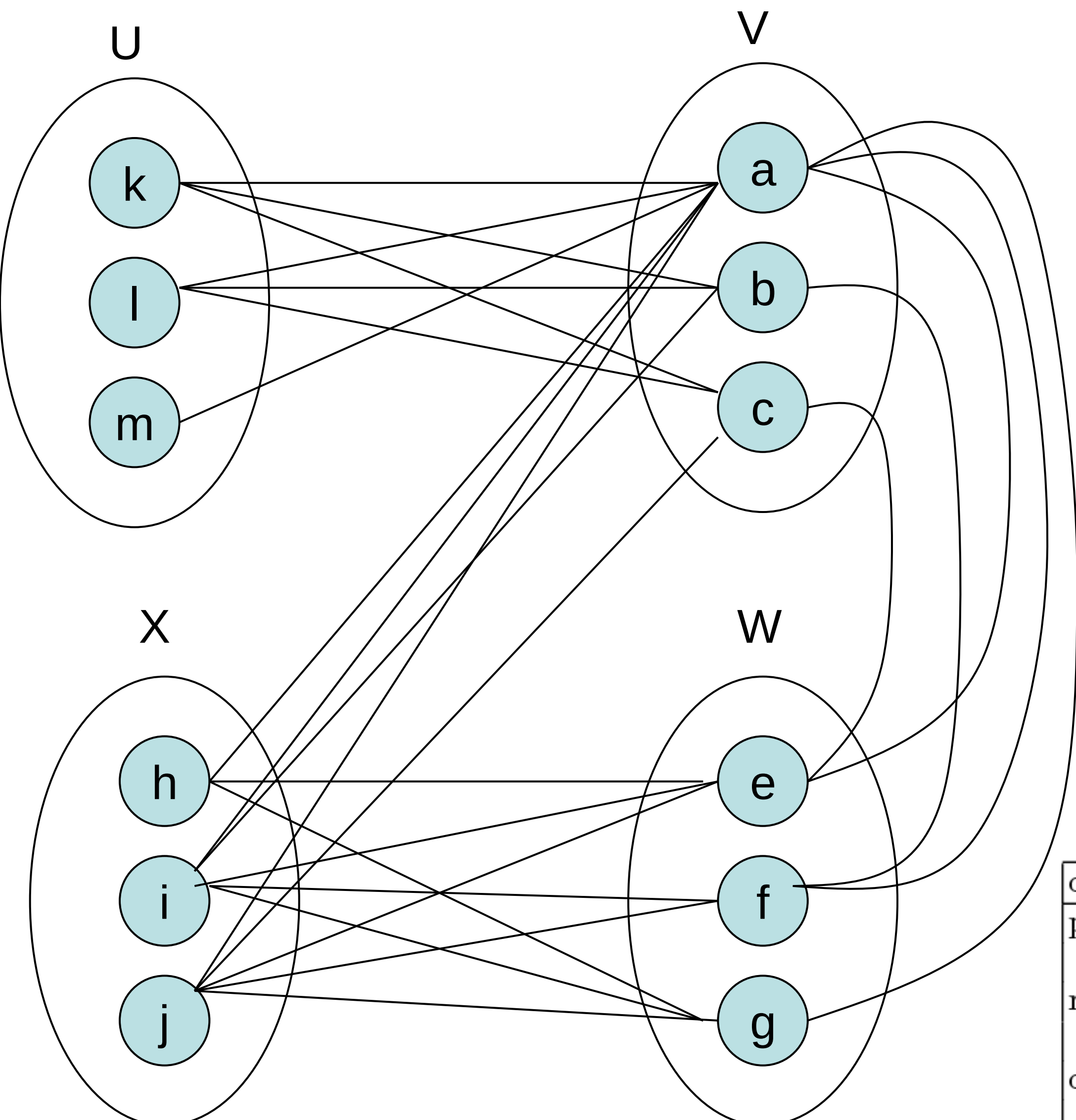
- Goal: minimize size of search space explored
- Principle:
  - given that we have already chosen the next variable to instantiate, choose first the values that are most likely to succeed
  - The most ***promising*** value

Measure promise of a value as follows

- count the number of supports in adjacent domain
- take the product of this value
- choose the value with the highest amount
- the most promising

A dual viewpoint (Geelen)
Choose the least promising variable
Assign it the most promising value

Microstructure & promise



domain values	a	b	c	e	f	g	h	i	j	k	l	m
promise	27	2	2	6	4	3	2	6	6	3	3	1
normalised	1	$\frac{2}{27}$	$\frac{2}{27}$	1	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{3}$	1	1	1	1	$\frac{1}{3}$
discrepancy	0	$\frac{25}{27}$	$\frac{25}{27}$	0	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	0	0	0	0	$\frac{2}{3}$

Table 1. Promise of domain values, giving discrepancy values

Can FF show Promise?

Might FF actually be promising?

If FF is on path to a solution we would prefer promise to failure
But does FF actually do this?

Experiments using probing suggest FF shows promise

Domain Specific Heuristics

- Golomb ruler
    - index order (!)
- Stable marriage (maybe not a heuristic)
    - value ordering!
- Jobshop/Factory scheduling
    - texture based heuristics
    - slack based heuristics
- Car Sequencing Problem
    - various (see literature)
- Bin packing
    - first-fit decreasing
- … the quest goes on

But remember, heuristic can play havoc with symmetry breaking

Domain Specific Heuristics

- Consider HC
  - different models
  - different heuristics?

AR33: section 5 (pages 27-29) and section 8 (pages 47-49)

Big question: why do heuristics work?

Is a heuristic similar to an umbrella lent to you by the bank?