

thrashing

## What's a csp?

$\langle V, D, C \rangle$

- a set of variables
- each with a domain of values
- a collection of constraints (I'm going to assume binary for the present)
- assign each variable a value from its domain to satisfy the constraint

Consider the following problem (csp5)

- variables  $V[1]$  to  $V[10]$
- uniform domains  $D[1]$  to  $D[10] = \{1,2,3\}$
- constraints
  - $V[1] = V[4]$
  - $V[4] > V[7]$
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How will search proceed?

A solution is 3--3--2--1

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# BT Thrashes!

past variable  $v[h]$



p  
a  
s  
t

conflict with  $v[h]$



current variable  $v[i]$



f  
u  
t  
u  
r  
e

future variable  
 $v[j]$



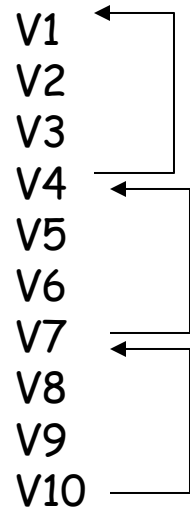
*Thrashing:*

Slavishly repeating the same set of actions  
with the same set of outcomes.

Can we minimise thrashing?



## Recording conflicts



Cause for conflict in  $csp_5$

- When we hit a dead end on  $V[7]$  we should jump back to  $V[4]$ 
  - the deepest conflicting variable for  $V[7]$  is  $V[4]$
  - if there are no more values for  $V[4]$  jump back to  $V[1]$ 
    - the deepest conflicting variable for  $V[4]$  or  $V[7]$ , (excluding  $V[4]$ )
- and so on

