Abstract

The paper investigates the relationship between the realisation conditions of the aggregate social capital and the dynamics of individual capitals. It introduces a conceptual framework based on phase plane diagrams for reasoning about the relationship between the gearing ratio and the rate of accumulation of individual capitals. Using this it argues that the dynamics of financial crises produce a polarisation process of capital that precipitates rentiers at one pole and bankrupts at the other.
Realisation Crises and the Polarisation of Capital

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Introduction

It is impractical to attempt to specify all the factors operating at the level of commodity circulation that enter into the determination of profits. To progress it is necessary to resort to simplification and abstraction. The simple adequate model for the determination of profits was given by M. Kalecki [2]. He showed that in an abstract capitalist economy with only two classes, no government and subsistence wages, profits are jointly determined by the levels of capitalist consumption and investment. We give an algebraic demonstration of this in the following section, but the basic argument is of disarming simplicity.

In the absence of workers’ savings and taxation the capitalist class will exactly cover its wage bill by selling consumer goods to workers. Since the only other sales are of investment goods and capitalists’ consumer goods, it follows that profits, as the only other form of revenue, must be derived from these sales. Any money that capitalists spend on commodities other than labour power is a revenue for another capitalist. Since wages have been accounted for it follows that investment and capitalist consumption must instantaneously determine gross profits. Taken collectively capitalists’ expenditures determine their own revenues. Whilst the existence of credit allows for wide fluctuations of investment and capitalist consumption independent of past profits, current profits are absolutely determined by investment and capitalist consumption. As collective owners of the means of production, capitalists finance their own appropriation of the surplus product.

They have the ability to appropriate the entire surplus product, but this ability belongs to capitalists as a class. The units of economic calculation are individual enterprises and capitalist households, and there is no reason to suppose that the quantity of commodities appropriated by them will coincide with the available surplus product. An investigation of the determinants of profit will therefore re-
quire some conception of the determinants of capitalists’ expenditure, particularly investment expenditure. To do this you have to make assumptions about the forms of property in existence and the types of economic calculation accompanying them. In particular this involves assumptions about the existence of financial institutions and about the possibility of substitution between real and fictitious capital assets, and this in turn requires assumptions about the establishment of a rate of interest.

Marx argued that a general theory of interest rates was impossible [3]. It is only because the conditions of production set limits to the rate of profit that it is possible to have a theory of the long run rate of profit that abstracts from specific property forms. Since there is no particular relationship between interest rates and conditions of production, any assumptions that we make about the determinants of interest must be related to the specific institutions that determine the interest rates. In our analysis we assume that interest rates are determined by the economic calculations of bank capital.

We first develop a model of the reproduction of the conditions of existence of realisation crises, and then go on to analyse trajectories of development following on from a particular form of resolution of such crises, and the reproduction of realisation crises under new conditions. At times the argument is rather technical and the mathematical exposition inelegant; these problems can doubtless be overcome if the analysis is considered to have contributed anything worthwhile.

Model 1: Direct Personification of the Enterprise, no Rentiers, no Banks

Our starting point is the simple Kalecki model of realisation, wherein

\[ \text{Profits} = \text{Investment} + \text{Capitalist Consumption} \]

expressed symbolically:

\[ P = I + C \]  \hspace{1cm} (1)

Two points must be noted. It implicitly assumes only two classes, workers and capitalists. It is assumed that subsistence wage rates result in there being no saving by workers, capitalists are sole owners of their enterprises and there is no capital market. Secondly, the order of determination is not what might initially be thought. Common-sense tells us that capitalists receive a certain sum of profits, part of which they consume and another part of which they invest. The level of profits seems to determine the level of capitalist consumption.
and investment. In fact the reverse is the case, investments and capitalist consumption are the determinants of profits, as is explained in Kalecki’s article ‘The Determinants of Profits’ (in Selected Essays on the Dynamics of the Capitalist Economy, Cambridge 1971). This means that profits are necessarily equal to the real appropriation of value as elements of constant capital or articles of consumption by the bourgeoisie. Real appropriation determines profits rather than vice-versa. Profit as a monetary expression of real commodity movements is ultimately determined by them.

If we were to include the whole national income in our calculations the following would apply:

\[ P + W = I + C + W_c \]  \hfill (2)

Where
- \( W \) = wages
- \( W_c \) = workers consumption

If wages equal workers consumption then, 2 reduces to 1. To obtain the net value product we must subtract consumption of constant capital, i.e., depreciation.

\[ V = P + W - \Delta \]  \hfill (3)

Where
- \( V \) = net value product
- \( \Delta \) = depreciation

If we assume that there are no unproductive workers then variable capital is identical to total wages and the rate of surplus value is given by:

\[ s' = \frac{P - \Delta}{W} \]  \hfill (4)

where
- \( s' \) = rate of surplus value

and the net rate of profit will be given by:

\[ p' = \frac{P - \Delta}{K + W.t} \]  \hfill (5)

where
- \( p' \) = net rate of profit
- \( K \) = stock of constant capital
\( t = \) turnover time of variable capital

Since the turnover time of variable capital is very short, its effect is negligible so that by substituting 1 into 5 we obtain the net rate of profit as a function of consumption and investment by capitalists.

\[
p' = \frac{I + C - \Delta}{K} \tag{6}
\]

Given the skeletal assumptions we have made about property forms in this model we cannot go on to say anything credible about the determinants of investment. This requires a more elaborate model.

Model 2: Complete Separation of Money Capital from Industrial Capital, Existence of Rentiers and Banks

We change the property relations in the new model to:

1. consider enterprises as abstract juridical personalities rather than individual capitalists;

2. assume that the entire capitalist class have become rentiers, making capitalist consumption rentiers’ consumption;

3. assume the existence of credit money with banks providing the main financial intermediary, between rentiers and enterprises, but allowing for a marginal market in direct loans from rentiers to enterprises.

Such a set of property forms has never existed in pure form, but corresponds to the hypothetical presuppositions for the complete dominance of finance capital in the specific form of bank capital, but without the merger of bank with industrial capital [6].

In the previous model the sole forms of revenue were wages and profits. Interest now appears as an additional category, so that profit is divided into interest and profit of enterprise [5].

For profit of enterprise we thus obtain:

\[
E = P - R - \Delta \tag{7}
\]

where

\( E = \) profit of enterprise
\( R = \) total interest
Now the level of total interest payments is a function of the rate of interest \( (r) \) and the outstanding debts of enterprise \( (D) \) since we assume enterprises to be the only net debtors in the system. It follows that:

\[
R = D \cdot r
\]  

(8)

Similarly we may divide the total capital of the enterprises into two parts, one of which belongs to the enterprise, and the other matching the enterprise’s outstanding debts to the banks.

\[
K = D + H
\]  

(9)

where  
\( H = \text{enterprise capital} \)

On the basis of the original model we can now give the determinants of profit of enterprise.

\[
E = I + C - D \cdot r - \Delta
\]  

(10)

If rentiers consumption is given by \( C \), any income that they get in excess of this must be their savings, or accumulation of money capital. On the other hand, since enterprises are pure juridical personalities they have no personal consumption. Any excess of profit over interest payments constitutes their internal accumulation. This may either take the form of the acquisition of new elements of constant capital (again assuming variable capital to be insignificant in stock terms) or as money capital. Given our assumption that there is no market in loan capital other than through the banking system, this means that individual enterprises must divide its internal accumulation between purchases of elements of constant capital and the accumulation of bank deposits. Purchases of elements of constant capital are the active factor, accumulation of deposits the residual. If investment exceeds profits, then the enterprise has a negative accumulation of money capital. Either it runs down its deposits or it borrows from the bank. Conversely a positive accumulation of money capital may involve either an absolute rise in its deposits with the banks or a fall in its outstanding debt.

The two portions of total accumulation are defined as follows:

\[
A_e = E
\]  

(11)

\[
A_r = D \cdot r - C
\]  

(12)

\[
A_e + A_r = I - \Delta
\]  

(13)
where
\[ A_e = \text{accumulation of enterprise capital} \]
\[ A_r = \text{accumulation of rentier capital} \]

We will assume that the consumption of rentiers is determined both by their income and their money stocks. They are buffered from the immediate effects of fluctuations in their income by their holdings of money capital. In this they are different from workers whose consumption is directly related to their current wages. We can treat rentiers’ consumption as being made of two components:

\[ C = x.D + y.D.r \]  \hspace{1cm} (14)

where
\[ x \text{ and } y \text{ are constants of value less than 1.} \]

Substituting into 10 and 11 we can see that:

\[ A_e = E = I - \Delta + D[x + r(y - 1)] \]  \hspace{1cm} (15)

This shows that accumulation of enterprise capital is a decreasing function of the rate of interest and an increasing function of the coefficients of rentiers’ consumption. The higher is rentiers’ accumulation the lower is enterprise’s own accumulation. Here we see an instance of the contradiction between industrial capital and money capital. It is also clear that if the rate of interest falls sufficiently low (below \( \frac{x}{1-y} \) in our example), then it will not cover rentiers’ expenditure on consumption, and will lead to a negative level of accumulation by rentiers. Low interest rates accelerate the accumulation of enterprise capital whilst undermining the position of the rentiers.

**Accumulation of Constant Capital**

We will assume that accumulation of constant capital is a function of:

(i) total capital stock;

(ii) the gearing ratio of debts to total capital \( G = \frac{D}{K} \);

(iii) the rate of profit \( p' \);

(iv) the rate of interest \( r \);

(v) the rate of change of the rate of profit \( \dot{p}' \).
Redefining $I$ as net accumulation of constant capital we have

$$I = f(K, G, p', r, \dot{p}')$$  \hspace{1cm} (16)$$

This can be simplified if we deal with the proportionate rate of capital accumulation $\alpha$, rather than absolute capital accumulation, giving

$$\alpha = f(G, p', r, \dot{p}')$$  \hspace{1cm} (17)$$

What characteristics can we expect this function to have?

We examine each of the parameters in turn considering their effects. We can expect the proportionate rate of accumulation to be a negative function of the gearing ratio, since heavily indebted firms will find it more difficult to raise further investment capital. Thus we can assume that $\frac{\partial \alpha}{\partial G}$ will be negative. We can expect that the rate of accumulation will be a decreasing function of the rate of interest, since the higher the rate of interest, the fewer the number of investment projects capable of returning at least the rate of interest. Thus $\frac{\partial \alpha}{\partial r}$ will be negative. We can expect that accumulation will be an increasing function of the rate of profit, since the higher the rate of profit, the more will be the number of investment projects that will return at least the rate of interest. Thus $\frac{\partial \alpha}{\partial p'}$ will be positive. When the rate of profit is rising, we can expect investment to increase in anticipation of future profits, so $\frac{\partial \alpha}{\partial \dot{p}'}$ will be positive.

**Interest and the Grounds for Compensation of Bank Capital**

Most bourgeois theories of the rate of interest base themselves upon the notion of supply and demand for money and upon the subjective preferences of the individual wealth holder (presented as an idealised rentier). The ideological and idealist nature of the latter aspect of the theory is obvious and does not need criticism. It is worth saying a few things about the supply and demand for money however.

The first point is that the concepts of supply and demand first developed in bourgeois economics for the analysis of commodity markets, where the notions refer to supply of and demand for commodities as a function of their prices. In this context, the notion of an upward-sloping long run supply curve is very dubious, for the majority of commodities at least, since under capitalist relations of production most commodities are produced under conditions of increasing marginal returns to scale. The main exceptions are agricultural and mineral products where diminishing marginal returns to scale allow for the existence of differential rents. But here an upward sloping supply curve is an
expression of the combination of certain natural preconditions with the existence of private property in land, rather than an expression of the general features of commodity production. So even in the domain of their initial establishment the notions of supply and demand are of doubtful reliability.

Secondly, money is not always a commodity. Credit money or paper money are not whereas gold and silver coins exchanging at their full metallic value are. If we take the latter then their equilibrium exchange values are determined by the prices of production of precious metals, and ultimately by the amount of labour required for their production. Insofar as these are produced under conditions of declining marginal returns to scale, then they do indeed have upward sloping supply curves. But what these determine are the exchange values of gold and silver, and therefore the general price level, not the rate of interest.

Thirdly, as a commodity money is not an object of demand in the same way as other commodities. The demand for commodities is the amount of money offered for their purchase at a given exchange value between money and the commodity in question. Since money cannot be purchased, there is no demand for it in this sense, unless one is dealing with international currency markets. Otherwise the only possible meaning for the demand for money is the total supply of commodities on the market, as these are all offers for money.

Fourthly, money as the measure of price has always got a price of unity. Interest is not the price of money, but the form in which surplus value is appropriated by money capital. As a form of surplus value, its existence cannot be deduced at the level of the social relations operating in simple exchange, it is a phenomenon of a different order.

Fifthly, the concept of the supply of money is vacuous. A particular total can be arrived at by adding currency to bank deposits, but the technical practice of producing this total does not produce the concept of it. For the concept of money supply to be used in a supply and demand analysis, one would have to establish either that money supply was a constant, or that it was a function of the rate of interest. But any bank loan constitutes an addition to the money supply, since it creates additional deposits. But demand for bank loans is an instance of the ‘demand for money’. Hence it would seem that demand for money creates its own supply. If so, how can supply and demand for money be treated as independent functions jointly determining the rate of interest?

We would contend that in the simple system that we have outlined, the rate of interest would be determined by the grounds for compensation of bank capital. As Marx argues [4] the establishment of an average rate of profit gives rise to particular forms of economic calu-
lation which compensate for differences between economic branches in determining prices.

"After average prices, and their corresponding market prices, become stable for a time it reaches the consciousness of individual capitalists that this equalisation balances \textit{definite differences}, so that they include them in their mutual calculations. The differences exist in the mind of capitalists and are taken into account as grounds for compensating" ([3], 209). Marx is here talking about industrial capital and the prices of industrial commodities, but he later goes on to extend the same argument to other fields of employment of capital such as trade. We would argue that interest can be treated as being determined by a similar form of economic calculation, and that its level will be determined by actuarial factors taking into account the borrower's position and the position of the bank making the loan. Differences between different classes of borrowers give rise to different rates of interest for different classes of loans. The process whereby higher risk borrowers pay higher rates is straightforward and needs no further elaboration. We will concentrate on how the condition of the bank making the loan can affect the interest rate.

We will treat the average rate of interest for industrial borrowers as a function of the reserve ratio of the banks, such that:

\[ r = f\left( \frac{m}{Q} \right) \]  

and \( f'(\frac{m}{Q}) = -ve \)

where

\( m = \) cash reserves of banks,

\( Q = \) total bank deposits.

The possibility of profitable banking is based upon the fact that banks face a probability function for net withdrawals in any small time period of the form \( q(w) \) as shown in figure 1. This shows that the probability of a large net withdrawal in any given time period is smaller than the probability of a small net withdrawal. This enables banks to keep reserves that make up only a fraction of total deposits, since the probability of withdrawals exceeding reserves, though finite, is small. In the event of withdrawals temporarily exceeding reserves, the bank will be forced to borrow from other banks to meet its obligations to them (the most significant portion of a bank's liabilities is always to other banks where credit money predominates). If forced to borrow to meet obligations it has to pay interest on the sum borrowed. This enables us to calculate the probable cost to a bank of such a loan, \( l \).
Figure 1: The function $q(w)$ shows the probability of withdrawals exceeding $w$.

\[ l. i + r. \int_{m-i}^{m} q(w). w \, dw \]  \hfill (19)

We know that $q$ is a decreasing function of $w$, so it follows that the cost of a decreasing function of cash reserves $m$, an increasing function of $r$, the rate payable to other banks for short term loans. Since the bank has to make a profit on its loan, the rate of interest it charges must be sufficient to cover the cost of the loan. So as $l$ tends to zero we get the following inequality:

\[ r' > i + r.q(m) \]  \hfill (20)

where
\begin{align*}
  r' & = \text{rate of interest charged to industrial borrowers} \\
  i & = \text{rate of interest paid on deposits} \\
  r & = \text{rate of interest paid on short term loans from other banks.}
\end{align*}

If we assume that $i$ is fixed by interbank competition and that there is a going rate of interest charged on interbank loans, then it follows that the rate of interest charged to borrowers will be a decreasing function of bank reserves for each individual bank. This still leaves the global values of $i$ and $r$ undefined. Where a central bank exists it can be assumed that it will fix the rate $r$ thereby exercising a control over $r'$. Otherwise we could treat $r$ as a linear function of $\bar{r}$, with the premium $\bar{r'} - r$ determined by the relative probabilities of banks and commercial borrowers defaulting (where $\bar{r'}$ is the market rate for commercial borrowers). We can assume that $i$ will be related to $r'$
in some lagged fashion, since if the difference between the two rates became too high, rentiers could by-pass banks by lending directly to industrial borrowers.

It follows from the above that a fall in reserve ratios will lead first to a rise in the profits of banks as a wider gap develops between rates charged to industrial borrowers and paid to depositors, followed by a general rise in interest rates as rates paid to depositors are adjusted upwards.

There is some empirical evidence for this sort of relationship between bank reserve ratios and the rate of interest. The evidence is particularly compelling for the earlier periods in the USA before active intervention by the Federal Reserve became a major factor determining the rates [1].

In order to obtain a value for the rate of interest it remains only to obtain a value for \( Q \), the total mass of bank deposits.

**Determination of the Mass of Bank Deposits**

For the banking system in our example the following basic relationship holds:

\[
\text{Growth in deposits} + \text{Bank profits} = \text{Growth in loans} + \text{Growth in monetary reserves}
\]

If we assume that all bank profits are distributed, i.e., are transferred to the accounts of that section of rentiers who are the nominal owners of bank capital then the left hand side becomes simply the growth of deposits.

In our example we have three classes of economic subjects, enterprises, rentiers, and workers. We are assuming that workers do not save. In consequence they contribute nothing to the growth of deposits, so we can say:

\[
\text{Growth in deposits} = \text{Net growth in rentiers deposits} + \text{net growth in enterprise deposits}
\]

Since we are assuming that bank deposits are the only instrument of debt that can be held by either rentiers or enterprises, then the net growth in rentiers' deposits will be the same as their savings. In the case of enterprises, they are both borrowers and lenders. We can divide them into four categories,

1. borrowers

2. those repaying loans
3. those accumulating deposits

4. those running down their deposits

Therefore the following relations must hold:

\[
\text{Net growth in bank loans} = \text{Gross borrowing} - \text{Repayments}
\]

and

\[
\text{Net change in enterprise deposits} = \text{Gross deposits by enterprises} - \text{Withdrawals by enterprises}
\]

Therefore, if we assume as before that the level of monetary reserves is determined by the stock of currency and the requirements of circulation, it follows that we have only one independent variable, the net change in enterprise deposits.

If an enterprise’s profit of enterprise exceeds its net investment then it will either accumulate deposits at the bank or repay its debts to the banks. If its net investment exceeds its profit of enterprise it will either have to borrow from the banks or run down its deposits. The relationship between profit of enterprise and investment thus determines the change in a company’s relative indebtedness.

\[
e = \frac{E}{K}
\]  

(21)

where

\[e \equiv \text{rate of profit of enterprise}
\]

\[
j = a - e
\]  

(22)

where

\[j \equiv \text{rate at which an enterprise is borrowing relative to its employed capital (flow of funds into the enterprise)}.
\]

Any company’s borrowing position can be described by the value of two variables \(j\) and \(G\), i.e., the rate of change of indebtedness and the gearing ratio. If we plot two axes, of \(G\) against \(j\) as in figure 2, then the values of an enterprise’s \(G\) and \(j\) define a point on the surface of the plane given by the two axes. There will be a point with coordinates \(\bar{j}, \bar{G}\) which corresponds to the mean values of \(j\) and \(G\) for all enterprises. The coordinates of this point are given by the total saving of rentiers and the total deposits of rentiers such that
(2) Repaying loans  

(1) Borrowing  

(3) Accumulating Deposits  

(4) Withdrawing deposits  

\[ \bar{j} = \frac{A_r}{K} \]  

\[ \bar{G} = \frac{D}{K} \]  

Figure 2: Phase plane of gearing ratio against relative change in indebtedness  

This point may be considered as the center of mass of the total social capital distributed about the plane. We may consider the social capital as a two dimensional fluid or gas with its particles comprised by enterprises each with a pair of \( j \) and \( G \) coordinates. The distribution can be described by a real valued function \( V(j, G) \) such that:

\[ \int_{-\infty}^{+1} \int_{-\infty}^{+\infty} V(j, G) dj dG = 1 \]  

The function gives the probability that any randomly chosen enterprise will be found on a particular point on the plane. Since no enterprise can have a gearing ratio greater than 1 (by the way we have defined the gearing ratio) the integral over all \( j \) and for all \( G \) up to 1 must be unity, as shown in 25.

As figure 2 shows all enterprises falling within the first quadrant are not borrowers. If we assume positive savings by rentiers, and if all enterprises were restricted to a sufficiently small neighbourhood around \( (\bar{j}, \bar{G}) \), then there would be no enterprises accumulating deposits; they would all be borrowers. Under these circumstances the growth of bank deposits would be exclusively determined by rentiers’ savings, and we
would have closed our system. However, there is no reason to suppose that such a restriction of the distribution of enterprises holds. In principal we could assume that the integral of $V$ over any of the four quadrants will yield a positive result. To get any further with the problem of the determinants of the growth of bank deposits we will have to deal with the dynamics of the distribution of enterprises in the plane.

**Distribution of Capital in the Phase Plane**

The diagram we have constructed is a form of phase plane diagram. On one of these one axis denotes the value of a variable whilst the other denotes the rate of change of the variable with respect to some third variable, usually time. In our case we are plotting the relative change in an enterprise’s gearing ratio against its absolute gearing ratio.

We are interested in the relation between the distribution of capital in the phase plane and the accumulation of debt (deposits). This is given by two integral over $V$.

\[
\text{Growth of inter-enterprise debt} = B_e = \int_{-\infty}^{0} V(j, G) G dG \quad (26)
\]

\[
\text{Total inter-enterprise debt} = D_e = \int_{-\infty}^{0} V(j, G) G dG \quad (27)
\]

\[
\text{Total inter-enterprise debt at time } t = \int_{0}^{t} B_e dt = D_e(t) \quad (28)
\]

These are the gross features of the distribution that we are interested in. Because of the gross relationship given in 28 we only have to specify $B_e$ to obtain those features of the distribution of interest to us. To do this we have to look at the dynamics of individual enterprises.

It is a property of the phase plane that an enterprise’s rate of change of $G$ is determined by its $j$, in other words its vertical movement is determined by its horizontal position as follows.

\[
\frac{dG}{dt} = j - G \cdot \alpha \quad (29)
\]

Since

\[
\frac{dG}{dt} = \frac{d(\frac{P}{K})}{dt} = \frac{dD}{dt} \cdot \frac{dK}{dt} \cdot K^2
\]

\[
\frac{dG}{dt} = \frac{dD}{dt} \cdot K \cdot \frac{dK}{dt} \cdot \frac{1}{K^2}
\]

15
\[
= \alpha - \epsilon - \frac{P_0 \alpha}{k} \quad \text{by (22)}
\]

\[
= j - G \alpha \quad \text{by (23)}
\]

It follows that the rate of change of the gearing ratio is a result of the gearing ratio itself, the rate of accumulation, and the rate of profit of enterprise. More importantly the equation enables us to discover the equilibrium value of the gearing ratio as a simple function of the rate of accumulation and the rate of profit of enterprise. By definition an equilibrium value of a variable is the value at which its rate of change with respect to time is zero. Thus at the equilibrium gearing ratio:

\[
j - G \alpha = 0
\]

thus

\[
G_e = \frac{j}{\alpha} \quad (31)
\]

where

\(G_e = \text{equilibrium gearing ratio}\)

As figure 3 shows, if the rate of accumulation \(\alpha\) is held constant then the equilibrium gearing ratios fall on a straight line sloping upwards through the origin. Not surprisingly all capitals with a positive \(j\) (i.e., a net inflow of funds from the banks) have positive equilibrium gearing ratios. Thus any capital with a rate of profit of enterprise less than its rate of accumulation will have a positive equilibrium gearing ratio (by 23).
Figure 4: Equilibrium gearing ratios with constant rate of profit of enterprise \( \epsilon \).

If on the other hand we assume that the profit of enterprise for any given capital is exogenously determined by market conditions, wage levels and the pre-existing gearing ratio, then the rate of change of the gearing ratio becomes a function of accumulation. If we hold a constant then the equilibrium gearing ratio becomes a function of the form shown in figure 4. Again the result is not particularly startling. The function shows that if positive profits of enterprise are assumed, then all enterprises with \( \alpha \) greater than \( \epsilon \) have equilibrium gearing ratios less than 1 but greater than zero. Enterprises with rates of accumulation less than their rates of profit of enterprise on the other hand have negative equilibrium gearing ratios. Those with zero accumulation rates have an equilibrium gearing ratio of minus infinity, i.e., they go on accumulating money capital indefinitely. However, we are still not able to describe the trajectory that would be followed by a given capital from an initial point on the phase plane. To do this we have to assume that the net rate of profit for any given capital is exogenously determined. The rate of profit of enterprise is then endogenously determined by the gearing ratio and rate of interest. Thus:

\[
\frac{de}{dG} = -r \tag{32}
\]

and

\[
\frac{df}{dG} = r + \frac{d\alpha}{dG} \tag{33}
\]

and
\[ G_e = \frac{(\alpha - p')}{(\alpha - r)} \] (34)

For any single capital the rate of interest is an exogenous variable and may be assumed to be constant when investigating its trajectory. For total industrial capital it is far from being an exogenous variable, since the distribution of industrial capital in the phase plane determines the growth of deposits and thereby the rate of interest. But for an individual capital it is valid to hold it constant.

**Trajectories of Capitals with Rate of Profit Greater than Rate of Interest**

Figure 5 shows the function giving the equilibrium gearing ratio under these circumstances. As can be seen it is similar to that shown in figure 4. It has two asymptotes, one parallel to the x axis and tends towards \( G = 1 \), the other of which is at an angle \( \Theta \) to the y axis passing through the x axis at \( f = -(p' - r) \) such that \( \tan \Theta = r \).

Consider a capital in the position labelled \( a \). Since it lies above the equilibrium gearing ration curve, its gearing ratio will tend to fall down towards the curve. However, its final gearing ratio will depend upon the slope of its trajectory. The figure shows three trajectories, labelled \( i, j, k \). If \( i \) is followed then the capital comes to rest in the first quadrant as an equilibrium net debtor. If \( k \) is followed, then it ends up as an equilibrium net creditor. If \( j \) were followed the capital would have no finite equilibrium gearing ratio, and would go on accumulating deposits indefinitely.

Consider a capital in position \( B \). Since it lies below the equilibrium curve it must tend to rise towards it. In this case it can only follow the two types of trajectories \( s \) and \( t \). \( s \) takes it into the first quadrant, \( t \) into the third quadrant. Unlike those capitals above the curve all below it tend to move back onto it.

It should be noted that the function \( G_e \) has another set of solutions lying above \( G_e = 1 \) and obtained by reflecting the curve we have drawn about its asymptotes. As all these imply a gearing ratio of more than one we can assume that they cannot be occupied by a real enterprise. In addition they are unstable equilibria, (ie, an enterprise in the neighbourhood of one of these curves would tend to move away from it rather than towards it), so that we ignore them both in this and the following section.
Figure 5: Equilibrium gearing ratio with constant net rate of profit and constant interest rate, $\tan \theta = r, p' > r$. 
Effects of Changes in Rates of Profit and Interest on Trajectories of Individual Capitals

In order to make any more specific analyses of these trajectories it is necessary to make further explicit assumptions about the relationship between profit, interest and accumulation of constant capital by enterprises. We assume that accumulation of constant capital tends to zero as the rate of profit for an enterprise falls below the rate of interest. The logic of this assumption is obvious enough.

This means that there is a major difference between those enterprises whose profits exceed and those whose profits fall short of the rate of interest. Those earning more than the rate of interest have more control over their cash flow situation. They can improve it by cutting back on accumulation of constant capital should the need arise. In our model an enterprise’s cash flow is shown by $j$. Positive $j$ signifies a net outflow of cash and vice versa for negative $j$. In terms of the standard phase plane diagram relating cash flow to gearing ratio, only those enterprises with rates of profit greater than the rate of interest can control their horizontal movements on the plane by cutting accumulation.

Consider figure 6. The lines marked as $P_0$ to $P_5$ show the relation-
ship between cash flow and gearing ratios for successively higher rates of profit. Consider the line $P_0$. All capitals whose rate of profit is equal to the rate of interest $r$ must lie on this line. (We have assumed the rate of interest to be fixed for the purpose of the diagram). We know that the line must cut the $x$ axis at $-r$ because:
\[ j = \alpha - \epsilon \]
\[ \alpha = 0 \text{ since } p' = r \]
\[ G = 0 \text{ since we are on the } x \text{ axis} \]
therefore
\[ j = -r \]  \hspace{1cm} (22)

Since accumulation is zero the line has a gradient of $\frac{1}{r}$ and will intersect the $y$ axis where $G = 1$. This enables us to define the trajectory of all capitals with a rate of profit equal to the rate of interest. From our previous argument we know that the equilibrium gearing ratio function for capitals with a rate of profit equal to the rate of interest is a straight line through the origin, of gradient $\frac{1}{r}$. This is shown on figure 6 by the line labelled $E_0$. Since $P_0$ is parallel to but above $E_0$, it follows that all capitals on $P_0$ will move down it to minus infinity.

The lines $P_1$ to $P_3$ represent the sets of possible positions for capitals of successively higher rates of profit. Capitals with rates of profit $P_3$ lie on the line $P_3$. It can be seen that they will have rates of ac-

Figure 7: Trajectory of capital as profit rate changes
cumulation above the rate of profit and will in consequence either be borrowers or else running down deposits.

To each rate of profit line there corresponds an equilibrium gearing ratio curve $E_i$ such that $P_i$ corresponds to $E_i$. It follows that for each rate of profit greater than the rate of interest there is a unique stable equilibrium point given by the intersection of $P_i$ and $E_i$. The intersection of these two sets of curves produces the curve $Ge_r$ shown in figure 7. This curve is the set of possible equilibrium gearing ratios for rate of interest $r$ and rates of profit greater than $r$. This enables us to define the trajectories resulting from a change in the rate of profit. Consider a capital initially at $P_5$ on figure 2a. If the rate of profit falls to that corresponding to the curve $P_3$, the capital will first move over to point $q$ on $P_3$. We treat this as an effectively instantaneous response to the changed rate of profit, brought about by a reduction in the enterprise’s accumulation rate. It then moves down the trajectory given by the line $P_3$ until reaching the point where this intersects the equilibrium gearing ratio. If the rate of profit rose again to $P_5$ the capital would describe trajectory $P_5, q_1, P_5$.

There will exist a set of such curves $Ge_r$, one for each rate of interest. Figure 8 shows such a set. The curve $G_{r_0}$ corresponds to the lowest rate of interest whilst $G_{r_0}$ corresponds to the highest. As is to be expected, the higher the rate of interest, the lower the equilibrium gearing ratio for capitals that are net borrowers. As $r$ rises, the equilibrium point for a capital with a given rate of profit moves down and to the left.

Suppose a capital with a rate of profit $P_4$ has an initial equilibrium at $q_0$ when the rate of interest was $r_0$. As the interest rate rises to $r_1$, the equilibrium point for this capital shifts to $q_1$, when the rate of interest reaches $r_2$ the equilibrium point shifts to $q_2$ etc. In figure 8 the lines $P_0$ to $P_4$ pass through the equilibrium points for capitals of successively higher rates of profit. The lines correspond to the trajectories that would be described by a capital in dynamic equilibrium as the rate of interest slowly rose or fell.

**Capitals with Rate of Profit less than Rate of Interest**

It can be readily demonstrated that there exists no stable equilibrium gearing ratio for such capitals.

$$Ge = \frac{\alpha - p'}{\alpha - r}$$  \hspace{1cm} (35)

but since we know that for $p'$ less than $r, \alpha = 0$
Figure 8: Trajectory of capital as interest rate changes: \((p' > r)\)

\[ Gc = \frac{\ell'}{r} \]

This gives us the \(y\) coordinate of the equilibrium point, we now need its \(x\) coordinate or \(j\)

\[ j = \alpha - p' + G \cdot r \]

& (29))

thus

\[ J = -p' + (\frac{\ell'}{r}) \cdot r = 0 \]

Thus the equilibrium gearing ratio for all capitals with rate of profit less than rate of interest lies on the \(y\) axis. We have already shown that the trajectories of such capitals have a slope of \(\frac{1}{r}\) (from (33). On such trajectories any capital in the neighbourhood of the \(y\) axis will tend to move away from it. All capitals to the left of the axis will move down and to the left, all capitals to the right will move up and to the right. The equilibrium is therefore unstable.
Effect of Changing Interest Rates on Capitals with Different Profit Rates

Figure 9 shows the three classes of response that can be provoked by a rise in the rate of interest. In the diagram, the curve $G \epsilon_{r_0}$ is the equilibrium gearing ratio curve corresponding to the initial, lower rate of interest. In the initial period, three capitals, $P_0, P_1, P_2$, having progressively higher rates or profit are at rest on the curve $G \epsilon_{r_0}$. All three capitals have profit rates above $r_0$, the initial rate of interest. Consequently they are all at stable equilibrium points produced at the intersection of trajectories $t_0, t_1, t_2$ and the curve $G \epsilon_{r_0}$.

Suppose the rate of interest now rises to $r_1$, a level greater than the rates of profit of the first two capitals: $P_0$ and $P_1$. As a result of the change in the rate of interest, the equilibrium gearing ratio curve shifts. As we have argued earlier this results in the equilibrium gearing ratio being lower in the first quadrant and higher in the third quadrant. The new curve is labelled $G \epsilon_{r_1}$. The effect of this shift is different for each of the three capitals.

The capital with the highest rate of profit cuts its rate of accumulation in response to the higher rate of interest. As a result its level of borrowing is reduced and it moves to the left along the line $T_0'$, Its old gearing ratio is now above the new equilibrium ratio for its new lower rate of accumulation. As a result its gearing ratio falls and it moves down the trajectory $T_2$, until it meets the equilibrium gearing ratio curve.

The capital with the next highest rate of profit $P_1$ also shifts rapidly to the left along $T_1'$ as its rate of accumulation is also reduced. Since however, the new rate of interest is above its rate of profit, i.e., $r_1$ is greater than $P_1$, the trajectory it ends up on, $T_1$, has a positive gradient of $\frac{1}{r_1}$, i.e., it slopes in the opposite sense to $T_2$. Since $T_1'$ intersects $T_1$ to the left of the $y$ axis, capital $P_1$ slides down into the third quadrant. Since as we have already shown capitals with a rate of profit below the rate of interest have no stable equilibrium gearing ratio, this implies that it does on accumulating deposits indefinitely, so long as there is no fall in the interest rate.

The capital with the lowest rate of profit $P_0$, finds that even after reducing accumulation to zero it is unable to meet the interest on its debts. It moves onto the trajectory $T_0$, which like $T_1$ has gradient $1/r$. In this case, however, it moves in the opposite direction. Its gearing ratio rises as its profits prove insufficient to meet its debt charges. In due course this would lead to bankruptcy unless its profit rate rose relative to the interest rate.
Figure 9: Polarisation effect of interest rate changes
Financial Crises and the Polarisation of Capital

To analyse a possible mechanism for financial crises, we will assume an initial situation in which capital is distributed about the phase plane in proximity to the equilibrium gearing ratio curve. For this purpose the equilibrium gearing ratio curve may be considered as a one dimensional potential well. If the variance of the rate of profit does not vary and the rate of interest remains constant the system would be structurally stable. If we assume that the probability function relating withdrawals to the level of deposits does not change, which implies the existence of stable grounds for compensation on the part of bank capital, then a stable interest rate implies a stable growth of deposits relative to the growth of reserves. If we assume the reserves to be metallic, (so that their growth is set by technical conditions in mining) we can assume that the rate of growth of the reserves is an exogenously determined constant. The growth of deposits is determined by rentiers’ savings and enterprises’ accumulation of deposits given by

\[ \int_{-\infty}^{0} V(j, G) \cdot j \cdot dG \]

Suppose that for some reason the balance between the rate of growth of deposits and the rate of growth of reserves, breaks down. Will the system remain stable or will it progressively become more unstable?

Suppose the growth of reserves falls behind the growth of deposits, i.e., the growth of debt outstrips the growth in the means of payment for that debt. The immediate effect that we would expect to see is a rise in interest rates. This produces effects at two levels:

1. Because of the rise, those whose revenue consists of interest - the rentiers - experience a rise in their income. As a result their savings rise, accentuating the disproportion between reserves and deposits.

2. The rise in the interest rate shifts the equilibrium gearing ratio curve down in quadrant 1 and to the left in quadrant 3. Capitals already in the third quadrant will shift down and to the left along curves of the form \( P_0 \) and \( P_1 \) as shown in figure 6. This signals an increase in enterprise savings which in turn accelerates the growth of deposits.

Capitals in the first quadrant cut back their accumulation in an attempt to reduce their borrowing. This attempt inevitably fails. We have already shown that the net borrowing of the enterprise sector is determined by the net savings of the rentier sector. This means that
the center of mass of capital on the phase plane is determined outside the enterprise sector. Therefore the capitals in quadrant 1 could only shift to the left and reduce their borrowing if capitals in other sectors moved to the right, as compensation. Since we have already said that rentiers' savings are increasing (point 1), and that capital in quadrant 3 is moving to the left (point 2), it follows that capital in quadrant 1 must, in aggregate, move to the right. In other words, despite their attempt to reduce their borrowing they are constrained to increase it. Why?

The reason is that the measure that they take to improve their cash flow position produces an aggregate worsening of it. By reducing their accumulation of constant capital, ie, reducing investment, they reduce profit levels (as shown in our first model). In consequence their rate of profit will fall faster than their rate of accumulation. This relation holds for the capitals in quadrant 1 taken in aggregate. In effect, as we showed in the previous section, this population of capitals will split up into three subgroups according to their rates of profit. Some will succeed in reducing their borrowing and go on accumulating constant capital, albeit at a lower rate than before. Some will reduce their accumulation of constant capital so much that they become net savers, moving into the third quadrant. A third group will start to make losses and as a result move up and to the right.

The net result of this is a polarisation of the population of capitals, into debtors and creditors. A portion of the population is shifted down into the third quadrant while another portion is shifted up in the first quadrant. This signifies the growth of inter-enterprise debts and produces a further acceleration of the growth of claims relative to the means of payment of these claims. The accelerated rise of deposits relative to reserves further increases the rate of interest. The system is thus unstable. It is subject to positive feedback such that a small destabilisation is reproduced with magnified effects. The oscillatory tendencies that this produces have to be damped by changes in property holding.

**Concentration of Capital**

The above mechanism enables us to give a possible explanation for the evolution of rentiers and financial enterprises, and for the centralisation of industrial capital.

Suppose we have a system in which enterprises are predominantly directly personified by individual owners. During a period of rising interest rates or falling profit rates, there will exist a set of entrepreneurs whose rate of profit falls below the rate of interest, and whose gearing ratio is low enough for them to follow a trajectory of the form $P_1$ in
What does this movement signify if not that they have become rentiers? Their relationship to the process of extraction of surplus value has changed. As their gearing ratio becomes increasingly negative their main source of revenue ceases to be profit and becomes interest. The process of polarisation of capital secretes rentiers.

If we had started out with a population of joint stock companies rather than directly personified enterprises, the same process would convert some of them into financial companies or pseudo-rentiers.

The same process of polarisation of capital provides the financial precondition for the centralisation of capital. The polarisation of the population of capitals produces some with markedly positive and some with marked negative gearing ratios. The possibility therefore exists for the two to cancel each other out. Highly liquid enterprises with negative gearing ratios are in a position to buy up those with positive gearing ratios. Enterprises with high positive gearing ratios go bankrupt and are bought up by those with negative gearing ratios. The resulting concentration and centralisation of capital annihilates a portion of the inter-enterprise debt and reduces the polarisation of capital. In turn this provides the preconditions for a reduction in interest rates and a resumption of investment and real capital accumulation.

Conclusion

The model developed here is very skeletal. It assumes a very simplified version of capitalist property relations. It only allows the existence of four categories of economic subject: enterprises, rentiers, banks and workers. It shows that such a system has an inbuilt tendency towards realisation crises. It shows that these proceed via the polarisation of capital to the centralisation of capital. It shows that the system tends to reproduce the existence of a class of rentiers or pseudo-rentiers, and thereby to reproduce the contradiction between financial capital and productive capital.

The abstractness and generality of the model have certain advantages. They show that these contradictions leading to realisation crises are independent of the particular historical form in which these categories of subjects are constituted. It does not matter whether the rentiers are individuals or companies, it does not matter whether the banks are publicly or privately owned. The taxing out of existence of a particular historically constituted class of rentiers would not eliminate the tendency of the system to secrete pseudo-rentiers. A change in the ownership of banks unaccompanied by a change in their forms of practice and economic calculation would do nothing to remove these contradictions.
To be a useful tool for analysing the current situation the model would have to be extended to look at: the relationship between state finance and the realisation process of surplus value; the relationship between the declining tendency of the rate of profit defined at the level of value relations, and the realisation process; tendencies introduced into the development of property relations by state counter cyclical actions; the possible effects of different types of restructurings at the level of the circulation process of capital.

References


