MATCH-UP: Matching Under Preferences – Algorithms and Complexity

Satellite workshop of ICALP 2008

Further background information

Many practical situations give rise to large-scale matching problems involving sets of participants – for example pupils and schools, school-leavers and universities, applicants and positions – where some or all of the participants express preferences over the others. A large class of matching problems that involve preferences is the class of stable matching problems.

Such problems involve a set of agents, each of whom ranks a subset of the other agents in order of preference. The task is to find a *stable matching*, i.e., a set of pairs of acceptable agents such that no two agents could improve their assignment by becoming matched to each other. Since their introduction by Gale and Shapley in 1962, stable matching problems (and the classical stable marriage problem in particular) have attracted the attention of many researchers in the area of the design and analysis of algorithms (one of the major topics of ICALP Track A) as well as in other fields, such as discrete mathematics, game theory and economics. Important developments prior to the 1990s include the fundamental Gale-Shapley algorithm, algorithms for many-to-one hospitals-residents problems [7], and algorithms for non-bipartite stable roommates problems [13], as well as the identification, analyses and exploitation of lattice and poset structures underlying stable matchings [25, 16, 8, 17, 6]. The monographs of Gusfield and Irving [9] and Roth and Sotomayor [29] are comprehensive sources for aspects of these problems studied during that period.

Stable matching problems have important practical applications; the most famous example is the NRMP (National Resident Matching Program) in the US, for which Gale-Shapley type algorithms have been used for more than 50 years. Similar applications, to medical and other domains, also exist in Canada, Scotland, Japan, and many other countries. A variety of new problems arising from such applications, such as consideration of couples, have also been studied.

Since the mid-1990s, the trend has been towards extensions and variants of the basic problem in several different directions. These have included different notions of stability for the case that preference lists may include ties [14, 19, 24], many-to-many versions [4, 26], game-theoretic analyses [5, 11, 12], dynamic systems and equilibrium analyses [3], and approximation algorithms for hard variants of stable matching [20, 10, 21, 22, 23, 18]. In addition, new research into matching problems where preferences are expressed on only side has been productive, including studies of so-called *Pareto optimal matchings* [1], *rank-maximal matchings* [15] and *popular matchings* [2, 28, 27]. Thus developments in the field in the last few years have been impressive and the time seems to be right for researchers to come together.

This workshop will mainly focus on these new developments, with keywords "matching" and "preferences". We are interested in not only on-going developments as above but also in future directions for the research community, e.g., towards more game-theoretic and financial aspects of the problems. We also hope that this meeting will provide an ideal opportunity to bring together researchers from the computer science and economics communities, whose efforts in this field hitherto have tended to follow different paths.

References

[1] D.J. Abraham, K. Cechlárová, D.F. Manlove, and K. Mehlhorn. Pareto optimality in house allocation problems. In *Proceedings of ISAAC 2004: the 15th Annual International Symposium on Algorithms and Computation*, volume 3341 of *Lecture Notes in Computer Science*, pages 3–15. Springer, 2004.

[2] D.J. Abraham, R.W. Irving, T. Kavitha, and K. Mehlhorn. Popular matchings. In *SODA '05: Proceedings of the sixteenth annual ACM-SIAM symposium on Discrete algorithms*, pages 424–432. Society for Industrial and Applied Mathematics, 2005.

[3] D.J. Abraham and T. Kavitha. Dynamic matching markets and voting paths. In *Proceedings of SWAT 2006: the 10th Scandinavian Workshop on Algorithm Theory*, volume 4059 of *Lecture Notes in Computer Science*, pages 65–76. Springer, 2006.

[4] V. Bansal, A. Agrawal, and V.S. Malhotra. Stable marriages with multiple partners: efficient search for an optimal solution. In *Proceedings of ICALP '03: the 30th International Colloquium on Automata, Languages and Programming*, volume 2719 of *Lecture Notes in Computer Science*, pages 527–542. Springer, 2003.

[5] K. Cechlárová and J. Hajduková. Stable partitions with $\mathcal{W}$-preferences. *Discrete Applied Mathematics*, 138:333–347, 2004.

[6] T. Feder. A new fixed point approach for stable networks and stable marriages. In *Proceedings of 21st ACM Symposium on Theory of Computing*, pages 513–522, 1989.

[7] D. Gale and L.S. Shapley. College admissions and the stability of marriage. *American Mathematical Monthly*, 69:9–15, 1962.

[8] D. Gusfield. Three fast algorithms for four problems in stable marriage. *SIAM Journal on Computing*, 16(1):111–128, 1987.

[9] D. Gusfield and R.W. Irving. *The Stable Marriage Problem: Structure and Algorithms*. MIT Press, 1989.

[10] M. Halldórsson, K. Iwama, S. Miyazaki, and H. Yanagisawa. Improved approximation of the stable marriage problem. In *Proceedings of ESA 2003: the Eleventh European Symposium on Algorithms*, volume 2832 of *Lecture Notes in Computer Science*, pages 266–277. Springer, 2003.

[11] C.-C. Huang. Cheating by men in the gale-shapley stable matching algorithm. In *Proceedings of ESA '06: the 14th Annual European Symposium on Algorithms*, volume 4168 of *Lecture Notes in Computer Science*, pages 418–431. Springer, 2006.

[12] C.-C. Huang. Two's company, three's a crowd: stable family and threesome roommate problems. In *Proceedings of ESA '07: the 15th Annual European Symposium on Algorithms*, volume 4698 of *Lecture Notes in Computer Science*, pages 558–569. Springer, 2007.

[13] R.W. Irving. An efficient algorithm for the "stable roommates" problem. *Journal of Algorithms*, 6:577–595, 1985.

[14] R.W. Irving. Stable marriage and indifference. *Discrete Applied Mathematics*, 48:261–272, 1994.

[15] R.W. Irving, T. Kavitha, K. Mehlhorn, D. Michail, and K. Paluch. Rank-maximal matchings. In *Proceedings of SODA '04: the 15th ACM-SIAM Symposium on Discrete Algorithms*, pages 68–75. ACM-SIAM, 2004.

[16] R.W. Irving and P. Leather. The complexity of counting stable marriages. *SIAM Journal on Computing*, 15(3):655–667, 1986.

[17] R.W. Irving, P. Leather, and D. Gusfield. An efficient algorithm for the "optimal stable" marriage. *Journal of the ACM*, 34(3):532–543, 1987.

[18] R.W. Irving and D.F. Manlove. Approximation algorithms for hard variants of the stable marriage and hospitals/residents problems. *Journal of Combinatorial Optimization*, to appear, 2008.

[19] R.W. Irving, D.F. Manlove, and S. Scott. Strong stability in the Hospitals/Residents problem. In *Proceedings of STACS 2003: the 20th Annual Symposium on Theoretical Aspects of Computer Science*, volume 2607 of *Lecture Notes in Computer Science*, pages 439–450. Springer, 2003.

[20] K. Iwama, D. Manlove, S. Miyazaki, and Y. Morita. Stable marriage with incomplete lists and ties. In *Proceedings of ICALP '99: the 26th International Colloquium on Automata, Languages, and Programming*, volume 1644 of *Lecture Notes in Computer Science*, pages 443–452. Springer, 1999.

[21] K. Iwama, S. Miyazaki, and K. Okamoto. A $\left(2 - c\frac{\log n}{n}\right)$-approximation algorithm for the stable marriage problem. In *Proceedings of SWAT 2004: the 9th Scandinavian Workshop on Algorithm Theory*, volume 3111 of *Lecture Notes in Computer Science*, pages 349–361. Springer, 2004.

[22] K. Iwama, S. Miyazaki, and N. Yamauchi. A 1.875–approximation algorithm for the stable marriage problem. In *Proceedings of SODA 2007: the Eighteenth ACM/SIAM Symposium on Discrete Algorithms*, pages 288–297, 2007.

[23] K. Iwama, S. Miyazaki, and H. Yanagisawa. Approximation algorithms for the sex-equal stable marriage problem. In *Proceedings of WADS 2007: the Tenth International Workshop on Algorithms and Data Structures*, volume 4619 of *Lecture Notes in Computer Science*, pages 201–213. Springer, 2007.

[24] T. Kavitha, K. Mehlhorn, D. Michail, and K. Paluch. Strongly stable matchings in time $O(nm)$ and extension to the Hospitals-Residents problem. In *Proceedings of STACS 2004: the 21st International Symposium on Theoretical Aspects of Computer Science*, volume 2996 of *Lecture Notes in Computer Science*, pages 222–233. Springer, 2004.

[25] D.E. Knuth. *Stable Marriage and its Relation to Other Combinatorial Problems*, volume 10 of *CRM Proceedings and Lecture Notes*. American Mathematical Society, 1997. English translation of *Mariages Stables*, Les Presses de L'Université de Montréal, 1976.

[26] V.S. Malhotra. On the stability of multiple partner stable marriages with ties. In *Proceedings of ESA '04: the Twelfth Annual European Symposium on Algorithms*, volume 3221 of *Lecture Notes in Computer Science*, pages 508–519. Springer, 2004.

[27] D.F. Manlove and C.T.S. Sng. Popular matchings in the Capacitated House Allocation problem. In *Proceedings of ESA '06: the 14th Annual European Symposium on Algorithms*, volume 4168 of *Lecture Notes in Computer Science*, pages 492–503. Springer, 2006.

[28] J. Mestre. Weighted popular matchings. In *Proceedings of ICALP '06: the 33rd International Colloquium on Automata, Languages and Programming*, volume 4051 of *Lecture Notes in Computer Science*, pages 715–726. Springer, 2006.

[29] A.E. Roth and M.A.O. Sotomayor. *Two-sided matching: a study in game-theoretic modeling and analysis*, volume 18 of *Econometric Society Monographs*. Cambridge University Press, 1990.