What is at stake in the debate on value

Paul Cockshott
Allin Cottrell

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1 Introduction

New Lanark is one of the big tourist attractions in the south of Scotland. Designated a world heritage site by the UN it is an almost perfectly preserved model industrial village run by Robert Owen two hundred years ago. It has a large and elegant school building, an 'Institute for the Formation of Character', workers houses and the original mills. An exhibition and theme ride puts accross some of the basic facts about Owen, the pioneer of Socialism, Trades Unionism and the Co-operative movement. The shop sells his Millenium address, dedicated 200 years ago to our generation. Posters around the site display his humanist philosophy, and explain how he established the first nursery school for worker's children and set up the first system of sickness benefits.

Owen, an idealistic employer, at first thought that he could persuade other capitalists to reform society and abolish poverty, but was ridiculed by his class.

The New Lanark exhibition is rather coy about what Owen did next. It does not explain how he went on to become a socialist revolutionary - organising the Grand National Consolidated Union, the objective of which was to hold a general strike then sieze the factories and mills for the workers. The union was suppressed by the government and Owen eventually went into exile to form the commune of New Harmony in America.

He is remembered as the founder of co-ops and of industrial Trades Unionism. His plans for socialism have been largely ignored.

He proposed to abolish money and replace it with labour vouchers. Every worker would be paid an hour's worth of labour tokens for each hour they worked. Shops would sell goods at prices marked in labour so that if you worked an hour you could buy goods that had taken an hour to make.

Contrast this to today's society. With an hour's wages, the average British worker can buy goods that took about 35mins to make. The rest goes as profits to the rich. In this very little has changed since Owen's day.

His idea would have abolished exploitation and poverty at a stroke. But at the same time it would have abolished profit and the incomes of the rich.

Owen's ideas were later taken up and systematised by Karl Marx, whose book Capital expalined just how labour was exploited. Marx's ideas for socialism were similar to Owen's. Marx put forward the labour theory of value, showed that labour was the source of all value, and like Owen, Marx was adamant that socialism required the abolition of money:

> ”At this point I will only say that Owen’s ”Labour money”, for instance is no more money than a theatre ticket is. Owen presupposes directly socialised labour, a form of production diametrically opposed to the production of commodities. The certificate of labour is merely evidence of the part taken by the individual in the common labour and of his claim to a certain portion of the common product which has been set aside for consumption. But Owen never made the mistake of proposing the production of commodities, while, at the same time, by juggling with money, trying to circumvent the necessary conditions of that form of production.
>
> (Capital 1 p 188, penguin edition)

In his pamphlet Critique of the Gotha Program, Marx advocated exactly the same system as Owen. The first step of socialism was to be to get rid of money and replace it with labour vouchers. Taxes on peoples labour incomes would be used to pay for the sick, the disabled, social insurance.

> the individual producer receives back from society – after the deductions have been made – exactly what he gives to it. What he has given to it is his individual quantum of labor. For example, the social working day consists of the sum of the individual hours of work; the individual labor time of the individual producer is the part of the social working day contributed by him, his share in it. He receives a certificate from society that he has furnished such-and-such an amount of labor (after deducting his labor for the common funds); and with this certificate, he draws from the social stock of means of consumption as much as the same amount of labor cost. The same amount of labor which he has given to society in one form, he receives back in another.” (Critique of the Gotha Program).

The labour theory of value thus enters into two key areas of controversy: that over exploitation under capitalism, and the debate on the feasiblity of a socialist alternative to capitalism. When Marx based his analysis of capitalist exploitation on the labour theory of value he claimed to provide a scientific explanation for profit and disclose the latters exploitative roots. Because the labour theory of value touches on such hot issues it has, not surprisingly, been controversial.

These controversies have come on two fronts. On the one hand, opponents argued that the theory was not a scientific account of contemporary society, on the other they disputed the feasibility of using it to reshape the social order.

2 Its scientific status

The principle thrust of criticism of the labour theory of value within orthodox economics has been from the dominant subjectivist theory of value which locates the origin of prices in the relative subjective utility of commodities to the consumer. This is what is taught in all elementary economics textbooks, and the rise of this school of value theory can be seen as a late 19th or early 20th century response to the political influence of Marxian socialism.

A subsequent round of criticisms (Steedman 1981, Samuelson 1973, Roemer 1986) claimed that the labour theory was not so much wrong as redundant, since the work of Sraffa (Sraffa 1960) apparently showed that a non-subjectivist theory of price could be formulated without recourse to labour value.

If a theory purports to be scientific rather than a dogma, it must produce testable predictions. It must be possible to make observations or carry out procedures that would either confirm or undermine it. In this sense the labour theory of value starts out from a much stronger position than the subjectivist theory. Whilst there may be some questions of how one measures labour input, these pale to insignificance compared to the problem of providing an objective measure of subjective utility. One can propose mechanisms for the labour theory to be confronted with evidence which might refute it. It is much harder to see how the same might be done with the utility theory of value, whose scientific status is thus questionable. The alleged discrediting of the labour theory of value in orthodox economics has entirely been based on a-priori theoretical arguments. It has not been discredited by the the discovery of empirical evidence that was inconsistent with the theory. In science competing theories are supposed to be evaluated on the basis of their ability to explain observed data. Economics does not proceed in this way. The practical political implications of different economic theories are so great that it is very difficult for scientific objectivity to take hold. Whilst people build political parties on the basis of different economic theories, they dont fight in the same way over alternative theories of galactic evolution.

It was not until the 1980s that a serious scientific effort was made to test whether or not the labour theory of value actually held in practice. The pioneering work was done by Anwar Shaikh (1984, 1998) and his collaborators (Petrovic, 1987; Ochoa, 1989) at the New School in New York. Following this, there is now a considerable body of econometric evidence in favour of the proposition that relative prices and relative labour values are highly correlated, or in other words, in favour of the law of value.

2.1 Method of calculation

The key to testing the labour theory of value has been the use of input-output tables. An input-output table is a way of showing the structural interaction of different industries. These tables are periodically constructed by government statistical departments for the leading economies of the world. The idea behind them can be grasped by looking at the example in Table 1. This shows in a very aggregate fashion the structure of an economy with 4 main industries labeled A,B, C, D. The columns corresponding to the industries show how much of the output of each other industry is used up by a given industry. Thus industry A uses 100 from B and 10 from D. The numbers would refer to quantities of money, for now we can think of them as being billions of dollars. At the bottom we have rows showing the total amount of wages and profits earned in each industry and the total final sales of the industry. The final sales row is the sum of the wages, profits, and indirect inputs above.

Table 1: Example input output table

industry	A	B	C	D	final consumption
A		100	100	10	100
B	100				100
C		20			280
D	10		20		10
Wages	100	45	85	14	
Profits	100	35	95	16	
Sales	310	200	300	40	

Table 2: How to calculate labour values of an industy's output using an I/O table

A	B	C	D	Description
100.000	45.000	85.000	14.000	direct labour
0.322	0.225	0.283	0.350	labour/$ estimate 1
126.000	82.924	124.258	17.225	total labour estimate 1
0.406	0.414	0.414	0.430	labour/$ estimate 2
145.768	93.929	134.258	18.064	total labour estimate 2
0.470	0.469	0.447	0.451	labour/$ estimate 3
151.480	100.972	141.054	18.702	total labour estimate 3
0.488	0.504	0.470	0.467	labour/$ estimate 4
155.161	103.268	143.215	18.886	total labour estimate 4
0.500	0.516	0.477	0.472	labour/$ estimate 5
156.355	104.599	144.495	19.005	total labour estimate 5
0.504	0.522	0.481	0.475	labour/$ estimate 6

It is possible to use input output tables to work out how many hours of labour went into producing the total output of each industry.

We start up by simply adding up the number of units of labour that were directly employed in each industry. This is shown in row 5 of our initial Table 1, and in the first row of Table 2.

If we divide the directly utilised labour by the dollar value of the industry's output, we get an initial figure for the amount of labour in each dollar of the output. For industry A we see that 0.32 units of labour go directly into each dollar of output. Since we already know the number of dollars worth of A's output used by every other industry, we can use this to work out the amount of indirect labour used in each industry when it spends a dollar on the output of industry A.

This gives a second estimate for the labour used in each industry, which in turn gives us a better estimate for the number of units of labour per dollar output of all industries. We can repeat this process many times and as we do so, our estimates will converge on the true value. This process is illustrated in Table 2. If the labour theory of

Table 3: Average percentage deviations between market prices and labour values for the USA over selected years. Figures extracted from (Shaikh 1998).

Year	Deviation
1947	10.5%
1958	9.0%
1962	9.2%
1967	10.2%
1972	7.1%
Average	9.2%

value is empirically correct, then if you buy a dollar's worth of any product you would get back roughly the same quantity of labour. In other words, the figures for labour/$ for each industry would be very similar as shown in the final line of Table 2.

2.2 Results

Our example is very small and uses completely fabricated data. What happens when you look at a real economy?

Well for a start the tables are much larger, typically with around a hundred industries listed. But the same method can be applied, it just requires more computational effort. The work of calculation would have been daunting prior to the ready availability of computers for economic research. This may be why nobody seriously investigated the matter until the 1980s. But when Shaikh and others tried, they obtained results very similar to our toy example.

The general procedure in these studies has been to use data from national input–output tables to calculate the total labour content of the output of each industrial sector, and then to see how closely the aggregate money value of sales from each industry match their total labour content. Various different ways have been devised to measure the correspondence between the prices and the values. Shaikh (1984) explains the details of the process, and also offers a theoretical argument in favour of a logarithmic specification of the price–value regressions. Table 3 shows some results from Shaikh and his collaborators.

As you can see, the average error you get when predicting United States prices using the labour theory of value is only about 9%. This has proven to be the case accross many industries and several decades.

An alternative way of measuring the similarity of prices to labour values is to draw a scatter plot relating the two and then try to fit a straight line to the data. If the labour theory of value is true, then the observations will tend to fall close to this line, and the line will pass through the origin. How close the observations are to the line is measured by what is termed the R^2 value of the data. If the $R^2 = 1$ then all points fall on the line and the line perfectly predicts the results. If the $R^2 = 0$ then the line is of no use at all in predicting the observations.

Table 4: Comparing the correlation of prices to labour values in different countries

Country	R^2	Source
United States	0.974	(Ochoa 1989)
United Kingdom	0.955	(Cockshott, Cottrell and Michaelson 1995)
Greece	0.942	(Tsoulfidis and Maniatis 2002)
Sweden	0.971	(Zachariah 2004)

Studies—utilizing data from the United States, Sweden, Greece, Italy, Yugoslavia, Mexico and the UK—have produced remarkably consistent results, with strong correlations observed: R^2s of well over .90. It also seems to be the case from the literature that the larger the population of the country, the closer is the fit between observed prices and labour values, (Table 4). This may be an example of the way that statistical regularities become more apparent the larger the population on which the observations are performed.

2.3 Production prices

One group of critics of the labour theory of value (Steedman 1981, Samuelson 1973) have avered that it is redundant since :

1. Production prices(Marx 1971) provide a better estimate of real prices.
2. Production prices can themselves be derived without recourse to labour values(Sraffa 1960).

Whatever the validity of the second point, the first is an empirical claim, and one which has now been refuted. A series of studies (Petrovic 1987, Ochoa 1989, Shaikh 1998, Cockshott and Cottrell 1998) have shown that the labour theory of value is at least as good a predictor of real prices as price of production theory.

One of the key tenents of price of production theory was that the rate of profit would be equalised accross industries. This has turned out to be false, and indeed it turns out that rates of profit are higher in industries with a low organic composition of capital(Cockshott and Cottrell 1998, Cockshott and Cottrell 2003).

3 Critiques of the evidence

3.1 Podkaminer

One objection (Podkaminer 2005) to this says that the finding of a statistically significant inverse relation between organic composition and rate of profit is *consistent* with Marx's intuition but is, on the other hand, not very surprising since one could predict this from the fact that s/c and c/v are functionally related. Thus even if s, c and v are pairwise independent random variables the correlation between s/c and c/v would be expected to be negative and relatively large.

Podkaminer misunderstands the theoretical point. The existence of an inverse correlation between rate of profit and organic composition is definitely not consistent with Marx's position. In Vol III of Capital he suggested that the organic composition would be uncorrelated with the rate of profit. Marx argued that industries will all tend towards an equal rate of profit independent of their capital composition. This assumption has been the basis of the the entire school price of production theories, from Marx(Marx 1971) through Sraffa(Sraffa 1960) and Samuelson(Samuelson 1973) to Steedman(Steedman 1981). The negative correlation observed would indeed be unsurprising if one assumed that s, c, v were statistically independent random variables. What makes it surprising is the large body of economic literature which assumes that the s, c, v are not indpendent random variables, but on the contrary, that the three variables are functionally related as:

$$s = r(c + v)$$

whith r is the mean rate of profit.

In a series of papers (Cockshott and Cottrell 1997b, Cockshott, Cottrell and Michaelson 1995, Cockshott and Cottrell 1998, Cockshott and Cottrell 2003) we have argued that Marx's theory of prices of production is no better as a predictor of prices than the simple labour theory of value. The aim of our publications has been to support Farjoun and Machover's (Farjoun and Machover 1989) critique of deterministic price theories. These authors pointed out that since Boltzman physicists had been able to make useful predictions about the aggregate behaviour of systems which, at a small scale appear random and chaotic.

At a small scale the movements of molecules in a gas or a liquid are random, and this random movement is even visible, as Einstein pointed out in 1905, in the form of Brownian motion - the jiggling about of small particles like pollen grains in water observed under the microscope. But at a large scale these random motions even out, allowing useful generalisations: the gas laws, the laws of thermodynamics. Farjoun and Machover avered that economists were stuck with an early 19th century model of causality. If this was dropped then quite different modes of reasoning about the economy would become possible. Dispensing almost completely with orthodox economic theory the authors derived a series of interesting generalisations about capitalist economies. One of these was a prediction that market prices would be closely correlated with labour values.

3.1.1 Farjoun and Machover's argument

In science, predictions always seem more convincing than postdictions. The fact that Farjoun and Machover's theoretical results were rapidly confirmed by empirical research(Shaikh 1984, Petrovic 1987, Ochoa 1989, Shaikh 1998) lends their results weight, the more so when one considers that their predictions ran counter to recieved opinion in economics. We can not hope to give a full account of their theory here. Instead we will offer a simplified account, missing most of the mathematical rigour, but which should still give an intuitive understanding of the mechanism they proposed for the operation of the law of value.

Consider all of the commodities sold by firms in one country over the course of a week. These will constitute a vast array of different goods and services, some expensive and some cheap. Some will require a lot of labour to make, some a little. Suppose that the law of value holds and prices of commodities are closely proportional to their labour content. How should we measure this?

Farjoun and Machover introduce a random variable Ψ which stands for the average price of an hour's worth of embodied labour. The idea is that we express all of the national production of different goods: A380 airbuses, chocolate digestive biscuits, disposable nappies etc in terms of their labour content. We then divide this up into units of one hour each, and imagine that we randomly select an hours worth from this huge aggregate. We then look at how much that hours worth sells for in money terms.

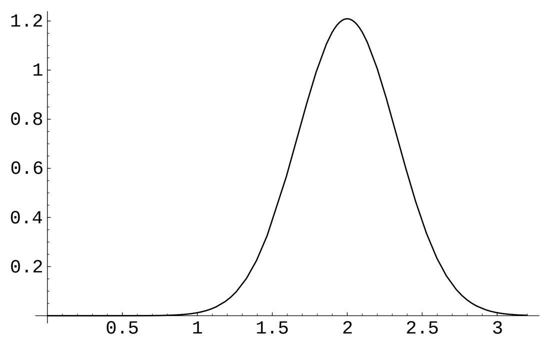

Figure 1: Farjoun and Machovers predicted form of Ψ the relation between labour value and price

They predicted that if one were to graph the frequency of occurence of different values of Ψ that one observed over a sufficiently large sample of commodities then the distribution would be normal and look like Figure 1.

A normal distribution arises, wherever the feature you are measuring is the result of summing up a large number of random, independently operating causal processes. A normal distribution $\mathcal{N}(\mu, \sigma)$, is characterised by two numbers its mean (μ) or average, which goes through the peak of the distribution, and its standard deviation (σ) which describes how wide the bell curve is. Farjoun and Machover predicted that the plot of prices to labour values would have a mean of 2 and a standard deviation of less than $\frac{1}{3}$. How did they arrive at this conclusion?

First why did they say that one would expect the mean to be 2, in otherwords that one would expect the average price of a commodity to be twice its labour value?

Well this is partly a matter of the unit of measurement they chose. As soon as you try to construct a theory of prices you are confronted with the question of the unit of account. When we want to measure distance we can do it in meters, which in their turn are defined in terms of a constant of nature - the wavelength of a particular type of light. This gives us a standard that is unvarying across space and time. But when it comes to measuring price, what do we use?

If we use money, should it be dollars, euros or yen?

If we stick to a single national currency, how do we account for inflation?

To get round this Farjoun and Machover use a technique favoured by Adam Smith and Maynard Keynes. They use the average hourly wage as their unit of account. Smith, as we have seen, held that the real price of any commodity was the amount of labour that it would command. Farjoun and Machover are slightly more precise and say that the real price of a commodity that took one hour's labour to produce is the number of hours of labour at the average hourly wage that it would command. Suppose a 4 kilo cod took an hour of direct and indirect labour to bring to the market. Suppose further that this cod sold for £15 and the average hourly wage was £6. In Farjoun and Machovers terminology then

$$\Psi_{cod} = \frac{\frac{\pounds 15/4kilo}{\pounds 6/1hour}}{1hour/4kilo} = \frac{\frac{15/4kilo}{6/1hour}}{1hour/4kilo} = \frac{\frac{15hour/4kilo}{6}}{1hour/4kilo} = \frac{15hour/6}{1hour} = \frac{15}{6} = 2\frac{1}{2}$$

We would expect Ψ to be > 1 since the selling price of any commodity can, as Smith showed, be decomposed into a part that pays wages and a part that pays profit[1]. The sale price goes to pay wages, profit and raw material costs. But the raw material costs likewise decomopose into wages, profits and a residuum of raw material costs. As you push the process back more and more stages one finds that the residual fraction of raw material costs tends to zero, so one can, to a good approximation, say the entire selling price goes ultimately to pay wages and profits[2]. Since Farjoun and Machover believed that in most capitalist countries value added was split 50/50 between wages and profits, it follows that the average price of the product of an hour's labour will be twice the average wage for an hour's labour.

That explains why they expect the mean of $\Psi = \frac{\mathbf{Price}}{\mathbf{labourcontent}}$ to be 2. Why then do they settle on a standard deviation of $\frac{1}{3}$?

The argument here is very simple. They say that it is very rare for commodities to be sold so cheaply that the selling price would be insufficient to pay the direct and indirect wages needed to make it. The cutoff point here is a value of $\Psi = 1$. Below this, the production of the commodity would be unviable, as not even wage costs would be met. For the sake of argument they assume that there is only one chance in a thousand of a commodity selling this cheaply relative to its cost of production.

By consulting a table of the normal distribution, one finds that the likelyhood of events 3 standard deviations away from the mean is about 1/1000, hence they derive that $\sigma = \frac{\mu-1}{3}$, so for a μ of 2, then σ must equal $\frac{1}{3}$.

How do these predictions stack up against real data. Using data for the United Kingdom in 1984, the year after their book was published, we calculate[3] that Ψ can be

[1] Smith also allowed for a part to pay rent, but Farjoun and Machover ignore this as being less significant than in the 18th century.

[2] Marx objected to this saying that the residual element of raw materials costs never quite reached zero. As a mathematical objection this is not very serious since the residual raw material cost exponentially approaches zero as a limit. As a sociological objection it has some weight since capitalist production presupposes the existence of capitalists who own raw materials and means of production and hire labour. If the raw materials and means of production were not in the hands of capital, then the workers would simply produce on their own account and there would be no division into wages and profits. Accepting this sociological point, Smith's mathematical approximation was reasonable.

[3] Result derived from Cockshott and Cottrell 1998, with slight adjustment to bring the definition of Ψ used in that paper in line with the definition used by Farjoun and Machover.

pretty well approximated by a distribution with $\mu = 1.46$ and $\sigma = 0.151$.

At first sight this appears significantly different from the precdition they gave. But the difference is almost entirely due to the fact that in the UK in 1984, value added was split between profits and wages in the ratio one to two instead of the equal split assumed by Farjoun and Machover. The full form of their predition was that if e is the ratio of aggregate profit to aggregate wages, then $\Psi \approx \mathcal{N}(\mu, \sigma)$ with $\mu = 1 + e$ and $\sigma \leq \frac{e}{3}$. If we substitute the relevant value of e for the UK in 1984 into the equations, we find an almost exact fit.

An interesting consequence of their theory is that it predicts that the correspondence between prices and labour values will be closer when the share of profit in national income declines. If the share of profit in the national income declines, then relative market prices can be expected to approximate more closely to relative labour values. Profits allow room for prices to have a lower signal to noise ratio.

The distribution of Ψ is random, or entropic. One can calculate the entropy of a normally distributed random variable using an amended form of Shannons formula(Shannon 1948). Shannon gave the entropy of a signal as

$$\sum_i -p_i \log_2(p_i)$$

where i takes on a set of discrete values corresponding to recognisably different quantisations of the signal. A Normal distribution $\mathcal{N}(\mu, \sigma)$ is a Probability Density Function (PDF). It is a function over the reals such that

$$P(a, b) = \int_a^b \mathcal{N}(\mu, \sigma)(x)dx$$

specifies the probability that x will be in the interval between $a..b$. If we substitute this into the Shannon formula and numerically integrate, we can compute the entropy of a normal distribution with a given standard deviation.

We find is that normal distributions with a small standard deviation have a low entropy and ones with a large standard deviation have a large entropy. Figure 2 shows the distribution of Ψ predicted by Farjoun and Machover, compared with a normal distribution with the mean and standard deviation observed for the UK in 1984. The entropy of wider bell curve on the right is about 7.1 bits, whereas that on the left is about 5.9 bits.

From the standpoint of the thermodynamic approach to the economy, Ψ's entropy $H(\Psi)$, measures the disorder of price with respect to value. From the standpoint of information theory, $H(\Psi)$ measures how much information there is in the deviation of prices from values. Our computed values for $H(\Psi)$ tell us that the market price of a commodity gives around 6 bits of information distinct from the information provided by its value. This raises the question : what about the rest? How much of the information in prices comes from labour values?

The random variable $\Psi = \frac{\pi}{\lambda}$ gives the ratio of a price π to its labour value λ. It thus assumes that we know the value of a commodity as well as its price. Strictly speaking $H(\Psi)$ is a *conditional* entropy.

A conditional entropy written as $H(A\ |B)$ or the entropy of A given B, is defined on two random variables: $A,\ B$, and is the disorder of A with respect to B. In our case

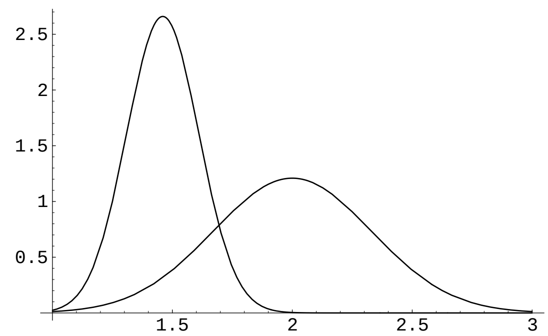

Figure 2: Farjoun and Machovers predicted Ψ (right) compared with a measured Ψ for the UK in 1984, (left).

we have $H(\pi\,|\lambda)$, or the entropy of price given value. The information shared by both A and B, which is called their *mutual* information, is given by $H(A) - H(A\,|B)$. We want to know the mutual information of prices and values $H(\pi) - H(\Psi)$. This will tell us how much information is common to both price and value.

To work it out we need some estimate of $H(\pi)$ the information content of prices. To do this accurately we would need to apply Shannons entropy formula to all prices so that $H(\pi) = \sum -p(\pi)\log_2(p(\pi))$.

This would involve knowing the probability distribution of prices. We would have to know how frequent prices of £1.00 were, how frequent prices of £2.00 were, etc, which must be done for all possible prices going from the lowest price at which a commodity can be bought - say 1 penny, up to the largest observed price, perhaps something like £1,000,000,000 for a large warship[4]. Although in principle this could be worked out, we don't have access to the data on real commodity prices required to get an answer, so we will use an alternative approach, based on coding theory, which will give us a rough estimate of the information content of prices.

Although prices can range from pennies to billions, a price in the billions will not be quoted down to the last penny. A shipyard are selling an aircraft carrier to the Navy need only quote to the nearest £million. If you are buying a cooker in the price range £100 to £500, you only look at the pounds and ignore the pennies. In general prices need not be quoted to more than 3 significant figures, the rest is just noise or a convention like the last 99 on a £34.99 pair of shoes. What we also need to know is the order of magnitude of the price: are the units, pennies, pounds, tens of pounds etc. This implies that for most purposes prices can be written in so called scientific notation as something like 1.47E3 to represent £1,470.00.

In a number with the format `x.xxEy` there are 4 digits that carry all the information. But 4 decimal digits can be encoded in just over 13 bits of information, so we can give a rough bound on the information content of a price as $H(\pi) < 14$. This implies that the mutual information shared between the price and value of a randomly selected

[4]To be compatible with the definition of Ψ we would have to weight the probabilities of each price with the amount of labour embodied in that price, but we need not be overly concerned with this technicality

Table 5: Price regressions for the UK in 1984

	(1)	(2)	(3)
constant	−0.055	−0.034	−0.046
	(−2.04)	(−1.79)	(−2.00)
labour value	1.024	1.014	1.024
	(46.55)	(63.38)	(51.20)
N	101	100	100
R^2	.955	.976	.964

Figures in parentheses are *t*-ratios. All variables in logarithmic form.
Data source: Central Statistical Office (1988).

commodity is likely to be $< 14 - H(\Psi)$ or roughly 6 to 7 bits: but not just any bits, the 6 to 7 most significant bits.

3.2 Are the correlations spurious?

Our presentation of how to calculate labour values from input output tables in section 2.1 said that you use the wages row of the input output table to estimate labour inputs to an industry. It could be argued that because this row is denominated in money, rather than in hours of labour, it is not really measuring labour inputs. It is possible to compensate for this by using data on hourly wage rates in the different industries. If we know the average hourly wage in an industry, we can translate that industries wage bill into actual hours worked.

The effects of doing this for the United Kingdom are shown in Table5.[5] In the published input–output tables, the labour input is expressed in £. Column (1) uses labour-value figures calculated on the assumption of a dummy wage-rate of £1 per hour for all industries. This is equivalent to assuming that any wage differentials across industries reflect differential rates of value-creation per clock hour. Column (2) is the same as (1) except for the exclusion of the oil industry, which is an outlier in the price–value regressions, presumably due to the high rent component (in the Ricardian sense) in oil extraction. Column (3) (which again excludes the oil industry) uses labour-value figures calculated using wages data from the *New Earnings Survey* to convert backwards from wages to hours for each industry—a correction relative to column (1) if (and only if) inter-industry wage differentials are the product of extraneous factors, and do not reflect differential rates of value-creation.

As can be seen from the equation (2) estimates, 'simple' labour values produce an R^2 of nearly 98% when the oil sector is excluded and the dummy uniform wage is adopted. The effect of adjusting for differentials in wage rates and using raw labour

[5]For further details regarding these estimates, see Cockshott, Cottrell and Michaelson 1995.

Table 6: Regressions of price on labour values and some alternative 'value-bases' for the UK.

	(5)	(6)	(7)	(8)	(9)	(10)
constant	−.056 (−2.06)	−0.169 (−2.425)	0.066 (3.15)	0.307 (3.16)	−0.067 −2.38	−0.263 (−2.47)
labour	1.030 (23.76)		0.904 (46.07)		1.048 (36.53)	
electricity	−0.009 (−0.19)	0.903 (14.60)				
oil			0.109 (7.43)	0.615 (13.29)		
iron and steel					−0.027 (−1.31)	0.445 (7.09)
Adjusted R^2	.953	.682	.984	.639	.954	.332

Figures in parentheses are *t*-ratios. All variables in logarithmic form. *Data source*: Central Statistical Office (1988).

ours in calculating the values gives a lower correlation of just over 96%. This is consistent with the hypotheses that :

1. Labour of higher skills produces more value per hour.
2. Inter-industry wage differentials at least partly reflect such skill differentials.

This suggests that the use of money wage bills as a surrogates for labour inputs to industries is valid.

3.3 Alternative value bases: empirical evidence

If the correlations between value and price are essentially spurious (Kliman 2002), then one could produce equally good results using something other than labour time as the 'basis' of value. But this turns out to be false as shown in Table 6. For the purposes of these regressions we used the Leontief inverse of the UK input–output tables (Central Statistical Office, 1988, Table 5) to calculate the total (direct plus indirect) electricity content, oil content and iron and steel content of the output of each industrial sector. Using the same methodology as in Table 5 (based on Shaikh, 1984), we then regressed aggregate price on these various 'values', both singly and in combination with labour values, in logarithmic form. The sample size is 100 for each of these regressions, the electricity industry being excluded from the equations including electricity-content, and similarly for oil and iron and steel.

From columns (6), (8) and (10) it can readily be seen than none of the alternatives, taken alone, performs anything like as well as labour. The highest R^2, at .682, is obtained for electricity content, as against .955 for labour in column (1) of Table 5.

Table 7: Regression of alternative value bases for Greece

Value Basis	R^2
Agriculture	0.174
Electricity	0.668
Oil	0.674
Chemicals	0.702
Labour	0.942

Data from Tsoufildis and Maniatis 2002

Columns (5), (7) and (9) show how the alternatives perform when entered alongside labour values, enabling us to address the question of whether the alternatives contain any independent information, or in other words offer any marginal predictive power over prices when labour content is given. Only oil content passes this test. From the *t*-ratios (in parentheses below the coefficient estimates) it can be seen that while labour content retains its statistical significance in all cases, electricity content and iron and steel content become statistically insignificant in the presence of labour content. The fact that oil content contains some independent information regarding prices is presumably linked to the element of rent in the price of oil. The North Sea fields are not marginal, which means that the labour time taken to extract North Sea oil is less than the socially necessary amount (on a world scale). The price of oil being determined on the world market, UK oil will then sell at a price above that which corresponds to its particular labour content. Table 7 shows similar results are obtained when analysing the Greek economy.

3.4 Steedmans objections

Steedman and Tomkins (Steedman and Tomkins 1998) raise two significant arguments against the measurements of correlation between labour values and market prices. It should be noted that both of their objections would also affect measurements of any other theory of value.

3.4.1 Physical units argument

Steedman argued that the use of regressions was unreliable since regressions can be sensitive to the physical units used to measure prices. Suppose we wanted to see if the price structure of consumer goods in Northern Ireland or in Scotland was more similar to the price structure in England. He argued that the correlation between the prices that we get will depend on the physical units used to measure commodities. In other words if in one case one used imperial units of volume and weight and in another one used metric units, the correlations between price vectors in the different countries would change. This is because one can change the weighting of prediction errors on particular commodities by chosing large or small units of measurement in ones study.

But it is questionable whether the concept of physical units is meaningfull in the studies in the literature since these use i/o tables and the sectors in the i/o tables, with the possible exception of electricty, produce a multiplicity of commodities which are aggregated to give a total output for the sector. There is no physical unit that can be used to measure the output of for example the farm sector. Is it bushels of wheat, tons of soya beans, thousands of apples? None of these make sense since the figure in the i/o tables is an aggregate. The studies compare the revenue of an industry to its direct and indirect labour inputs. Physical units of measure are not used at all.

3.4.2 Numeraire

Their next argument is that the result of correlation measures depend on the numeraire chosen - the commodity chosen to represent money.

Throughout the paper there is a working assumption that prices are exchange ratios between commodities and one particular commodity - money. This assumption informs the discussion, taken over from Steadman, of the variability of different measures with respect to the numeraire. In Steedmans discourse, this focus on the numeraire is part of his Ricardian background, and derives ultimately from Ricardo's commodity theory of money. Whilst there is obviously a considerably body of writers who subscribe to the commodity theory of money in the modern Post Keynesian literature this theory has been subjected to a sustained critique. For a summary of this see Ingham (Ingham 2004) or Wray (Wray 2004).

If we leave aside the problematic within which the discussion of the influence of the numeraire on correlation is placecd, there remain statistical problems with their objection.

If we introduce the numeraire as a new free variable in our function that evaluates the correlation of the value to price we have gone from a single measure to an ensemble of $n - 1$ measures. In moving from a single measure to an ensemble of measures the appropriate procedure is to look at the statistical properties of this ensemble. One would then have to look at the spread of the correlation as one systematically varied the numeraire. The hypothesis that market prices are well predicted by labour values would be consistent with the coefficient of variation of these correlations being relatively narrow.

4 Labour value and the Austrian critique of Socialism

The collapse of hithertooexisting socialism 15 years ago led to a revitalisation of the Austrian school of economics who had been in the ideological vanguard in criticising the very possibility of socialist economies.

The first proponent of the claim that socialist economic calculation was impossible was the economist of the Austrian school von Mises. In his book *Human Action* (von Mises 1949) he devoted a chapter to arguing against socialism. He had two main arguments: on the one hand he said that the socialists themselves could not agree on what socialism meant, on the other he tried show that economic calculation would be impossible without a market.

Mises notes that socialists have no uniform idea of what socialism is. Each socialist, or at least each group of socialists proclaims that only its view of socialism is right and that all others are misleaders, enemies of the people etc. Each socialist, he claims, implicitly assumes that the future socialist state will be headed by himself. True socialism is what he will decree. All other views are dangerous heresies best dealt with by the firing squad.

This seems to us to be a fairly accurate caricature of a substantial fraction of the socialist movement. Whilst the communist parties tended to have a fairly clear idea of what they wanted to achieve, based for the most part on an emulation of the USSR, other socialist parties have been loath to give a concrete view of how socialism should be organised. On all sides there has been a reluctance to examine the practical problems of organising a socialist economy. In a series of publications(Cottrell and Cockshott 1992, Cottrell and Cockshott n.d., Cockshott and Cottrell 1997a) we have put forward counter arguments to the Austrians. These arguments center on the use of labour values as a rational basis for socialist economic calculation.

> The director [6] wants to build a house. Now, there are many methods that can be resorted to. Each of them offers, from the point of view of the director certain advantages and disadvantages with regard to the utilization of the future building, and results in a different duration of the building's serviceableness; each of them requires other expenditures of building materials and labor and absorbs other periods of production. Which method should the director chose? He cannot reduce to a common denominator the items of various materials and various kinds of labor to be expended. Therefore he cannot compare them. He cannot attach either to the waiting time (period of production) or to the duration of serviceableness a definite numerical expression. In short, he cannot in comparing costs to be expended or gains to be earned, resort to any arithmetical operations. [7]

Mises is concerned above all with the issue of the choice of techniques to be used in the production process. The claim is that only a market, by reducing all costs and benefits to the common denominator money allows rational comparison of alternative possibilities.

He reviews various possible ways in which this could be done and rejects them all.

1. Calculation in kind is rejected because one can not add together quantities of different inputs unless one first converts them to a common unit of measurement like money. This is at first sight a reasonable argument but it involves certain presuppositions about the nature of calculation to which we will return.

2. Calculation in terms of the labour theory of value is rejected in a single sentence:

[6] The 'director' is von Mises term for the dictator of a socialist state: a peculiar adoption of capitalist corporate terminology that is perhaps understandable for a book published in 1940. His argument however is not dependent on the planning process being subordinated to the will of a single individual, but is more general so that for 'director' one could read: planning agency.

[7] **Human Action**, p694

> This suggestion does not take into account the original material factors of production and ignores the different qualities of work accomplished in the various labor-hours worked by the same and by different people. [8]

This is a somewhat brief treatment of the issue so our reply can also be concise. We have shown in (Cottrell and Cockshott 1992) that the labour theory of value does allow one to assign definite measures to the different value creating powers of labours of different degrees of skill. We also demonstrate that the complexity of such calculations are $\mathbf{O}N \log N$ and hence computationally tractable. The essence of the method is to cost the training of workers in terms of labour also and impute this to the work they do once they have been trained. As for the failure to take into account the original material factors of production, the classical theory of rent shows how the level of differential ground rent is governed by the marginal labour costs of production. There is no reason why this calculation can not be applied directly in a socialist economy.

3. He rejects what is essentially the market socialist approach on the ground that the market is essentially the pursuit of self interest and that its effective operation implies the existence of risk taking entrepreneurs. If one accepts that the pursuit of self interest through the market is necessary for economic calculation then it is inconsistent to try and exclude the function of the entrepreneur. In the view of what has happened in the former USSR after Gorbachov, this was a politically astute observation. Once the socialists have conceded the virtue of the market it is hard to denounce the vice of the exploiter clothed in the shining raiment of enterprise.

4. He argues against the use of "the differential equations of mathematical economics" as a technique of socialist economic calculation. It is not clear exactly which differential equations he means, but they appear to be those of comparative statics. Modern economics tends to assume that a differential equation will involve derivatives with respect to time, and thus that its function is to capture the dynamics of an economy. We assume that Mises means simply the differential calculus which is used in neo-classical economics to deduce static equilibrium conditions. The gist of his argument is that the equilibrium condition dealt with in comparative statics is an entirely abstract construction which never really occurs. The economy is constantly in a process of change and current resources available to it are always a hangover from the past unsuited to current wants.

   This is all true enough, but it does not prove that it is impossible to plan how best to use current resources to achieve a given future output. Our algorithm for plan balancing taking into account current stocks is one of probably many mathematical procedures that could be followed to achieve this end.

5. He also rejects what he calls the method of trial and error. This is the most interesting in our current context because it bears some relationg to what we advocate.

[8] Human Action,p 699.

> We may assume that in the socialist commonwealth there is a market for consumers goods and that money prices for consumers goods are determined on this market. We may assume that the director assigns periodically to every member a certain amount of money and sells the consumer goods to those bidding the highest prices. ... But the characteristic mark of the socialist system is that the producers' goods are controlled by one agency only in whose name the director acts, that they are neither bought nor sold, and that there are no prices for them. Thus there can not be any question of comparing input and output by the methods of arithmetic. [9]

This mechanism is similar to that which we advocate for the distribution of personal consumer goods. Mises again concentrates on the alleged impossibility of applying arithmetical methods to comparing inputs with outputs in the absence of markets for means of production. Our answer is simple, the planning agency knows:

(a) the labour contents of the different means of production,

(b) the number of labour tokens that each consumer good will fetch on sale to individuals

from this it is possible to compare the social cost of producing something with the valuation put on it by consumers. Dealing with producer goods is a little more complicated. In this case we have no market to give us a measure of demand for the good, but we do have the more direct information derived from input/output analysis. We know how much of each intermediate good will be required to meet a given mix of final consumer goods. We do not need a market in intermediate goods to determine how much should be produced.

Throughout, Mises identifies calculation with arithmetic. This is understandable since commercial calculation and arithmetic have been strongly associated. Calculation [10] and arithmetical operations are practically synonymous. But calculation can be seen as a particular instance of the more general phenomenon of computation or simulation. What a control system requires is the ability to compute. This is true whether the control system in question is a set of firms operating in a market, a planning agency, an autopilot on an aircraft or a butterfly's nervous system. But it is by no means necessary for this computation to proceed by arithmetical means.

The important thing is that the control system is able to model significant aspects of the system being controlled. Firms do this by means of the procedures of stock control and accountancy in which marks on paper model the location and movement of commodities. In preparing these marks the rules of arithmetic are followed. The applicability of arithmetic to the problem relies upon number theory being a model for the properties of commodities. A butterfly in flight has to control its thoracic muscles to direct its movement towards objects, flowers or fruit, that are likely to provide it with

[9] Human Action, p 701

[10] From **calculus** a pebble or stone used in counting.

energy sources. In doing this it has to compute which of many possible wing movements are likely to bring it nearer to nectar. As far as can be determined it performs these computations without the benefit of a training in arithmetic.

To use economic terminology the butterfly has many choices open to it. Different sequences of muscle movement have different costs in terms of energy consumption and bring different benefits in terms of nectar. Its nervous system has to try to minimise the costs and maximise the benefits using non-arithmetical methods of computation. The continued survival of butterflies is evidence of their computational proficiency. A planning agency is likely to make widespread use of arithmetic and indeed, if one wants to make localised decisions on the optimal use of resources by arithmetic means, then Mises arguments about the need to convert different products into some common denominator for purposes of calculation are correct. This is exactly the role played by labour values in our proposal: they allow engineers to have a good estimate of what is likely to be a cheap method of production.

If, however, one is wanting to perform global optimizations on the whole economy, other computational techniques having much in common with the way nervous systems are thought to work are appropriate: see (Cottrell and Cockshott 1992) chapter 6. These can in principle be performed without resort to arithmetic. Indeed Oskar Lange pioneered such approaches in the 1950's when he constructed a hydraulic model of the Polish economy for planning purposes. Mises, like many bourgeois theorists confuses the particular historical form in which a function is carried out with its essence. He reasons that :

1. Economies must optimize.
2. Arithmetic allows us to construct ordering relations over numbers, which can be used for optimization.
3. If one is to order numbers they must be of the same sort.
4. This requires conversion into a common unit of measure.
5. Money is a method of converting into a common unit of measure.
6. Hence all economies need money.

The problems with this argument lie in the step 5. While propositions 1.. 5 are true, they do not support conclusion 6. To reach that conclusion we sould need a stronger claim:

$5'$. Money is the *only* practical metric.

As we have shown, these stronger claims are false: there are non arithmetical methods of optimsation and money is not the only method of converting into a common unit of measure. Labour values are a viable alternative.

5 Conclusion

The dispute over the labour theory of value relates both to the Marxian critique of capitalist exploitation, and the socialist vision of a non-market economy of the future.

Over the last 20 years, empirical research enables us to state with ever increasing confidence that the labour theory of value is a well supported hypothesis for the working of the price system under capitalism, and that, in consequence, Marx's claim to have a scientific explanation of capitalist exploitation was well grounded.

By relying on the results of information theory and computational complexity theory one can also demonstrate that claims relating to the impossibility of using labour values in a socialist economy are unjustified.

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