Weakly-Popular and Super-Popular Matchings

Gergely Csáji, KRTK KTI, csaji.gergely@krtk.hun-ren.hu

Motivation

Matching markets have countless applications from school choice to kidney exchange. An important concept in such markets is *popularity*. However, a *key problem* with popularity is that a popular matching *may fail to exist* in the presence of ties. *We propose new versions of popularity, such that one of them always exits and study how to find them.*

Advantages of popular matchings

- It can be defined for both one-sided and two-sided markets
- Stronger than Pareto-optimality
- Weaker than stability, but allows much larger matchings, while still preserving global stability.

New popularity versions

Motivated by the fact that an agent may prefer not to change partners, if the new partner is similarly good, we define a slightly altered version of popularity. Let M, N be two matchings and v be an agent. Agent v cast a vote such that

$$vote_v(M,N) = \begin{cases} +1 & \text{if } M(v) \succeq_v N(v) \\ -1 & \text{if } N(v) \succ_v M(v) \\ 0 & \text{if } M(v) = N(v). \end{cases}$$

We say that M is *weakly popular*, if $\sum_v vote_a(M,N) \geq 0$ for all matchings N. We can define *γ-popularity* similarly, if we have γ_e^v numbers that define what improvement is enough to vote for N and change.
An opposite way to change the definition is when all agents are willing to help others to deviate and improve, so they vote with -1, even when $N(v) \succeq_v M(v)$. This leads to *super-popularity*.

The Algorithm

Step 1. *Create* an instance I' of the stable marriage problem with strict preferences by making *parallel copies* of each edge and create strict preferences over the created edges.

Step 2. *Run the Gale-Shapley* algorithm to obtain a stable matching M' in the new instance I'

Step 3. *Take the projection* M of M' to I by taking an edge e inside M, whenever one of the parallel copies of e was inside M'.

Results

MAX-γ-PM
Input: A bipartite graph $G = (U, W; E)$, $p_v(e)$ preference functions for each $v \in U \cup W$, numbers $0 < \gamma_e^v$ for each pair $(e, v) \in E \times (U \cup W)$ such that $v \in e$.
Output: A maximum size γ-popular matching M.

Let I be an instance of MAX-γ-PM. For each edge e, we create parallel copies $a(e), b(e), c(e), x(e), y(e), z(e)$.
Then, we create strict preferences as follows. For $u \in U$, we rank the copies according to the rule

$$a \succeq^\gamma b \succ c \succ x \succeq^\gamma y \succ z$$

For $w \in W$, we rank the copies according to the rule

$$z \succeq^\gamma y \succ x \succ c \succeq^\gamma b \succ a$$

Theorem. MAX-γ-PM can be *$\frac{4}{3}$-approximated* in polynomial-time.

Theorem. Assuming the Strong-UGC or the Small Set Expansion Hypothesis (SSEH), there is *no $(\frac{4}{3} - \varepsilon)$-approximation* algorithm for MAX-WEAK-PM, for any $\varepsilon > 0$.

Theorem. Deciding if there exists a *super-popular* matching is *NP-hard*, even if only two agents have ties.