Automatic SIMD/MIMD Compilation of tensor expressions

Paul Cockshott

University of Glasgow

June 22, 2009
<table>
<thead>
<tr>
<th>CPU</th>
<th>Year</th>
<th>Registers</th>
<th>Clock</th>
<th>Clock/Instr</th>
<th>Cores</th>
<th>Speed (Mips)</th>
<th>Data Rate (MB/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4004</td>
<td>1971</td>
<td>4</td>
<td>0.1</td>
<td>8</td>
<td>1</td>
<td>0.0125</td>
<td>0.00625</td>
</tr>
<tr>
<td>8080</td>
<td>1974</td>
<td>8</td>
<td>2</td>
<td>8</td>
<td>1</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>8086</td>
<td>1978</td>
<td>16</td>
<td>5</td>
<td>8</td>
<td>1</td>
<td>0.33</td>
<td>0.66</td>
</tr>
<tr>
<td>386</td>
<td>1985</td>
<td>32</td>
<td>16</td>
<td>3</td>
<td>1</td>
<td>5.0</td>
<td>20</td>
</tr>
<tr>
<td>MMX</td>
<td>1997</td>
<td>64</td>
<td>200</td>
<td>0.5</td>
<td>1</td>
<td>400</td>
<td>3,200</td>
</tr>
<tr>
<td>Harpertown</td>
<td>2007</td>
<td>128</td>
<td>3400</td>
<td>0.25</td>
<td>4</td>
<td>54,400</td>
<td>870,400</td>
</tr>
<tr>
<td>Larrabee</td>
<td>2010</td>
<td>512</td>
<td>2000</td>
<td>0.5</td>
<td>16</td>
<td>64,000</td>
<td>4096,000</td>
</tr>
</tbody>
</table>

- **Instruction speed** $s_i = \frac{pc}{i}$ where $p$ is processor cores, $c$ is the clock and $i$ is clocks per instruction.

- **Data throughput** $d = s_i w$ where $w$ is the register width in bytes.
Growth of clock speed versus maximum throughput

Log plot of Intel performance

Note how much of the increase in performance comes from increasing data parallelism.
Importance of Graphics Operations

The driving force in processor data throughput over the last decade has been graphics. We can see 4 stages in this evolution:

1. Intel introduce saturated parallel arithmetic for working on pixel arrays with the MMX instruction set.
2. AMD and Intel introduce parallel operations on 32 bit floats for working on co-ordinate transformations for 3D graphics in games.
3. Nvidia and ATI develop programmable Multi-core GPUs able to operate on 32 bit floats for games graphics.
4. Sony\textsuperscript{1} and Intel\textsuperscript{2} respond by developing general purpose multicore CPUs optimised for 32bit floating point vector operations.

\textsuperscript{1}Cell
\textsuperscript{2}Larrabee
Use the right types!

To get the best from current processors you have to be able to make use of the data-types that they perform best on: 8 bit saturated integers, and 32 bit floats. Parallel operations are possible on other data-types but the gain in throughput is not nearly so great.
**Operate on whole arrays at once**

The hardware is capable of operating on a vector of numbers in a single instruction.

<table>
<thead>
<tr>
<th>processor</th>
<th>byte</th>
<th>int</th>
<th>float</th>
<th>double</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Vector Lengths</td>
</tr>
<tr>
<td>MMX</td>
<td>8</td>
<td>2</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>SSE2</td>
<td>16</td>
<td>4</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>Cell</td>
<td>16</td>
<td>4</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>Larrabee</td>
<td>64</td>
<td>16</td>
<td>16</td>
<td>8</td>
</tr>
</tbody>
</table>

Thus a programming language for this sort of machine should support whole array operations. Provided that the programmer writes the operation as operating on a whole array the compiler should select the best vector instructions to achieve this on a given architecture.

*Use multiple cores*

If the CPU has multiple cores the compiler should parallelise across these without the programmer altering their source code.
Working with Pixels

When operating with 8 bit pixels one has the problem that arithmetic operations can wrap round. Thus adding two bright pixels can lead to a result that is dark. So one has to put in guards against this. Consider adding two arrays of pixels and making sure that we never get any pixels wrapping round in C:

```c
#define LEN 6400
#define CNT 100000
main()
{
    unsigned char v1[LEN], v2[LEN], v3[LEN];
    int i, j, t;
    for(i=0; i<CNT; i++)
        for (j=0; j<LEN; j++) { t = v2[j] + v1[j]; if( t > 255) t = 255; v3[j] = t;}
}
```

[wpc@maui tests]$ time C/a.out
real  0m2.854s
user  0m2.813s
sys   0m0.004s
Doing it with the hardware

Intel provide an instruction PADDUSB which can add 8 pixels in one cycle simultaneously ensuring that there is no wrap around. If we code the same program up in assembler we get much better performance.
SECTION .text ;
  global main
LEN equ 6400
main: enter LEN*3,0
  mov ebx,100000 ; perform test 100000 times for timing
l0:
  mov esi,0 ; set esi registers to index the elements
  mov ecx,LEN/8 ; set up the count byte
l1: movq mm0,[esi+ebp-LEN] ; load 8 bytes
    paddusb mm0,[esi+ebp-2*LEN] ; packed unsigned add bytes
    movq [esi+ebp-3*LEN],mm0 ; store 8 byte result
    add esi,8 ; inc dest pntr
    loop l1 ; repeat for the rest of the array
  dec ebx
  jnz l0
  mov eax,0
  leave
  ret
[wpc@maui tests]$ time asm/a.out
real 0m0.209s
user 0m0.181s
sys 0m0.003s
Why the difference?

- Semantic gap between source language and hardware capabilities.
  - C is a von Neuman single word at a time language.
  - Machine is a vector machine.
- Compiler tends to select the scalar instructions not the vector ones.
- Operator set of the language does not match the operator set of the hardware – it is less powerful than the hardware.
Now lets use an array language compiler

```pascal
program vecadd;
type byte=0..255;
var v1,v2,v3:array[0..6399]of byte;
i:integer;
begin
  for i:= 1 to 100000 do v3:=v1 +: v2;
  { +: is the saturated add operation } 
end.
[wpc@maui tests]$ time vecadd
real 0m0.094s
user 0m0.091s
sys 0m0.005s

So the array language code is about twice the speed as the assembler.
```
I will focus on the language Vector Pascal, an extension of Pascal that allows whole array operations, and which both vectorises these and parallelises them across multiple CPUs. It was developed specifically to take advantage of SIMD processors whilst maintaining backward compatibility with legacy Pascal code. It stands in a similar relationship to ISO Pascal as FORTRAN 95 stands to FORTRAN 77.
Extend array semantics

Standard Pascal allows assignment of whole arrays. Vector Pascal extends this to allow consistent use of mixed rank expressions on the right hand side of an assignment. For example, given:

\[
\begin{align*}
  r1 & : array[0..7] \text{ of real;} \\
  r2 & : array[0..7,0..7] \text{ of real}
\end{align*}
\]

then we can write:

1. \(r1 := 1/2;\)
2. \(r2 := r1*3;\)
3. \(r1 := r1+r2[1];\) \{\(\circ\text{ reduces using operator }\circ\}\)
4. \(r1 := r1+r2[1];\)

Line 1 assign 0.5 to each element of \(r1\).
Line 2 assign 1.5 to every element of \(r2\).
In line 3, \(r1\) gets the totals along the rows of \(r2\).
In line 4, \(r1\) is incremented with the corresponding elements of row 1 of \(r2\).
Data reformatting

Given two conformant matrices \(a, b\) the statement

\[
a := \text{trans} \ b;
\]

will transpose the matrix \(b\) into \(a\).

For more general reorganisations you can permute the implicit indices thus

\[
a := \text{perm}[1,0] \ b; \{ \text{equivalent to } a := \text{trans} \ b \}
z := \text{perm}[1,2,0] \ y;
\]

In the second case \(z\) and \(y\) must be 3 d arrays and the result is such that \(z[i,j,k] = y[j,k,i]\).

Given \(a: \text{array}[0..10,0..15] \text{ of } t;\) then

\[
\begin{align*}
a[1] & \quad \text{array } [0..15] \text{ of } t \\
a[1..2] & \quad \text{array } [0..1,0..15] \text{ of } t \\
a[][1] & \quad \text{array}[0..10,0..0] \text{ of } t \\
a[1..2,4..6] & \quad \text{array}[0..1,0..3] \text{ of } t
\end{align*}
\]
Equivalent loops

These are defined to be equivalent to the following standard Pascal loops:

1’. for $\iota_0 := 0$ to 7 do $r1[\iota_0] := 1/2$;
2’. for $\iota_0 := 0$ to 7 do
   for $\iota_1 := 0$ to 7 do $r2[\iota_0, \iota_1] := r1[\iota_1] \times 3$;
3’. for $\iota_0 := 0$ to 7 do
   begin
      $t := 0$;
      for $\iota_1 := 7$ downto 0 do $t := r2[\iota_0, \iota_1] + t$;
      $r1[\iota_0] := t$;
   end;
4’. for $\iota_0 := 0$ to 7 do $r1[\iota_0] := r1[\iota_0] + r2[1, \iota_0]$;

The compiler has to generate an implicit loop. In the above $\iota_0, \iota_1, t$
are temporary variables created by the compiler. The implicit
indices $\iota_0, \iota_1$ etc are accessible to a coder using the syntax
$iota[0], iota[1]$ etc.
Maps are implicitly defined on both operators and functions. If $f$ is a function or unary operator mapping from type $T_1$ to type $T_2$ and $x : \text{array of } T_1$ then $a := f(x)$ assigns an array of $T_2$ such that $a[i] = f(x[i])$. Similarly if we have $g(p, q : T_1) : T_2$, then $a := g(x, y)$ for $x, y : \text{array of } T_1$ gives $a[i] = g(x[i], y[i])$. 
Method of translation

pascal source $\rightarrow$ ILCG tree $\rightarrow$ machine specific assembler
ILCG

Intermediate language for code generation. It is a machine level array language which provides a semantic abstraction of current processors.

1. We can translate source code into ILCG.
2. We can describe hardware in ILCG too.

This allows the automatic construction of vectorising code generators.
Translation from source to ILCG

Pascal

\[ v3 := v1 +: v2; \]

ILCG

\[
\text{mem}(\text{ref uint8 vector (6400)}, +(\text{PmainBase}, -25600)) := \\
+:(^{\text{mem}(\text{ref uint8 vector (6400)}, +(\text{PmainBase}, -12800))), \\
\quad ^{\text{mem}(\text{ref uint8 vector (6400)}, +(\text{PmainBase}, -19200)))}
\]

Note that all operations are annotated with type information, and all variables are resolved to explicit address calculations in ILCG—hence close to the machine, but it still allows expression of parallel operations.

\(^\) is the dereference operation.
Key instruction specifications in ILCG

These are taken from the machine specification file gnuPentium.ilc

saturated add

**instruction pattern** PADDUSB(mreg m, mrmaddrmode ma)

**means** [(ref uint8 vector(8))m :=

(uint8 vector(8))+:((uint8 vector(8))^m),

(uint8 vector(8))^ma)]

**assembles** ['paddusb 'ma ',' m];

vector load and store

**instruction pattern** MOVQL(maddrmode rm, mreg m)

**means** [m := (doubleword)^rm]

**assembles** ['movq ' rm ',' m'

prefetchnta 128+’rm];

**instruction pattern** MOVQS(maddrmode rm, mreg m)

**means** [(ref doubleword)rm:= ^m]

**assembles** ['movq 'm ',rm];
Automatically build an optimising code generator

ILCG Compiler

Pentium.ilc → Pentium.java → Pentium.class
Opteron.ilc → Opteron.java → Opteron.class

Java Compiler

To port to new machines one has to write a machine description of that CPU in ILCG. We currently have the Intel and AMD machines post 486 plus Beta versions for the PlayStation 2 and PlayStation 3.
Vectorisation process

Basic array operation broken down into strides equal to the machine vector length. Then match to machine instructions to generate code.

ILCG input to Opteron.class

\[
\text{mem}(\text{ref uint8 vector ( 6400 ), } +(\text{PmainBase, -25600}) ) := \\
\quad +: (\wedge (\text{mem}(\text{ref uint8 vector ( 6400 ), } +(\text{PmainBase, -12800}))), \\
\quad \wedge (\text{mem}(\text{ref uint8 vector ( 6400 ), } +(\text{PmainBase, -19200}))))
\]

Assembler output by Opteron.class

```
leaq 0,%rdx ; init loop counter
l1: cmpq $ 6399, %rdx
   jg l3
   movq PmainBase-12800(%rdx),%MM4
prefetchnta 128+PmainBase-12800(%rdx) ; get data 16 iterations ahead into cache
   paddusb PmainBase-19200(%rdx),%MM4
   movq %MM4,PmainBase-25600(%rdx)
   addq $ 8,%rdx
   jmp l1
l3:
```
Extend to Multi-cores

Vectorisation works particularly well for one dimensional data in which there is locality of access, since the hardware wants to work on adjacent words. But newer chips have multiple cores. For the Opteron, the $\beta$ version of our compiler will parallelise across multiple cores if the arrays being worked on are of rank 2 rather than 1.
2 D example.

procedure sub2d;
type range=0..127;
var x,y,z:array[range,range] of real;
begin
    x:=y-z;
end;

Top level ILCG translation when compiled for a dual core Opteron

procedure(sub2d,
    procedure (label12 ... see below )
    post_job[label12,(%rbp),1];  /* send to core 1 */
    post_job[label12,(%rbp),0];  /* send to core 0 */
    wait_on_done[0];
    wait_on_done[1];
    )
Individual task procedure

The statement $x := y - z$ is translated into a procedure that can run as a separate task, the ILCG has been simplified for comprehensibility! Note the stepsizes of 2 and 4 in the loops rather than 1

```c
procedure (label12 /* internal label*/ ,
  for(mem(+(^(%rbp),-24)),^ (mem(+(^(%rbp),16))),127 , 2,
  /*iota [0] */ task number limit step*/
  var(mem(+(^(%rbp),-32))),/* iota[1] */
  for(mem(+(^(%rbp),-32)), 0 ,127, 4 ,
  /*iota [1] */ start limit step*/
  mem(ref ieee32 vector ( 4 ), /* x[iota[0],iota[1]] */
  +(*(^ (mem(+(^(%rbp),-24))),512),
    +(*(^ (mem(+(^(%rbp),-32))), 4),-131072)),
    ^ (mem(+(^(%rbp),-8))))):=
     ^(mem(ref ieee32 vector ( 4 ),/* y[iota[0],iota[1]] */
  +(*(^ (mem(+(^(%rbp),-24))),512),
    +(*(^ (mem(+(^(%rbp),-32))), 4),-196608)),
    ^ (mem(+(^(%rbp),-8))))),
  ^ (mem(ref ieee32 vector ( 4 ),/* z[iota[0],iota[1]] */
  +(*(^ (mem(+(^(%rbp), -24))),512),
    +(*(^ (mem(+(^(%rbp), -32))), 4),-262144)),
    ^ (mem(+(^(%rbp),-8))))))))},
)```
Memory organisation

Old context

Original stack for sub2d

Stack for task on core 0

Stack for task on core 1

Tasknum=0
Ret addr
display[1]
iota[0]
iota[1]

Rbp core0
Rsp core0

Tasknum=1
Ret addr
display[1]
iota[0]
iota[1]

Rbp core1
Rsp core1
Practical Example

The example is to write a parallel image blurring program in Vector Pascal.
The blurring program uses a simple 3x3 separable kernel applied first to the rows and then the columns.
It should use the type `pixel` for arithmetic since this maps to 8 bit fixed point numbers which can effectively use the MMX instructions.
It should be expressed in whole array operations if possible.
Horizontal blur

Recall we can blur horizontally if
\[ p'_i = 0.25p_{i-1} + 0.25p_{i+1} + 0.5p_i \]
So to form temporary images \( b, c \) from image \( a \) thus:
\[ b = 0.5a, c = 0.25a \]
the horizontal blurred image is then
\( b + c \) shifted left one + \( c \) shifted right one

**2D blur**

1. blur horizontally
2. transpose
3. blur horizontally
4. transpose
procedure blurplane(var inplane, outplane: plane);
  var temp:^plane;
begin
  new(temp, maxrow, maxcol );  {create buffer}
  horizblur(inplane, temp^);
  outplane := trans temp^;
  horizblur(outplane, temp^);
  outplane := trans temp^;
  dispose(temp);
end;
procedure horizblur(var inplane, outplane: plane);
  type tempplane = array[0..maxrow, 0..maxcol+2] of pixel;
  var temp:^tempplane;

  quarter, half: array[0..1024] of pixel;

begin
  quarter := 0.25;  (* create vectors of pixels *)
  half := 0.5;  (* to do parallel arithmetic *)
  new(temp);
  temp^[1.. maxrow+1] := inplane * quarter;
  outplane := temp^ + temp^[2.. maxcol+2]
               + inplane * half;
  dispose(temp);
end;