Notes on dynamic value

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The purpose of this is to see if I can come up with a formalisation of value in a dynamic context and a model of how value in this context will influence prices.

In what follows I will designate quantities of use values and dimensionless numbers by italic script thus M, o, p, quantities of labour values as bold script thus $\mathbf{K}, \mathbf{s}, \mathbf{v}$ and quantites of money in typewritter script $\mathtt{X}, \mathtt{y}, \mathtt{z}$.

1 INDUSTRIES

I start out by assuming that there exists a set of industries.

I will further assume that each industry produces a single undifferentiated commodity. I will label the types of different products P_i, for the product of the ith industry. It will be assumed that each type of use value implies its own natural units of measure so that $7P_3$ would be 7 units of product P_3.

I shall assume that industry 0 produces gold, and that this serves as the money commodity. Thus $256P_0$, might stand for 256 grams of gold.

The different enterprises within an industry will tend to have different production techniques. The value of a commodity will thus be the mean over the industry of the different production enterprise values.

Each industry i is assumed to have a stock of machinery used as fixed capital M_{ij}, where j ranges over the types of product. M is assumed to be in natural units of the the relevant product and is a stock quantity. The proportionate rate of physical depreciation of these stocks of fixed capital per second I will designate by D_{ij}.

The rate of change of the physical stocks of fixed capital will be the sum of the physical depreciation and the physical investment I_{ij}, thus

$$(\frac{d}{dt}M = I + DM)_{ij}. \quad (1)$$

In this of course I_{ij} is a flow of natural units of the jth product per second.

I will assume for simplicity that industries hold no stocks of raw material, using instead a just-in-time system of raw materials supply from other industries. So each industry holds only stocks of its output. These I will designate S_i for the stock held by the ith industry. These are again assumed to be in natural units.

Changes in the stocks of output held by the industry result from sales of output o_i, and new production p_i. I assume that these are all in natural units and that there is no joint production. Thus we have

$$\frac{d}{dt}S_i = o_i + p_i. \quad (2)$$

Let there be a set of industry specific rates of productive consumption c_{ij} which define the rate of consumption of raw material j per second in industry i. Thus the consumption of product 2 in industry 1 would be $c_{1,2}$. Let the number of workers employed in the industry be $\mathbf{n}_i$ persons. As a point of dimensionality, this is also the instantaneous rate of labour perfomance, since that is person seconds per second, which reduces to persons.

2 VALUES

In the standard labour theory of value, the value of a product is defined as the labour required for its reproduction. This involves accounting for the indirect labour required to produce raw materials and consumed fixed capital.

If the output of the ith industry is o_i, total value per second of this flow is the number of people directly or indirectly working to produce it. Let $\mathbf{V}_i$ denote the total value of the output of the ith product. Its dimension is number of people. The value per unit of P_i, the type of use value produced in the ith industry is then

$$\mathbf{v}_i = \frac{\mathbf{V}_i}{o_i} \quad (3)$$

Dimensionaly this is persons per (P_i per second), or person seconds per P_i.

The total value flow $\mathbf{V}_i$ is made up of those directly working in the ith industry $\mathbf{n}_i$, plus those working indirectly to keep it going. We can obtain the number working indirectly to keep it going by multiplying the consumption of raw materials and physical depreciation by the appropriate product values:

$$\mathbf{C}_i = \sum_k \mathbf{v}_k c_{ik} + \sum_l \mathbf{v}_l D_{il} M_{il}. \tag{4}$$

Thus the total value flow of the ith industry is given by

$$\mathbf{V}_i = \mathbf{n}_i + \mathbf{C}_i. \tag{5}$$

Clearly if we sum over all industries to get the gross value product flow for the economy $\mathbf{V} = \sum_i \mathbf{V}_i$, this must

1. be a number of persons,

2. exceed the total number of persons employed in the economy $\sum_i \mathbf{n}_i$.

This excess arises because we are considering the gross value product flow. If we subtract from this $\mathbf{C} = \sum_i \mathbf{C}_i$, the value of the means of production consumed per second, we get the net value product flow which is identical to the number of workers $\mathbf{n}$.

3 PRODUCTION OF SURPLUS VALUE

Let $\mathtt{w}_i$ be the wage in the ith industry, measured in grams of gold paid per worker per second, or P_0 per person per second.

The the money wage $\mathtt{w}_i$ paid in the ith industry,can be multiplied by the value of gold, to give

$$f_i = \mathtt{w}_i \mathbf{v}_0 \tag{6}$$

This is of type (P_0 per person per second)×(person seconds per P_0), a dimensionless number, which will be less than unity if exploitation occurs. This mean that a fraction f_i of one person would be required to produce the flow of gold being paid to each worker. The rate of exploitation is then $\frac{1-f_i}{f_i}$. The mean rate of exploitation for the economy as a whole, f is then the weighted average of the rates in the individual industries: $f = \frac{1}{\mathbf{n}} \sum_i f_i \mathbf{n}_i$.

4 VALUE RATE OF PROFIT

I will now consider the current account value rate of profit in flow and stock terms. At this stage we abstract from whether the profit is realised or not. The surplus value produced per second in the ith industry is clearly $\mathbf{n}_i(1 - f_i)$.

The flow rate of profit $\mathbf{r}_i$ is obtained by dividing through by the total value flow of the industry:

$$\mathbf{r}_i = \frac{\mathbf{n}_i(1 - f_i)}{\mathbf{V_i}} = \frac{\mathbf{p_i}}{\mathbf{V_i}} \tag{7}$$

Where $\mathbf{p_i} = \mathbf{n}_i(1 - f_i)$ is the profit value-flow, or, the number of workers producing the surplus product as opposed to those producing the necessary product. This is not identical with Marx's formulation since it expresses it as a percentage of the output rather than the input values, but the two are related, and this is slightly simpler to deal with. As would be expected for a flow rate of profit this is a dimensionless number, since the numerator and denominator are both numbers of people.

A stock rate of profit is an exponential operator with dimension t^{-1} being the growth rate of the capital stock.

Let capital stock in value terms of the ith industry be denoted by $\mathbf{k}_i$. This is the sum of the value of the fixed capital, and of the unsold stocks. Thus

$$\mathbf{K}_i = \mathbf{v}_i S_i + \sum_j \mathbf{v}_j M_{ij} \tag{8}$$

Thus the current account rate value rate of profit on stock must be:

$$R_i = \frac{\mathbf{p}_i}{\mathbf{K}_i} \tag{9}$$

which has the required dimension.

5 Technical change and devaluation

Changes in technology or changing natural conditions can alter the amount of labour required to produce things. Such changes would have all sorts of implications for flows of products between branches of production etc.

The changes that have to be considered are as follows:

1. Changes in the direct labour inputs $\frac{d}{dt}\mathbf{n}_i$.
2. Changes in the output of a given industry $\frac{d}{dt}o_i$.
3. Changes in the rate of consumption of raw materials $\frac{d}{dt}c_{ij}$.

4. Changes in the rate of physical depreciation of fixed capital $\frac{d}{dt}D_i$.

From these and equation 4 we can derive

$$\frac{d}{dt}\mathbf{C}_i = \sum_k(\frac{d}{dt}\mathbf{v}_k c_{ik} + \mathbf{v}_k\frac{d}{dt}c_{ik}) + \sum_l(\frac{d}{dt}\mathbf{v}_l D_{il}M_{il} + \mathbf{v}_l\frac{d}{dt}D_{il}M_{il}). \quad (10)$$

which gives the rate of change of the constant capital flow in value terms as a function of the rates of change of values and technical coefficients. Combining (10) with (5) I obtain the change in the gross rate of value production per industry:

$$\frac{d}{dt}\mathbf{V}_i = \frac{d}{dt}\mathbf{n}_i + \frac{d}{dt}\mathbf{C}_i \quad (11)$$

If I feed this into equation (3) determining individual commodity values we obtain:

$$\frac{d}{dt}\mathbf{v}_i = \mathbf{V}_i\frac{d}{dt}\frac{1}{o_i} + \frac{\frac{d}{dt}\mathbf{V}_i}{o_i} \quad (12)$$

which defines the rate of change of individual commodity values as a decreasing function of changes in output and an increasing function of changes in gross value flow. What effect will this have on profits?

5.1 Flow profit rate

Consider first the flow rate of profit given in (7). We can express its rate of change as

$$\frac{d}{dt}\mathbf{r}_i = \frac{\frac{d}{dt}\mathbf{p_i}}{\mathbf{V_i}} + \mathbf{p_i}\frac{d}{dt}(\frac{1}{\mathbf{V_i}}) \quad (13)$$

This is an increasing function of increases in the flow of surplus value expressed in people, and a negative function of increases in the gross value-flow.

5.2 Stock profit rate

The stock rate of profit has to take into account changes in the value of the capital stock due to changes in value. Let us call this stock appreciation $\mathbf{a}_i$. It will be equal to the change in the value of the stock of fixed capital and finished goods brought about by revaluations:

$$\mathbf{a}_i = (\frac{d}{dt}\mathbf{v}_i)S_i + \sum_j(\frac{d}{dt}\mathbf{v}_j)M_{ij} \quad (14)$$

which gives us a modified definition for the rate of profit derived from (9) as

$$\rho_i = (\mathbf{p}_i + \mathbf{a}_i)\mathbf{K}_i^{-1} \tag{15}$$

where ρ_i is the rate of profit of the ith industry taking into account stock appreciation. The change in the rate of profit with respect to time is then

$$\frac{d}{dt}\rho_i = \mathbf{K}_i^{-1}(\frac{d}{dt}\mathbf{p}_i + \frac{d}{dt}\mathbf{a}_i) + (\mathbf{p}_i + \mathbf{a}_i)\frac{d}{dt}\mathbf{K}_i^{-1} \tag{16}$$

which, differentiating $\mathbf{K}_i^{-1}$ gives

$$\frac{d}{dt}\rho_i = \frac{\mathbf{K}_i(\frac{d}{dt}\mathbf{p}_i + \frac{d}{dt}\mathbf{a}_i) - (\mathbf{p}_i + \mathbf{a}_i)\frac{d}{dt}\mathbf{K}_i}{\mathbf{K}^2} \tag{17}$$

6 Kliman's counter example

If this formula is correct, we should be able to work out the effect on the flow and stock rates of profit of Kliman's example which involves a uniform rate of change k of all values, all physical quantities except labour inputs remaining constant. Under this assumption we can in general say that $\frac{d}{dt}\mathbf{x} = k\mathbf{x}$ for all terms $\mathbf{x}$ which are a simple sum of values. We thus obtain from (13) that

$$\frac{d}{dt}\mathbf{r}_i = \frac{k\mathbf{p_i}}{\mathbf{V_i}} - \mathbf{V}_i^{-2}\mathbf{p_i}k\mathbf{V_i} \tag{18}$$

for the flow rate of profit and from (17) that

$$\frac{d}{dt}\rho_i = \frac{\mathbf{K}_i(k\mathbf{p}_i + k\mathbf{a}_i) - (\mathbf{p}_i + \mathbf{a}_i)k\mathbf{K}_i}{\mathbf{K}^2} \tag{19}$$

for the stock rate of profit. Distributing the ks we clearly have the result that $\frac{d}{dt}r_i = \frac{d}{dt}\rho_i = 0$.