Affirmative Action through Minority Reserves: An Experimental Study on School Choice

Flip Klijn, Joana Pais, and Marc Vorsatz

Glasgow, April 16, 2015

Background and Motivation

School choice: model of many–to–one matching markets where only one side is strategic.

Background and Motivation

School choice: model of many–to–one matching markets where only one side is strategic.

Concern for diversity has led to attempts to implement affirmative action in school choice programs.

Background and Motivation

School choice: model of many–to–one matching markets where only one side is strategic.

Concern for diversity has led to attempts to implement affirmative action in school choice programs.

Abdulkadiroğlu and Sönmez (2003) and Abdulkadiroğlu (2005) propose a cap or **maximum quota** on the number of students from the same group that a school can admit.

Background and Motivation

Kojima (2012) shows that affirmative action policies based on **maximum quotas** can be detrimental to the very minorities they are supposed to help.

Background and Motivation

Kojima (2012) shows that affirmative action policies based on **maximum quotas** can be detrimental to the very minorities they are supposed to help.

Intuition: if there is a school that is mostly wanted by majority students, it may end up with unfilled seats and unassigned majority students may create competition for seats at other schools, thus hurting minority students.

Background and Motivation

Kojima (2012) shows that affirmative action policies based on **maximum quotas** can be detrimental to the very minorities they are supposed to help.

Intuition: if there is a school that is mostly wanted by majority students, it may end up with unfilled seats and unassigned majority students may create competition for seats at other schools, thus hurting minority students.

Possible solution: **minority reserves** (Hafalir et al., 2013).

Background and Motivation

Kojima (2012) shows that affirmative action policies based on **maximum quotas** can be detrimental to the very minorities they are supposed to help.

Intuition: if there is a school that is mostly wanted by majority students, it may end up with unfilled seats and unassigned majority students may create competition for seats at other schools, thus hurting minority students.

Possible solution: **minority reserves** (Hafalir et al., 2013).

Minority reserves: school gives higher priority to minority students up to the point when minorities fill their "reserved seats." However, a school may assign some of its reserved seats to majority students provided that no minority student is interested.

Background and Motivation

Hafalir et al. (2013) adapt GS and TTC to minority reserves and show that strategy-proofness is preserved (i.e., no student can ever benefit by misrepresenting her preferences).

Background and Motivation

Hafalir et al. (2013) adapt GS and TTC to minority reserves and show that strategy-proofness is preserved (i.e., no student can ever benefit by misrepresenting her preferences).

Moreover, when all students tell the truth there is a clear sense in which minority reserves present an improvement over GS and TTC with/without majority quotas.

Background and Motivation

Hafalir et al. (2013) adapt GS and TTC to minority reserves and show that strategy-proofness is preserved (i.e., no student can ever benefit by misrepresenting her preferences).

Moreover, when all students tell the truth there is a clear sense in which minority reserves present an improvement over GS and TTC with/without majority quotas.

For instance, Hafalir et al. (2013) show that

- GS with minority reserves (weakly) Pareto dominates GS with majority quotas and, considering minority students only, is not strictly Pareto dominated by the standard GS.
- for minority students, TTC with minority reserves is not strictly Pareto dominated by the standard TTC.

Background and Motivation

Hafalir et al. (2013) adapt GS and TTC to minority reserves and show that strategy-proofness is preserved (i.e., no student can ever benefit by misrepresenting her preferences).

Moreover, when all students tell the truth there is a clear sense in which minority reserves present an improvement over GS and TTC with/without majority quotas.

For instance, Hafalir et al. (2013) show that

- GS with minority reserves (weakly) Pareto dominates GS with majority quotas and, considering minority students only, is not strictly Pareto dominated by the standard GS.
- for minority students, TTC with minority reserves is not strictly Pareto dominated by the standard TTC.

But: experimental literature on school choice shows that despite strategy-proofness, students very often do not submit their true preferences.

Background and Motivation

If two strategy-proof mechanisms are perceived differently, then they may give rise to very different types and levels of non-truthful behavior.

Background and Motivation

If two strategy-proof mechanisms are perceived differently, then they may give rise to very different types and levels of non-truthful behavior.

Therefore, whether or not affirmative action policies actually benefit minority students may well depend on how agents perceive the different mechanisms...

Background and Motivation

If two strategy-proof mechanisms are perceived differently, then they may give rise to very different types and levels of non-truthful behavior.

Therefore, whether or not affirmative action policies actually benefit minority students may well depend on how agents perceive the different mechanisms...

In our experimental study: four mechanisms, namely

- standard GS (GSs) and its counterpart with minority reserves (GSm);
- standard TTC ($TTCs$) and its counterpart with minority reserves ($TTCm$).

The school choice problem

Three schools: s_1, s_2, and s_3. Each school offers exactly two seats.

Six students look for a seat at one of three schools.

Majority group (students): M_1, M_2, M_3, and M_4.
Minority group (students): m_1 and m_2.

	Preferences						Priorities		
	M_1	M_2	M_3	M_4	m_1	m_2	s_1	s_2	s_3
Best match	s_1	s_1	s_3	s_3	s_2	s_2	m_1	M_3	M_1
Second best match	s_2	s_2	s_1	s_1	s_3	s_3	m_2	M_4	M_2
Third best match	s_3	s_3	s_2	s_2	s_1	s_1	M_4	M_2	m_2
Fourth best match							M_3	M_1	m_1
Fifth best match							M_1	m_1	M_3
Sixth best match							M_2	m_2	M_4

Centralized market: 1. students submit rank order lists, 2. a clearinghouse that uses one of the four mechanisms to assign students to schools.

For the mechanisms *with minority reserves*: each school *reserves one seat* for the minority group.

Gale-Shapley mechanisms (GS**s** and GS**m**)

Step 1. Each student sends an application to the school she ranked first.

Gale-Shapley mechanisms (GS**s** and GS**m**)

Step 1. Each student sends an application to the school she ranked first.

Step 2. Each school that receives at least 1 application acts as follows.

(*GSs*) It temporarily accepts the applicant with the highest priority. It also temporarily accepts the applicant with the highest priority among all remaining applicants (if any). The rest of the applicants (if any) are rejected.

Gale-Shapley mechanisms (GS**s** and GS**m**)

Step 1. Each student sends an application to the school she ranked first.

Step 2. Each school that receives at least 1 application acts as follows.

(*GSs*) It temporarily accepts the applicant with the highest priority. It also temporarily accepts the applicant with the highest priority among all remaining applicants (if any). The rest of the applicants (if any) are rejected.

(*GSm*) If the school receives no application from minority students, proceed as in Step 2 of *GSs*. If the school receives at least one application from minority students, then it temporarily accepts the minority applicant with the highest priority; it also temporarily accepts the applicant with the highest priority among all remaining (majority or minority) applicants (if any); the rest of the applicants (if any) are rejected.

Gale-Shapley mechanisms (GS**s** and GS**m**)

Step 3. Whenever a student is rejected by a school, she applies to the next highest ranked school.

Gale-Shapley mechanisms (GS**s** and GS**m**)

Step 3. Whenever a student is rejected by a school, she applies to the next highest ranked school.

Step 4. Each school that receives at least 1 new application acts as follows.

(*GSs*) Among all new and retained applications, the school temporarily accepts the applicant with the highest priority. Among the remaining (new and retained) applications (if any), it also temporarily accepts the applicant with the highest priority (if any). The rest of the applicants (if any) are rejected.

Gale-Shapley mechanisms (GS**s** and GS**m**)

Step 3. Whenever a student is rejected by a school, she applies to the next highest ranked school.

Step 4. Each school that receives at least 1 new application acts as follows.

(*GSs*) Among all new and retained applications, the school temporarily accepts the applicant with the highest priority. Among the remaining (new and retained) applications (if any), it also temporarily accepts the applicant with the highest priority (if any). The rest of the applicants (if any) are rejected.

(*GSm*) The school considers the new and the retained applications. If none of these applications is from a minority student, proceed as in Step 4 of *GSs*. If there is at least one application from a minority student, the school temporarily accepts the minority applicant with the highest priority; it also temporarily accepts the applicant with the highest priority among all remaining (majority or minority) applicants (if any); the rest of the applicants (if any) are rejected.

Gale-Shapley mechanisms (GS**s** and GS**m**)

Step 3. Whenever a student is rejected by a school, she applies to the next highest ranked school.

Step 4. Each school that receives at least 1 new application acts as follows.

(*GSs*) Among all new and retained applications, the school temporarily accepts the applicant with the highest priority. Among the remaining (new and retained) applications (if any), it also temporarily accepts the applicant with the highest priority (if any). The rest of the applicants (if any) are rejected.

(*GSm*) The school considers the new and the retained applications. If none of these applications is from a minority student, proceed as in Step 4 of *GSs*. If there is at least one application from a minority student, the school temporarily accepts the minority applicant with the highest priority; it also temporarily accepts the applicant with the highest priority among all remaining (majority or minority) applicants (if any); the rest of the applicants (if any) are rejected.

Step 5. Steps 3 and 4 are repeated until there are no more rejections. Matching becomes final.

Top Trading Cycles mechanisms (TTC**s** and TTC**m**)

Step 1. Each student points to the school she ranked first.

(*TTCs*) Each school points to the student with the highest priority.
(*TTCm*) Each school points to the minority student with the highest priority.

There is at least one cycle of students and schools.

Each student in any of the cycles is matched to the school she is pointing to and the school's number of available seats is reduced by one.

Top Trading Cycles mechanisms (TTC**s** and TTC**m**)

Step 2. Each unmatched student points to the school she ranks highest among all schools that still have available seats.

Top Trading Cycles mechanisms (TTC**s** and TTC**m**)

Step 2. Each unmatched student points to the school she ranks highest among all schools that still have available seats.

(*TTCs*) Each school with at least one available seat points to the student with the highest priority among all remaining students.

Top Trading Cycles mechanisms (TTC**s** and TTC**m**)

Step 2. Each unmatched student points to the school she ranks highest among all schools that still have available seats.

(*TTCs*) Each school with at least one available seat points to the student with the highest priority among all remaining students.

(*TTCm*) A school that was not matched to a minority student before points to

- the unmatched minority student if there is one; otherwise it points to
- the majority student with the highest priority among all remaining students.

A school that was already matched to a minority student points to the student (minority or majority) with the highest priority among all remaining students.

Top Trading Cycles mechanisms (TTC**s** and TTC**m**)

Step 2. Each unmatched student points to the school she ranks highest among all schools that still have available seats.

(*TTCs*) Each school with at least one available seat points to the student with the highest priority among all remaining students.

(*TTCm*) A school that was not matched to a minority student before points to

- the unmatched minority student if there is one; otherwise it points to
- the majority student with the highest priority among all remaining students.

A school that was already matched to a minority student points to the student (minority or majority) with the highest priority among all remaining students.

There is at least one cycle of students and schools. Each student in any of the cycles is matched to the school she is pointing to and the school's number of available seats is reduced by one.

Top Trading Cycles mechanisms (TTC**s** and TTC**m**)

Step 2. Each unmatched student points to the school she ranks highest among all schools that still have available seats.

(*TTCs*) Each school with at least one available seat points to the student with the highest priority among all remaining students.

(*TTCm*) A school that was not matched to a minority student before points to

- the unmatched minority student if there is one; otherwise it points to
- the majority student with the highest priority among all remaining students.

A school that was already matched to a minority student points to the student (minority or majority) with the highest priority among all remaining students.

There is at least one cycle of students and schools. Each student in any of the cycles is matched to the school she is pointing to and the school's number of available seats is reduced by one.

Step 3. Repeat Step 2 until all students are matched.

Procedures

- Eight sessions.
- Each session: two phases.
- In the two phases: different mechanisms. (So, 1 session= 2 mech's.)

Procedures

- Eight sessions.
- Each session: two phases.
- In the two phases: different mechanisms. (So, 1 session= 2 mech's.)
- Subjects knew that
  - decisions in 1st phase would not affect their payoffs in 2nd phase;
  - they would not receive feedback nor information regarding the decisions of any other player.

Procedures

- Eight sessions.
- Each session: two phases.
- In the two phases: different mechanisms. (So, 1 session= 2 mech's.)
- Subjects knew that
  - decisions in 1st phase would not affect their payoffs in 2nd phase;
  - they would not receive feedback nor information regarding the decisions of any other player.
- First mechanism is explained.
- Each subject plays two games under the first mechanism: once in her 'true' and once in a 'fictitious' role. Subjects do not know when.

Procedures

- Eight sessions.
- Each session: two phases.
- In the two phases: different mechanisms. (So, 1 session= 2 mech's.)
- Subjects knew that
  - decisions in 1st phase would not affect their payoffs in 2nd phase;
  - they would not receive feedback nor information regarding the decisions of any other player.
- First mechanism is explained.
- Each subject plays two games under the first mechanism: once in her 'true' and once in a 'fictitious' role. Subjects do not know when.
- Second mechanism is explained.
- Each subject plays two games under the second mechanism: once in her 'true' and once in a 'fictitious' role. Subjects do not know when.

Procedures

- Eight sessions.
- Each session: two phases.
- In the two phases: different mechanisms. (So, 1 session= 2 mech's.)
- Subjects knew that
  - decisions in 1st phase would not affect their payoffs in 2nd phase;
  - they would not receive feedback nor information regarding the decisions of any other player.
- First mechanism is explained.
- Each subject plays two games under the first mechanism: once in her 'true' and once in a 'fictitious' role. Subjects do not know when.
- Second mechanism is explained.
- Each subject plays two games under the second mechanism: once in her 'true' and once in a 'fictitious' role. Subjects do not know when.
- Either the 1st or 2nd phase & true roles is payoff relevant (3 euros fee + 6/9/12 euros). Randomly determined by computer at the end.

Null Hypothesis

Null Hypothesis:

In all four mechanisms, preferences are revealed truthfully. Hence, all four mechanisms generate the student-optimal stable matching and are Pareto-efficient.

Alternative Hypothesis 1 (on truth-telling)

	Preferences						Priorities		
	$\boldsymbol{M_1}$	M_2	$\boldsymbol{M_3}$	M_4	$\boldsymbol{m_1}$	m_2	s_1	s_2	s_3
Best match	$\boldsymbol{s_1}$	s_1	$\boldsymbol{s_3}$	s_3	$\boldsymbol{s_2}$	s_2	$\boldsymbol{m_1}$	$\boldsymbol{M_3}$	$\boldsymbol{M_1}$
Second best match	$\boldsymbol{s_2}$	s_2	$\boldsymbol{s_1}$	s_1	$\boldsymbol{s_3}$	s_3	m_2	M_4	M_2
Third best match	$\boldsymbol{s_3}$	s_3	$\boldsymbol{s_2}$	s_2	$\boldsymbol{s_1}$	s_1	M_4	M_2	m_2
Fourth best match							$\boldsymbol{M_3}$	$\boldsymbol{M_1}$	$\boldsymbol{m_1}$
Fifth best match							$\boldsymbol{M_1}$	$\boldsymbol{m_1}$	$\boldsymbol{M_3}$
Sixth best match							M_2	m_2	M_4

Alternative Hypothesis 1:

In the absence of minority reserves (i.e., GSs and TTCs),

- $\boldsymbol{M_1}$, $\boldsymbol{M_3}$, *and* $\boldsymbol{m_1}$ *have same levels of truthtelling.*
- M_2, M_4, *and* m_2 *have same levels of truthtelling.*

Alternative Hypothesis 2 (on truth-telling)

In case of minority reserves:

- Minority student m_1 will be assigned to her most preferred school in case she puts that school on the top of her reported preferences, independently of the behavior of the other students.
- As a consequence, minority student m_2 also experiences less strategic uncertainty. Moreover, still an advantage over majority students.

Alternative Hypothesis 2:
Minority students report preferences truthfully more often in the presence than in the absence of minority reserves.
In both GSm and TTCm, the level of truth-telling of student m_1 (m_2) is higher than the level of truth-telling of M_1 and M_3 (M_2 and M_4).

Alternative Hypothesis 3 (on truth-telling)

Alternative Hypothesis 3:

The mechanisms with minority reserves generate more truth-telling among all students than the corresponding mechanisms without minority reserves.

Alternative Hypothesis 4 (on stability)

Student	Stable matching				
	μ_1	μ_2	μ_3	μ_4	μ_5
M_1	s_3	s_3	s_2	s_1	s_1
M_2	s_3	s_2	s_2	s_2	s_1
M_3	s_2	s_2	s_1	s_3	s_3
M_4	s_2	s_1	s_1	s_1	s_3
m_1	s_1	s_1	s_3	s_2	s_2
m_2	s_1	s_3	s_3	s_3	s_2
Expected payoff (in ECU)	9	10.5	12	13.5	15

Table: Stable matchings.

Alternative Hypothesis 4 (on stability)

Student	Stable under minority reserves matching						
	μ_1	μ_2	μ_3	μ_4	μ_5	μ_6	μ_7
M_1	s_3	s_3	s_2	s_1	s_1	s_3	s_3
M_2	s_3	s_2	s_2	s_2	s_1	s_3	s_2
M_3	s_2	s_2	s_1	s_3	s_3	s_2	s_1
M_4	s_2	s_1	s_1	s_1	s_3	s_1	s_1
m_1	s_1	s_1	s_3	s_2	s_2	s_2	s_2
m_2	s_1	s_3	s_3	s_3	s_2	s_1	s_3
Expected payoff (in ECU)	9	10.5	12	13.5	15	10.5	12

Table: Stable under minority reserves matchings.

Alternative Hypothesis 4:

The probability of obtaining μ_4 or μ_5 (μ_4, μ_5, or μ_7) relative to the other stable (stable under minority reserves) matchings is higher in the presence than in the absence of minority reserves.

Alternative Hypothesis 5 (on expected payoffs)

In order to evaluate the success of the discriminatory policy, one definitively demands that the payoff to the minority students as a group is higher when minority reserves are present.

In our particular school choice problem, the effect of the minority reserves on the expected payoff of the majority group could be positive as well.

Alternative Hypothesis 5:
No student is harmed by the presence of minority reserves.

Truth-telling

Notation (2,3,1): ranking where a student lists her second most preferred school first, her least preferred school second, and her most preferred school last. (The other five strategies have similar interpretations.)

Mechanism	Submitted ranking					
	(1,2,3)	(1,3,2)	(2,1,3)	(2,3,1)	(3,1,2)	(3,2,1)
GSs	**0.39**	0.02	0.34	0.11	0.02	0.12
GSm	**0.39**	0.07	0.26	0.09	0.06	0.13
TTCs	**0.48**	0.05	0.31	0.07	0.03	0.06
TTCm	**0.34**	0.06	0.33	0.11	0.05	0.11

Truth-telling is salient, but much lower than predicted by theory (Null Hypothesis).

Truth-telling

Mechanism	Student					
	M_1	M_2	M_3	M_4	m_1	m_2
GSs	0.47	0.37	0.37	0.27	0.44	0.37
GSm	0.25	0.30	0.43	0.14	0.76	0.44
TTCs	0.53	0.55	0.50	0.33	0.58	0.39
TTCm	0.50	0.42	0.21	0.07	0.55	0.30

Result 1.a. *The proportion of truth-telling of the minority students is the same as that of their majority counterparts in the absence of minority reserves.*

Truth-telling

Mechanism	Student					
	M_1	M_2	M_3	M_4	m_1	m_2
GSs	0.47	0.37	0.37	0.27	0.44	0.37
GSm	0.25	0.30	0.43	0.14	0.76	0.44
TTCs	0.53	0.55	0.50	0.33	0.58	0.39
TTCm	0.50	0.42	0.21	0.07	0.55	0.30

Result 1.a. *The proportion of truth-telling of the minority students is the same as that of their majority counterparts in the absence of minority reserves.*

But:

Result 1.b. *There is a positive effect of minority reserves on the level of truth-telling of m_1 only under the Gale-Shapley mechanism.*
There are no spillover effects on other students.

Stability

Mechanism	Overall	Breakdown by stable matching (μ_1 worst, ... , μ_5 best)					
		μ_1	μ_2	μ_3	μ_4	μ_5	$\mu_4 + \mu_5$
GSs	0.3452	0.0125	0.1744	**0.5973**	0.2034	0.0125	**0.2158**
GSm	0.1318	0.0099	0.1366	0.0190	**0.8058**	0.0288	**0.8346**
TTCs	0.0460	0.0152	0.0022	0.0522	0.4696	0.4609	**0.9304**
TTCm	0.1157	0.0320	0.2740	0.1219	0.4538	0.1184	**0.5722**

Result 2.a.
The probability distribution over stable matchings obtained under GSm first-order stochastically dominates the one obtained under GSs. Conversely, the probability distribution over stable matchings under TTCs first-order stochastically dominates the one obtained under TTCm!

Stability under minority reserves

Mechanism	Overall	Breakdown by stable under minority reserves matching							
		μ_1	μ_2	μ_3	μ_4	μ_5	μ_6	μ_7	$\mu_4 + \mu_5 + \mu_7$
GSs	0.3928	0.0109	0.1533	0.5249	0.1833	0.0109	0.0120	0.1044	0.2986
GSm	0.3553	0.0037	0.0507	0.0070	0.2989	0.0107	0.0276	0.6015	0.9111
TTCs	0.0530	0.0141	0.0009	0.0450	0.4075	0.4000	0.1094	0.0226	0.8302
TTCm	0.2403	0.0154	0.1319	0.0587	0.2185	0.0570	0.0745	0.4449	0.7203

Result 2.b.

The probability of obtaining μ_4, μ_5, or μ_7 relative to the other four stable under minority reserves matchings is higher in GSm than in GSs.
Also, the probability distribution over stable under minority reserves matchings under TTCs first-order stochastically dominates the one obtained under TTCm.

Stability (2)

Mechanism	Students							
	M_1	M_2	M_3	M_4	m_1	m_2	Majority	Minority
GSs	0.1938	0.1494	0.3878	0.0543	0.2301	0.1988	0.1963	0.2024
GSm	0.5882	0.4245	0.2392	0.3063	0.0960	0.0328	0.3896	0.0644
TTCs	0.3471	0.4674	0.3716	0.2721	0.2695	0.4913	0.3646	0.3804
TTCm	0.4859	0.6090	0.3799	0.1565	0.2162	0.0852	0.4078	0.1507

Table: Average probability of belonging to a blocking pair.

Result 2.c.

Introducing minority reserves increases (decreases) the probability that the average majority (minority) student forms part of a blocking pair. Minority students are less likely to form part of a blocking pair when the GS mechanism is employed.

Expected payoffs

Mechanism	Overall	Students						
		M_1	M_2	M_3	M_4	m_1	m_2	$m_1 + m_2$
GSs	12.06	12.33	12.02	11.98	12.07	12.06	11.92	11.99
GSm	12.27	11.97	11.67	12.27	11.26	14.34	12.12	13.23
TTCs	12.70	13.44	12.24	13.10	12.01	13.52	11.99	12.71
TTCm	12.25	12.29	11.33	12.16	11.80	13.44	12.47	12.96

Result 3. *Minority reserves harm majority students, but benefit minority students inasmuch as the average payoff of this group increases. The distribution for m_1 under GSm first-order stochastic dominates the one under GSs.*

Summary

- TTC: adding minority reserves decreases levels of truthtelling of all types of students. It seems minority reserves increase the difficulty to understand the induced game.

Summary

- TTC: adding minority reserves decreases levels of truthtelling of all types of students. It seems minority reserves increase the difficulty to understand the induced game.
- GS: adding minority reserves only enhances truth-telling of some minority student.

Summary

- TTC: adding minority reserves decreases levels of truthtelling of all types of students. It seems minority reserves increase the difficulty to understand the induced game.
- GS: adding minority reserves only enhances truth-telling of some minority student.
- While there is almost no difference in the overall *efficiency* between the two mechanisms with minority reserves, GS mechanism tends to be more *stable* than TTC independently of whether minority reserves are employed or not.

Summary

- TTC: adding minority reserves decreases levels of truthtelling of all types of students. It seems minority reserves increase the difficulty to understand the induced game.
- GS: adding minority reserves only enhances truth-telling of some minority student.
- While there is almost no difference in the overall *efficiency* between the two mechanisms with minority reserves, GS mechanism tends to be more *stable* than TTC independently of whether minority reserves are employed or not.
- Moreover, when minority reserves are introduced in GS, the better stable matchings are reached more often.

Summary

- TTC: adding minority reserves decreases levels of truthtelling of all types of students. It seems minority reserves increase the difficulty to understand the induced game.
- GS: adding minority reserves only enhances truth-telling of some minority student.
- While there is almost no difference in the overall *efficiency* between the two mechanisms with minority reserves, GS mechanism tends to be more *stable* than TTC independently of whether minority reserves are employed or not.
- Moreover, when minority reserves are introduced in GS, the better stable matchings are reached more often.
- Price to pay for introducing minority reserves in GS:
  (1) stability decreases and
  (2) majority students are hurt (which should not happen in our particular setting).