The tetrahedral principle in kite design, revisited

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Abstract

I review an article by Alexander Graham Bell on the tertahedral principle in kite design, first prublished in 1903. Bell claims that as the tetrahedral kite is scaled up its density remains constant. I show, that if we assume that each tetrahedral cell is composed of six edges and that the joints have non-zero mass, then the density falls as scale increases. This implies that the principle leads us to large efficient structures.

1 Introduction

For a long time now (more than 15 years) I have had a fascination with solid geometry, most notably the geodesic domes of R.B. Fuller, and the tetrahedron. This interest was sparked off during my time as an undergraduate, studying Computer Science, and graph theory in particular. This in turn led me into building kites, and in particular tetrahedral kites based on the work of A.G. Bell.

In June 1903 Alexander Graham Bell published a paper in the National Geographic Magazine, titled *The Tetrahedral Principle in Kite Structure* [1]. This is a remarkable paper, and I quote from it extensively below. I'm doing this for a number of reasons. It gives us (fellow kite fliers) a picture of technology circa 1900's. There have been numerous scientific goals, as we look back in time: TOE (theory of everything), decoding the genome, room temperature fusion (or was it fission?) to take a man to the moon and return him safely to earth, the atomic bomb, etc. In Bell's time the problem was *the flying-machine problem and the goal of aerial locomotion.* Bell believed that

> ... a flying-machine should be capable of being flown as a kite; and conversely, that a properly constructed kite should be capable of use as a flying-machine when driven by its own propellers.

The other reason why I quote so much from this paper is that it gives us a picture (maybe only a sketch, but worth having) of one of the great Victorian Scientists. And finally, he was a fellow kiteflier/builder. He took the phone off the hook, and got down to the serious business! Oh ... I nearly forgot. One last thing. He was a Scotsman, born in Edinburgh to be exact.

2 Bell's Tetrahedral Principle

In his experiments in kite design Bell adopted the following criteria:

> I have not cared to ascertain how high a kite may be flown or to make one fly at any great altitude. The point I have specially in mind is this: That the equilibrium of the structure in the air should be perfect; that the kite should fly steadily, and not move about from side to side or dive suddenly when struck by a squall, and that when released it should drop slowly and gently to the ground without material oscillation. I have also considered it important that the framework should posses great strength with little weight.

At the time

> ... *the recent developments in the art* have been largely due to the efforts of one man - Mr Laurence Hargrave, of Australia ... and Mr Eddy and others in America have taken the Malay tailless kite as a basis for their experiments.

Bell made the following observations on Hargrave's work, the box kite.

> This represents, in my opinion, the high-water mark of progress in the nineteenth century; and this form of kite forms the starting point of my own researches (...in the Hargrave's box kite). The front and rear cells are connected together by a framework, so that a considerable space is left between them. This space is an essential feature of the kite: upon it depends the fore and aft stability of the kite. The greater the space, the more stable is the equilibrium of the kite in a fore and aft direction. Pitching motions or oscillations are almost entirely suppressed when the space between the cells is large. The greater the extent of the vertical sides, the greater is the stability in the lateral direction, and the less for the kite to roll, or move side to side, or turn over in the air.

A number of points from above are worth noting: the evolution of language, along with the progress of technology. *State of the art* seems to be modern jargon, but here we have Bell talking of *recent developments in the art.* And can any of us imagine the state of a world where the box kite is the pinnacle of technology? In addition, Bell recognises that Eddy's work was based on a known principle, namely the Malay tailless kite, and he does not consider it to be novel. Clearly, Bell is a Hargrave's fan, and rightly so.

In his paper Bell explained the superiority of the tetrahedral principle, with respect to the *box* principle of Hargrave. Bell showed that as the Hargrave's box kite gets bigger, it requires heavier structural elements and increased internal bracing. This results in

> ... adding to the drift of the kite, without contributing to the lift of the kite

Apparently the word *drag* had not yet appeared in the literature. He then goes on to show the superiority of the tetrahedron, and in particular:

> The weight relatively to the wing surface remains the same, however large the compound kite may be. This, at first sight, appears to be somewhat inconsistent with certain mathematical conclusions announced by Prof. Simon Newcomb in an article entitied *Is the Air-Ship Coming* published in McClure's Magazine, for September 1901 - conclusions which

> led him to believe that *the construction of an aerial vehicle which could carry even a single man from place to place at pleasure requires the discovery of some new metal or some new force.*
>
> The process of reasoning by which Professor Newcomb arrived at this remarkable result is undoubtedly correct. His conclusion, however, is open to question, because he has drawn a general conclusion from restricted premises. He (Newcomb) says:
>
> > *Let us make two flying machines exactly alike, only make one double the scale in all its dimensions. We all know that the volume, and therefore the weight, of two similar bodies are proportional to the cubes of their dimensions. The cube of two is eight: hence the large machine will have eight times the weight of the other. But surfaces are as the square of the dimensions. The square of two is four. The heavier machine will therefore expose only four times the wing surface to the air, and so will have a distinct disadvantage in the ratio of efficiency to weight.*
>
> From which he concludes that *if we make our structure large enough it will be too heavy to fly.* This is certainly true, as far as it goes, and it account for any failure to make a giant kite that should lift a man - upon the model of the Hargrave box kite. When the kite was constructed with two cells, each about the size of a small room, it was found that it would need a hurricane to raise it into the air.

I enjoy reading the above. One Victorian scientist talking of another. And it raises a number of points. Bell constructed a box kite with cells *each about the size of a small room.* I would imagine, in Victorian times, that the *size of a small room* might be pretty big by today's standards! Bell then goes on to show why Newcomb's conclusion is incorrect, and in particular, Bell reiterates the claim:

> Upon the tetrahedral plan the weight relatively to the wing surface remains the same however large the compound kite may be.

Bell concludes his paper as follows:

> It is not my object in this communication to describe the experiments that have been made in my Nova Scotia laboratory, but simply to bring to your attention the importance, of the tetrahedral principle in kite construction.

This is truly generous. Bell really believed that he was making a significant contribution to knowledge, and like Hargrave he placed that knowledge into the public domain. Aerial locomotion was considered to be of such import, personal gain was put aside. We could argue that Bell had quite enough material worth already, but I would prefer not to take that (cynical) view. Clearly, Hargrave made little, if anything, from his research.

3 The Tetrahedral Principle Revisited

And now an update, from another Scotsman. It is not my object in this communication to describe the experiments that have been made in my Scotican laboratory, but simply to bring to your attention some oversight in the previously reported tetrahedral principle in kite construction.

First, let me say that my fellow countryman, Mr Alexander Graham Bell, made a slight error in his claim, namely that *The weight relative to the wing surface remains the same however large the compound kite may be.* I suggest that Mr Bell be more precise, and make his assumptions explicit. His claim holds true so long as we assume that the cost of joining tetrahedra together is zero. That is, the joints have zero mass. From the photographs in his paper we can see that this is not the case. From the table below, and the following discussion I will show the remarkable fact that *when the cost of joining tetrahedra together is not zero, the weight relatively to the wing surface decreases as we subdivide the kite.*

Assume we have a tetrahedral cell of unit length. That is, each of its six edges are of size 1. Assume we wish to build a composite tetrahedron out of these cells, such as we have a great tetrahedron, where each of its *outer* edges is of length D. That is, we have subdivided the outer edges by D[1]. Tabulated below is the subdivision of the great tetrahedron, a front view of the great tetrahedron, the number of cells that make up the structure, the number of joints (vertices) required to hold it together, and the number of edges in the entire structure. More detail of each column is given below that table.

Table 1
Subdivisions of the great tetrahedron

D	G	*NxL*	N	VxL	V	V/N	E
1	*	1x1	1	2	4	4.0	6
				2			
2	* *	2x1	4	3	10	2.5	24
	*	1x2		4			
				3			
3	* * *	3x1	10	4	20	2.0	60
	* *	2x2		6			
	*	1x3		6			
				4			
4	* * * *	4x1	20	5	35	1.75	120
	* * *	3x2		8			
	* *	2x3		9			
	*	1x4		8			
				5			
5	* * * * *	5x1	35	6	56	1.60	210
	* * * *	4x2		10			
	* * *	3x3		12			
	* *	2x4		12			
	*	1x5		10			
				6			

- D: the degree of Division of a large tetrahedron, giving a composite structure made up of cells (where each cell is a tetrahedron).
- G: a Graphic, from the front, showing that composite tetrahedron.
- NxL: the Number of tetrahedral cells at each Level within the composite tetrahedron (width times depth where depth = 1 is top row).
- N: the total Number of tetrahedral cells in the composite tetrahedron.

[1] ... and we might consider D to be the *frequency* of the tetrahedron

- VxL: the number of joints (Vertices) at each Level within the composite. Note: there is one more level of vertices than there are divisions within the composite structure.
- V: The total number of joints (Vertices) in the composite structure.
- V/N: The average number of joints (Vertices) in each tetrahedral cell. We might think of this as an indication of the density of the structure.
- E: (The Maximum) Number of Edges in the composite structure.

What is of interest here is the ratio V/N, the average number of vertices per cell within the composite structure. As N increases V/N quickly approaches 1. This is quite remarkable. When we have a single cell we require 4 joints to hold the structure together. This is making the assumption that the cell is constructed out of 6 edges[2]. When we have 4 cells we have 2.5 joints per cell. We can see that this quickly approaches 1 joint per cell. This is shown in table 2 below, for the first 100 tetrahedra. Again column D is the number of divisions of the great tetrahedron, N is the total number of cells within that tetrahedron and V/N is the average number of joints required to hold each cell together.

Table 2
The first 100 tetrahedra

D	N	V/N
1	1	4.0
10	220	1.3
20	1,540	1.15
30	4,960	1.1
40	11,480	1.075
50	21100	1.06
60	37,820	1.05
70	59,640	1.0428
80	88,560	1.0375
90	125,580	1.0333
100	171,700	1.03

Given D we can compute the number of cells, N, using equation (1):

$$N = \frac{D^3 + 3D^2 + 2D}{6} \tag{1}$$

The number of cells increases in the order O(D^3), but V/N rapidly approaches (but never reaches) 1. How does this happen? First of all we must realise that V, the total number of joints in the composite tetrahedron of division D, is the same as the number of cells in the composite tetrahedron of division D + 1. That is, the number of joints in a composite tetrahedron is the same as the number of cells in the next larger composite tetrahedron. For example, when D = 1, we have 1 cell and four vertices, and each vertex has valency 3 (3 edges are incident on a vertex). When D = 2 we have 4 cells, and 4 joints of valency 3 (the outer vertices), and 6 joints of valency 6 (10 joints in total). That is, vertices are shared between cells.

[2]Obviously the cell could be composed of a central atom with a valency of four, but this in itself might be considered as redundant. A more efficient implementation is possible, and I might describe this in a future article.

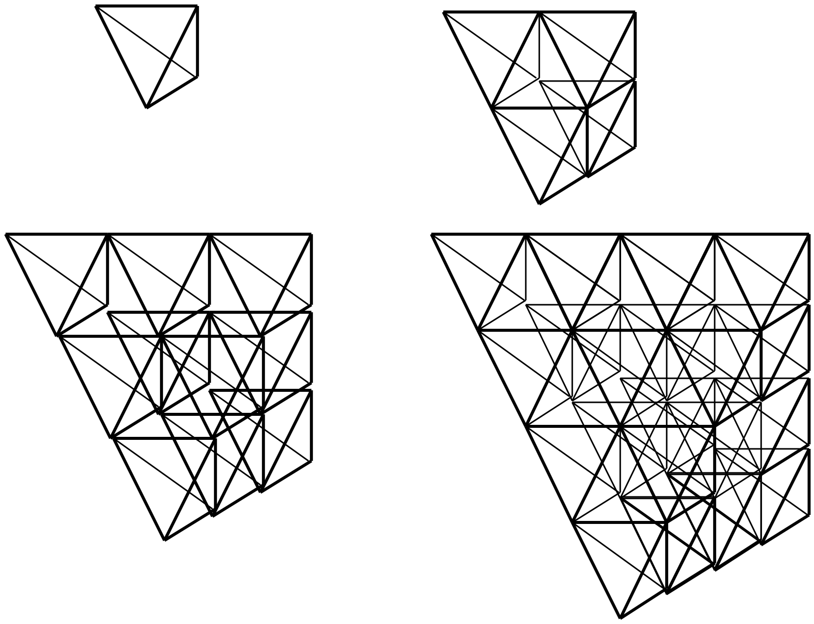

Figure 1: The first 4 great tertahedra

Ultimately, when D is large each vertex will tend to join together 4 cells. That is, the majority of joints will be of valency 12.

Figure 1 below shows diagramatically the first 4 great tetrahedra with respectively 1 cell, 4 cells, 10 cells, and 20 cells respectively.

4 Conclusion

As we increase D the density of the structure V/N falls, and if the heaviest component of the structure is a vertex, then this fall in density is significant. In plain words, *the bigger the division of the tetrahedral kite*[3] *the more efficient it becomes.*

Acknowledgements

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References

[1] Alexander Graham Bell, The Tetrahedral Principle in Kite Structure, National Geographic Magazine, vol 14, part 6, 1903, pages 219-251

[2] Patrick Prosser, Observations on tetrahedral kites The Kiteflier, Newsletter of the Kite Society of Great Britain, Issue 54, 20-24, January 1993

[3] Patrick Prosser, Beobachtungen zu tetraedischen Drachen, Drachen Magazine, Summer 1993

[3] ie. the greater the frequency.