

## Introduction

### Goals

- ✓ Analyse popular diversification objective functions to find common pattern
- ✓ Find a simple way to estimate/validate the hyperparameters for balanced relevance-diversity trade-off

### Contributions

- ✓ Popular diversification objectives can be unified under the scheme of maximizing submodular or modular functions from the class of parameterized concave or linear over modular functions.
- ✓ The total curvature of the objective functions provides insights about the relevance-diversity trade off
- ✓ The total curvature value serves as a ‘vehicle of validation’ to seek hyperparameters that balances the relevance-diversity.

## Background

### Definitions

**Definition 1** A function  $F$  defined on the subsets of a ground set  $\mathcal{Z}$  is called *submodular*, if for all subsets  $\mathcal{A}, \mathcal{B} \subseteq \mathcal{Z}$ ,

$$F(\mathcal{A}) + F(\mathcal{B}) \geq F(\mathcal{A} \cup \mathcal{B}) + F(\mathcal{A} \cap \mathcal{B}).$$

$F$  is *modular* if strict equality holds, while  $F$  is *monotone* if for every  $\mathcal{A} \subseteq \mathcal{B}$ ,  $F(\mathcal{A}) \leq F(\mathcal{B})$ .

**Definition 2** A real-valued function  $g$  on a convex set  $\mathcal{C}$  is said to be *concave* if, for any  $x$  and  $y \in \mathcal{C}$  and for any  $a \in [0, 1]$

$$g((1-a)x + ay) \geq (1-a)g(x) + ag(y)$$

## Diversification Objective Analysis

### Generic Form

- Re-ranking based diversification objective functions can be expressed in the generic form:

$$F(\mathcal{S}) = f(\mathcal{S}) + \beta g(h(\mathcal{S})),$$

- ✓  $f(\cdot)$  represents relevance of  $\mathcal{S}$ . It is a **modular** function.
- ✓  $g(h(\mathcal{S}))$  represents the diversity of  $\mathcal{S}$ . It is the composition of a **linear or concave** function,  $g(\cdot)$ , and a **modular** function  $h$ .
- ✓  $h(\cdot)$  is defined in terms of different diversification concepts such as item coverage, popularity bias, item novelty and long-tail recommendation.
- ✓ The  $\beta$  is the hyperparameter to be tuned for the relevance-diversity trade-off.

### Functional Form of $g(x)$

Algorithm/Method	$g(x)$
Onuma et al. [5], Su et al. [7]	$-\frac{1}{x}, x > 0$ ,
Oh et al. [4]	$\log(x), x > 0$
Puthiya Parambath et al. [6], Vargas et al. [8]	$x^\beta, x \geq 0, \beta \in (0, 1)$
Wu et al. [10]	$\frac{x}{1+x}, x \geq 0$
Wasilewski and Hurley [9]	$\beta x, \beta > 0$

### Monotonicity of the Objective

Algorithm/Method	Monotonicity
Carbonell and Goldstein [2], Su et al. [7]	No
Abdollahpouri et al. [1], Oh et al. [4], Onuma et al. [5]	Yes
Puthiya Parambath et al. [6], Vargas et al. [8]	Yes
Wasilewski and Hurley [9], Wu et al. [10]	Yes

## Total Curvature

**Definition 3** The total curvature  $\alpha$  of a non-decreasing submodular set function with respect to a set  $\mathcal{S}$  is defined as:

$$\alpha = \max_{j \in \mathcal{S}} \frac{F(\mathcal{S} \setminus \{j\}) + F(\{j\}) - F(\mathcal{S})}{F(\{j\})}$$

- ✓ *total curvature* measures how far  $F(\cdot)$  is from being modular i.e. it represents the distance of a monotone submodular function to the modularity.
- ✓ *total curvature* takes value between 0 and 1.
- ✓ *total curvature* is zero for modular functions and one for matroid rank function.
- ✓ *total curvature* relies only on the marginal gains obtained by adding an item and not on specific functional form of  $F(\cdot)$ .

### Optimality bound in terms of total curvature

The bounds that can be obtained using the greedy algorithm for submodular maximization can be tightened using *total curvature*. According to Conforti and Cornuéjols [3],

$$F(\mathcal{S}^*) \geq \frac{(1 - e^{-\alpha})F(\mathcal{S}^{opt})}{\alpha}$$

$\alpha$	Optimality Gap
0.1	$F(\mathcal{S}^*) \geq 0.95F(\mathcal{S}^{opt})$
0.5	$F(\mathcal{S}^*) \geq 0.79F(\mathcal{S}^{opt})$
0.9	$F(\mathcal{S}^*) \geq 0.66F(\mathcal{S}^{opt})$

- ✓ By changing the value of the hyperparameter  $\beta$  in the re-ranking objective function, one can effectively change the *total curvature* of the objective.

### Balanced Hyperparameter Tuning

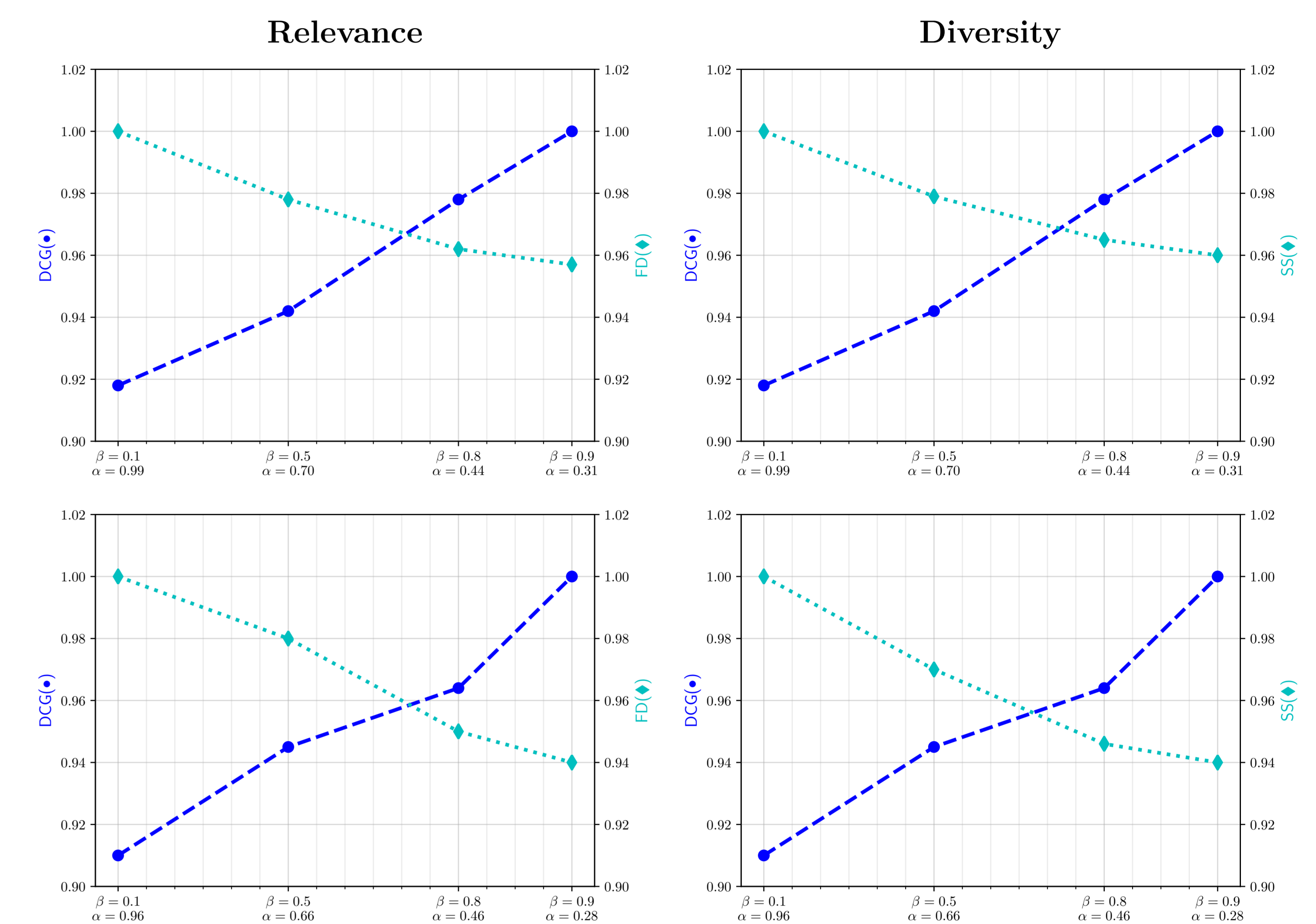
- ✗ In practice, the balanced value of  $\beta$  is found by the grid search.
- ✓ The balanced  $\beta$  value depends on the objective function  $F$ , whereas  $\alpha$  is independent of  $F$ .
- ✓ An  $\alpha$  value closer to 0.5 gives the balanced trade-off between being completely relevant and diverse.
- ✓ Practitioner can choose  $\beta$  such that the corresponding  $\alpha$  is closer to 0.5.

## Experiments

We used the benchmark MovieLens 20M dataset. We tested with two algorithms (i) the coverage maximization algorithm in [6] and *Binary xQuAD* algorithm in [1].

### Results

Relevance-Diversity trade-off for different values of  $\alpha$  and  $\beta$ . Top and bottom rows represent the relevance-diversity trade-off for the algorithms in [6] and [1], respectively.



## References

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