Algorithmics of Matching Under Preferences*

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Errata†

All page and line numbers are given with respect to the published (hard-copy) book.

Frontmatter

- Page ii, line 6: "Iwana" ⟶ "Iwama". (Due to Sofiat Olaosebikan.)

Preface

- Page vii, line -1 to Page viii, line 1: "polynomial-time algorithms)" ⟶ "polynomial-time) algorithms".

Foreword

- Page xiii, line -16: "became" ⟶ "become".

Chapter 1

- Page 5, line 8: "set pairs" ⟶ "set of pairs". (Due to Mechthild Opperud.)
- Page 6, line 3: add "who are not indifferent between the two matchings" after "preferred by the majority of the applicants".
- Page 17, line 13: "constrast" ⟶ "contrast". (Due to Mechthild Opperud.)
- Page 21, line 17: "attributes" ⟶ "attributed".
- Page 21, lines -4 to -3: change "pairs in which either (i) r_i is unassigned if she is unassigned in both M and M', or (ii)" to "pairs obtained as follows: for each resident r_i, r_i is unassigned if she is unassigned in both M and M', otherwise".
- Page 21, line -1: "join" ⟶ "meet". (Due to Didac Busquets.)
- Page 22, line 1: "meet" ⟶ "join". (Due to Didac Busquets.)
- Page 23, line 4: add "In an SM instance, any matching is automatically assumed to have size n."
- Page 33, line -18: "in an" ⟶ "is an". (Due to Mechthild Opperud.)

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- Page 35, line 9: "pairs" $\longrightarrow$ "distinct pairs". (Due to Mechthild Opperud.)
- Page 35, line 12: "all successors of $a_{i_{k-1}}$ from the list of a_{j_k}," $\longrightarrow$ "all successors a_r of $a_{i_{k-1}}$ from the list of a_{j_k}, and deleting a_{j_k} from the list of a_r,". (Due to Mechthild Opperud.)
- Page 39, line -8: add "Let A_M denote the set of applicants who are assigned in M." to the end of this paragraph.
- Page 41, lines -7 to -6: add 'who are not indifferent between the two matchings" after "preferred by the majority of the applicants".

Chapter 2

- Page 55, line 7: "admits least" $\longrightarrow$ "admits at least". (Due to Radosław Cymer and Mechthild Opperud.)
- Page 55, line 17: "Gusfield and Irving [261]" $\longrightarrow$ "Irving and Leather [319]".
- Page 56, line 5: "$(M(w), M(m)$" $\longrightarrow$ "$(M(w), M(m))$". (Due to Ciaran McCreesh.)
- Page 57, line 7: "D_{I_4}" $\longrightarrow$ "$D_{I'_4}$".
- Page 57, line -13: "D_I" $\longrightarrow$ "D_{I_k}". (Due to Mechthild Opperud.)
- Page 59, line -13: "for the finding" $\longrightarrow$ "for finding". (Due to Mechthild Opperud.)
- Page 61, line -5: "sex-equality measure measures" $\longrightarrow$ "sex-equality measures". (Due to Mechthild Opperud.)
- Page 66, line 15: "network!stability" $\longrightarrow$ "network stability". (Due to Radosław Cymer, Shuichi Miyazaki and Mechthild Opperud.)
- Page 79, line -15: "by the a" $\longrightarrow$ "by a". (Due to Mechthild Opperud.)
- Page 80, line -7: "paramters" $\longrightarrow$ "parameters". (Due to Mechthild Opperud.)
- Page 85, line -3: "$U \cup W) \setminus S$" $\longrightarrow$ "$(U \cup W) \setminus S$". (Due to Shuichi Miyazaki and Mechthild Opperud.)
- Page 98, line 1: "median stable matching in" $\longrightarrow$ "median of". (Due to Mechthild Opperud.)
- Page 98, line 2: "median stable matchings in" $\longrightarrow$ "medians of". (Due to Mechthild Opperud.)
- Page 99, line -16: "the the" $\longrightarrow$ "the".
- Page 113, line -2: "G in n-choosable" $\longrightarrow$ "G is n-choosable". (Due to Radosław Cymer.)
- Page 114, line 4: "the the line graph" $\longrightarrow$ "that the line graph". (Due to Mechthild Opperud.)

Chapter 3

- Page 138, Algorithm 3.1, add "Require: SMTI instance I", "Ensure: return a weakly stable matching M in I such that $|M| \geq \frac{2}{3}s^+(I)$".
- Page 139, Algorithm 3.2, under line "Require:", add "Ensure: w_j rejects m_i".
- Page 147, line -10: "prefers r_i to r_k" $\longrightarrow$ "prefers r_k to r_i". (Due to Mechthild Opperud.)
- Page 149, line 2: delete "Pareto".
- Page 149, line 3: "resident-Pareto" $\longrightarrow$ "resident-optimal weakly".
- Page 149, line 4: "resident-Pareto" $\longrightarrow$ "resident-optimal".
- Page 149, line 5: "matching M'" $\longrightarrow$ "weakly stable matching M'".
- Page 149, line 12: "resident-Pareto stable" $\longrightarrow$ "resident-optimal".
- Page 149, line 14: "resident-Pareto" $\longrightarrow$ "resident-optimal weakly".
- Page 149, line 20: "resident-Pareto" $\longrightarrow$ "resident-optimal weakly".
- Page 149, line 22: "resident-Pareto" $\longrightarrow$ "resident-optimal".
- Page 149, line 25: "resident-Pareto" $\longrightarrow$ "resident-optimal weakly".
- Page 149, line -7: add "Note that an instance of SM may not admit a stable matching that is Pareto optimal for the men – see Sec. 5.7.3."
- Page 158, line -6: "fewest" $\longrightarrow$ "minimum".
- Page 160, line -14: "super-stable in every" $\longrightarrow$ "stable in every". (Due to Mechthild Opperud.)

Chapter 4

- References to the Tan-Hsueh algorithm should be in the index.
- Similarly all references to the Roth–Vande Vate algorithm should be in the index (note that the term "Roth–Vande Vate Mechanism" is used in Chapter 2).
- Page 186, line 7: "exit conditions loop" $\longrightarrow$ "exit conditions". (Due to Mechthild Opperud.)
- Page 192, line -4: "given worked" $\longrightarrow$ "worked". (Due to Mechthild Opperud.)
- Page 197, line 5: "inSec." $\longrightarrow$ "in Sec." (Due to Mechthild Opperud.)
- Page 200, line -4: "$s + (I)$" $\longrightarrow$ "$s^+(I)$". (Due to Mechthild Opperud.)
- Page 216, line -4: "an many-many extension" $\longrightarrow$ "a many-many extension". (Due to Mechthild Opperud.)

Chapter 5

- Page 246, Definition 5.13: the first sentence of Case (3) should read "it involves a couple $(r_i, r_j) \in R_C$ and a pair of (not necessarily distinct) hospitals $h_k, h_l \in H$ such that $h_k \neq M(r_i)$, $h_l \neq M(r_j)$, (r_i, r_j) finds (h_k, h_l) acceptable, and either (r_i, r_j) is unmatched or prefers (h_k, h_l) to $(M(r_i), M(r_j))$, and *either*".
- Page 248, Theorem 5.15: "every distinct pair of hospitals" $\longrightarrow$ "every ordered pair of distinct hospitals".
- Page 251, line -10: "In fact, consistent preference lists need not be responsive" $\longrightarrow$ "In fact, responsive preference lists need not be consistent". (Due to Mechthild Opperud.)
- Page 254, caption of Figure 5.8: "HRIC" $\longrightarrow$ "HRS".
- Page 255, line -15: "all possible" $\longrightarrow$ "acceptable".
- Page 255, line -14: "the each" $\longrightarrow$ "each".
- Page 255, line -10: "in general exponential" $\longrightarrow$ "in the worst case exponential in".
- Page 259, line 8: "[498]" $\longrightarrow$ "[499]".
- Page 259, line 21, "mentiond" $\longrightarrow$ "mentioned".
- Page 264, line 18: "linear orders gives rise" $\longrightarrow$ "linear orders give rise". (Due to Mechthild Opperud.)
- Page 264, line 21: "*Algorithm* SPA-S-*student*" $\longrightarrow$ "*Algorithm* SPA-S-student. (Due to Mechthild Opperud.)
- Page 268, line 9: "generalistions" $\longrightarrow$ "generalisations". (Due to Sofiat Olaosebikan.)
- Page 277, line 8: "$deg_M(t)$" $\longrightarrow$ "$deg_M(t)$)". (Due to Radosław Cymer.)
- Page 278, line -12: after "Boros *et al.*", cite Ref. [109]. (Due to Radosław Cymer.)
- Page 286, line -5: "prefers $\{a_p, a_q\}$ and $\{a_r, a_s\}$" $\longrightarrow$ "prefers $\{a_p, a_q\}$ to $\{a_r, a_s\}$". (Due to Mechthild Opperud.)
- Pages 294-295: Algorithm 5.2, as it stands, may not produce a bistable matching. Instead of line 11, we should delete the pair (m_k, w_l) only if it belongs to M, otherwise the pair should be marked as *ineligible* (all man–woman pairs are initially *eligible*). If a man m_i proposes to a woman w_j where (m_i, w_j) is marked as ineligible, the procedure is as per lines 4-6 and 8-13 of Algorithm 5.2 (subject to the modifications to line 11 as described), but following any deletions and pairs being marked as ineligible, the pair (m_i, w_j) is not added to M but is instead deleted. This is as described in [585, Section 5]. (Due to Shuichi Miyazaki and Kazuya Okamoto.)
- Page 295, line 2: "instnance" $\longrightarrow$ "instance". (Due to Radosław Cymer.)

Chapter 6

- Page 311, lines -6 to -2: delete these lines as it is not true in general that $p^-(I) = \beta^-(G)$. However it is true that $p^-(I) \geq \beta^-(G)$ and $p^+(I) = \beta^+(G)$. (Due to Mechthild Opperud.)
- Page 312, lines 1-4: Theorem 6.6 should reference [18]. The second sentence in the theorem statement should be replaced by "The result holds even if each applicant finds at three houses acceptable." (Due to Mechthild Opperud.)
- Page 312, lines 5-6: "$p^-(I) = \beta^-(G)$" $\longrightarrow$ "$p^-(I) \geq \beta^-(G)$". (Due to Mechthild Opperud.)
- Page 312, lines 11-15: delete from "One way of proving this" up to the end of the paragraph, and replace with "A similar result holds for matchings in a graph: that is, a given graph G admits a maximal matching of size k, for each k such that $\beta^-(G) \leq k \leq \beta^+(G)$ [276]." (Due to Mechthild Opperud.)
- Page 313, line -13: insert "in" after "better off".
- Page 315, line 10: the case where $r = 1$ should be dealt with separately. In this case, each of a_{i_0} and h_k is unassigned, and $h_k \in A(a_{i_0})$. (Due to Baharak Rastegari.)
- Page 315, lines 24-27: replace by the following. Given an improving coalition C, let M' be the matching

  $$M' = (M \backslash \{(a_{i_j}, M(a_{i_j})) : 1 \leq j \leq r-1\}) \cup \{(a_{i_j}, M(a_{i_{j+1}})) : 0 \leq j \leq r-2\}.$$

  Then M'' is defined to be the matching obtained from M by *satisfying* C, where $M'' = (M' \backslash \{(a_{i_0}, M(a_{i_0}))\}) \cup \{(a_{i_{r-1}}, h_k)\}$ in the case of an alternating path coalition, $M'' = M' \cup \{(a_{i_{r-1}}, h_k)\}$ in the case of an augmenting path coalition and $M'' = (M' \backslash \{(a_{i_0}, M(a_{i_0}))\}) \cup \{(a_{i_{r-1}}, M(a_{i_0}))\}$ in the case of a cyclic coalition. (Due to Baharak Rastegari.)
- Page 317: the statement prior to Proposition 6.14 is incorrect. It is open as to whether the time complexity stated in Proposition 6.14 is true. However note that in an instance I of HAT in which every applicant's preference list comprises a single tie, the Pareto optimal matchings in I are precisely the maximum matchings in the underlying graph G. Thus an $O(m)$ algorithm for finding a Pareto optimal matching in I would imply an $O(m)$ algorithm to find a maximum matching in an arbitrary bipartite graph. (Due to Baharak Rastegari.)
- Page 320, lines 2-4: the sentence beginning "Also M is *trade-in-free*" should read "Also M is *trade-in-free* if there is no applicant–house pair (a_i, h_j) such that a_i is assigned in M, h_j is undersubscribed in M and a_i prefers h_j to $M(a_i)$." (Due to Andre Veski.)
- Page 321, lines 3 and 9 of Algorithm 6.3: A should be A_M. (Due to Zhiyuan Lin.)
- Page 322, after line 7 of Algorithm 6.4: add "if $(Q \neq \emptyset)$ then remove $head(Q)$ from L_k" – this is to prevent $a_t = head(Q)$ having h_k removed from its list, because a_t will be promoted to h_k at the next iteration of the while loop. (Due to Zhiyuan Lin.)
- Page 323, line 19: "who a_j envies" $\longrightarrow$ "whom a_j envies". (Due to Mechthild Opperud.)
- Page 325, line 16: "mxaimum" $\longrightarrow$ "maximum". (Due to Ágnes Cseh.)

Chapter 7

- Page 339, line 10: delete "indexsolvabilityprobability". (Due to Shuichi Miyazaki and Mechthild Opperud.)
- Page 339, line 10: replace "indexsolvability probability" by the corresponding invisible LaTeXcommand. (Due to Mechthild Opperud.)
- Page 339, lines 13 and 15: "proportion" $\longrightarrow$ "percentage". (Due to Mechthild Opperud.)
- Page 339, line 14: "1000" $\longrightarrow$ "100%" and "556" $\longrightarrow$ "55.6%". (Due to Mechthild Opperud.)
- Page 339, line 16: "1000" $\longrightarrow$ "100%" and "2" $\longrightarrow$ "0.2%". (Due to Mechthild Opperud.)
- Page 343, line 15: $s(T_i) \longrightarrow |s(T_i)|$. (Due to Shuichi Miyazaki.)
- Page 355, line 9: add the following text after "a contradiction": "Similarly if a house $h_j \in H$ is unassigned in M, let a_i be any applicant such that $h_j \in f(a_i)$. If a_i is unassigned in M, clearly $M \cup \{(a_i, h_j)\}$ is more popular than M, a contradiction. Hence let $h_k = M(a_i)$. Then $M' = (M \backslash \{(a_i, h_k)\}) \cup \{(a_i, h_j)\}$ satisfies $|P(M', M)| = |P(M, M')| = 0$, a contradiction." (Due to Mechthild Opperud.)
- Page 356, line -11: after "majority consensus" add "(among the applicants who are not indifferent between M and M')".
- Page 357, line -16: after "majority of the applicants" add "(who are not indifferent)".
- Page 361, line 4: "to case that" $\longrightarrow$ "to the case that" (Due to Mechthild Opperud.)
- Page 366, line -1: after "weighted majority of the applicants" add "who are not indifferent between the two matchings".
- Page 368, line 12: after "majority of the agents" add "who are not indifferent between the two matchings".
- Page 378, line 5: after "majority of the agents" add "who are not indifferent between the two matchings".
- Page 380, lines -8 to -7: "each of the problems of finding a popular matching and a maximum popular matching in the context of SRTI and SMTI" $\longrightarrow$ "the problem of finding a popular matching or reporting that none exists in the context of SRTI or SMTI"

Chapter 8

- Page 400, line 11: add "Note that the components in the profile of an alternating path can be negative, which is not true in the case of the profile of a matching."
- Page 401, Algorithm 8.3, line 3, 25 and 26: "O_r" $\longrightarrow$ "O_r^-, where $O_r^- = \langle p_1^-, \ldots, p_r^- \rangle$, $p_1^- = -n_1 - 1$ and $p_k^- = 0$ $(2 \leq k \leq r)$. (Due to Augustine Kwanashie.)
- Page 402, line 16: "O_r" $\longrightarrow$ "O_r^-, where $O_r^- = \langle p_1^-, \ldots, p_r^- \rangle$, $p_1^- = -n_1 - 1$ and $p_k^- = 0$ $(2 \leq k \leq r)$,". (Due to Augustine Kwanashie.)

- Page 404, line 8: "$1 \leq s < \beta$" $\longrightarrow$ "$1 \leq s \leq \beta$". (Due to Mechthild Opperud.)
- Page 404, lines -9 and -8: "Let O'_r be the vector $\langle p_1, \ldots, p_r \rangle$, where $p_k = 0$ $(1 \leq k \leq r-1)$ and $p_r = n+1$." $\longrightarrow$ "Let O_r^+ be the vector $\langle p_1^+, \ldots, p_r^+ \rangle$, where $p_k^+ = 0$ $(1 \leq k \leq r-1)$ and $p_r^+ = n_1 + 1$."
- Page 404, lines -8, -3: $O'_r \longrightarrow O_r^+$.
- Page 409, line -17: "such each paper" $\longrightarrow$ "such that each paper". (Due to Mechthild Opperud.)

Bibliography

- Page 419, reference 35: "Exchance-proofness" $\longrightarrow$ "Exchange-proofness". (Due to Mechthild Opperud.)
- Page 420, reference 50: In the order of authors, the order of Mitchell and Okamoto should be swapped.
- Page 426, reference 129: "How hard is to find" $\longrightarrow$ "How hard is it to find". (Due to Mechthild Opperud.)
- Page 430, reference 198: "Sjostrand" $\longrightarrow$ "Sjöstrand". (Due to Radosław Cymer.)
- Page 433, reference 238: "Maximale systeme unabhängiger kanten" $\longrightarrow$ "Maximale Systeme unabhängiger Kanten". (Due to Radosław Cymer and Mechthild Opperud.) Also "1965" $\longrightarrow$ "1964".
- Page 436, reference 272: The title should read " Improved approximation results for the stable marriage problem."
- Page 436, reference 274: "stable stable" $\longrightarrow$ "stable".
- Page 440, reference 330: The full title is "The stable fixtures problem – A many-to-many extension of stable roommates". (Due to Radosław Cymer.)
- Page 445, reference 394: The full title is "Mariages stables et leurs relations avec d'autres problèmes combinatoires". (Due to Mechthild Opperud.)
- Page 451, reference 462: "$o(n^3 \log n)$" $\longrightarrow$ "$O(n^3 \log n)$". (Due to Radosław Cymer.)
- Page 452, reference 476: "Die theorie der regulären graphs" $\longrightarrow$ "Die Theorie der regulären Graphs". (Due to Radosław Cymer.)
- Page 457, reference 556: "29" $\longrightarrow$ "30". (Due to Radosław Cymer.)
- Page 458, reference 578: "Tallin" $\longrightarrow$ "Tallinn". (Due to Radosław Cymer.)
- Page 459, reference 590: "18, 1" $\longrightarrow$ "38, 3". (Due to Radosław Cymer.)

Glossary

- Page 461, line -17: add notation for A_M, the applicants who are assigned in M (the context is HA).

Index

- Page 488, column 2, line 20: "FRee" $\longrightarrow$ "Free". (Due to Ágnes Cseh.)