Strategyproof Matching of Roommates to Rooms

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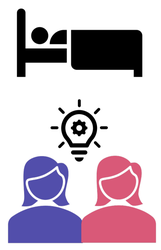
1. Problem

Given: n rooms and $2n$ agents, each with preferences over both roommates and rooms

Goal: Design **strategyproof** matching mechanisms that **maximize welfare**

2. Motivation

Much work on matching agents to agents OR items



Often have to match agents AND items

- Shared dorms/offices
- Group projects

Preferences over **both agent and item**

Most work only considers preferences over one type with additive valuations

3. Model

N-set of agents, R- set of rooms
$v_i(j)$ and $\hat{v}_i(r)$ value of agent i for agent j and room r

Roommate Matching: $\mu \subset N \times N \times R$

Utilities:

- **Additive**: $u_i(j,r) = v_i(j) + \hat{v}_i(r)$
  → Happy living with Darth Vader if it's in a mansion
- **Leontief**: $u_i(j,r) = \min\{v_i(j), \hat{v}_i(r)\}$
  → Unhappy with Darth Vader, even if it's in a mansion

Welfare: $\sum_{i \in N} u_i(\mu(i))$
Strategyproof (SP): No incentive to misreport v_i or $\hat{v}_i$

Binary Valuations: $v_i(j), \hat{v}_i(r) \in \{0,1\}$
Symmetric Valuations: $v_i(j) = v_j(i)$

4. Technical Contributions

- Introduce Leontief utilities to roommate matchings
- Study various maximal matching algorithms
- **Max Welfare Strategyproof mech under binary Leontief**

	General Valuations		Binary Valuations	
	Max Welfare	SP+ Max Welfare exists?	SP Mech (poly-time approx)	Upper Bound
Leontief Utilities	**APX-hard**	×	$\mathbf{1/3}$	$\mathbf{1}$
Additive Utilities	NPH[1]	×	$\mathbf{1/7}$	$\mathbf{2/3}$
Symmetric		×	$\mathbf{1/6}$	$\mathbf{3/4}$

1. Chan et al. Assignment and Pricing in Roommate Market. AAAI 2016

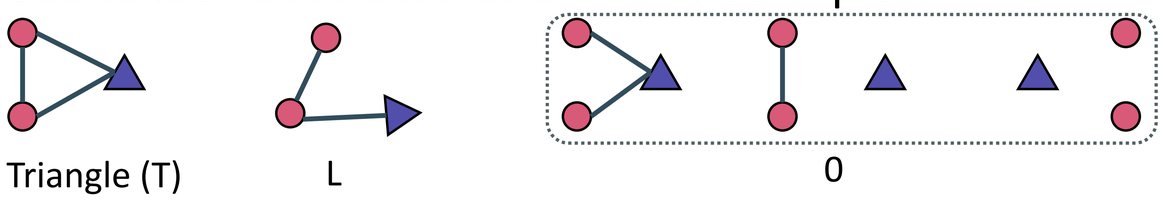
5. Approximate Max Welfare

Non-trivial to build maximal matchings.

Naïve approach: arb. match agent to pref agent/room

- 0-SW under Leontief + ¼-SW under additive
- Not strategyproof

Need to look at structures of matched triples:



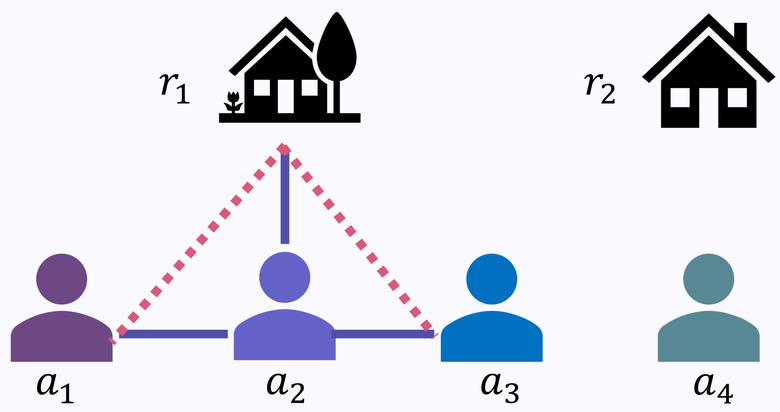
L/T maximal: No more L/T triples can be added

- $1/6$-SW for Leontief
- $1/7$-SW for additive
- Serial Dictatorship version: best known SP for additive

T-then-L Maximal Matching:

- $1/3$-SW
- Strategyproof for Leontief
- Best known poly-time SP mechanism for Leontief

6. Impossibility of Strategyproofness



Under true preference , two max welfare matchings
Both a_1 and a_3 have an incentive to misreport under any α-SW mechanism:

- General Additive/Leontief: for any $\alpha > 0$
- Binary Additive: for any $\alpha > 2/3$
- Binary Symmetric Additive: for any $\alpha > 3/4$

7. Max Welfare Strategyproof Mechanism for Binary Leontief

Welfare Set Reduction Mechanism:

- S_0 = ALL max welfare matchings
- For each agent $i \in [2n]$, $S_i = \text{argmax}_{\mu \in S_{i-1}} u_i(\mu)$
- Pick arbitrarily from S_{2n}

Can be improved using 3-SET PACKING algorithm, but not SP

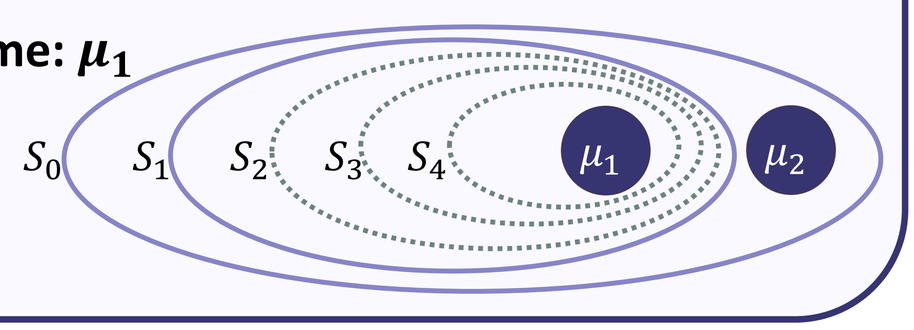
Precedence Based Search Mechanism:

- Pick an arbitrary precedence order on agents
- For each value w of max welfare in $2n$ to 1
  - Pick highest precedence subset N' of w agents not tried
  - Find a matching that gives N' value 1 and others 0 (using 3-SET PACKING)

Two max welfare matchings:
$\mu_1 = \{(a_1, a_2, r_1), (a_3, a_4, r_2)\}$ and
$\mu_2 = \{(a_2, a_3, r_1), (a_1, a_4, r_2)\}$.

Precedence order: $a_1 \succ a_2 \succ a_3 \succ a_4$

Outcome: μ_1



References: 1. Chan et al. Assignment and Pricing in Roommate Market. AAAI 2016
2. Gan, Li, Li. Your college dorm and dormmates: Fair resource sharing with externalities. JAIR 2023
3. Björklund et al. "Narrow sieves for parameterized paths and packings. JCSS 2017
4. Feng, Wang, Chen. Matching and weighted p2-packing: Algorithms and kernels. TCS 2014

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