

# On the Logical Interpretation of Sessions: The story so far

Bernardo Toninho  
`b.toninho@imperial.ac.uk`

Imperial College London

This document is an extended abstract proposing a talk at the upcoming final BETTY meeting. This contribution falls under the category of “Achievements during the project”, summarising the key results and contributions of the line of work connecting linear logic and session types.

## 1 Context

The advent of linear logic [7] triggered a number of research efforts aiming to connect logic (in the sense of proof theory) and concurrency theory. Notably, the work of Abramsky [1], later refined by Bellin and Scott [3], developed a connection between the proof theory of linear logic and a  $\lambda$ -calculus with parallel composition and a variant of the  $\pi$ -calculus, respectively. This line of work successfully identified the concurrent, or interactive, nature of linear logic specifications but failed to accurately identify a true propositions-as-types correspondence for linear logic. Notably, the work of Abramsky et al. [2] develops a notion of propositions-as-types for concurrency based on interaction categories – a categorical model of linear logic – but does not flesh out the identification of typed process calculi and linear logic *per se*.

This landscape remained mostly unchanged until the work of Caires and Pfenning [5], which identified linear logic propositions as session types, proofs as (typed) concurrent processes and proof reduction as communication, paving the way for a series of works relating typed concurrent phenomena with linear logic.

## 2 Session Types and Linear Logic

In this talk I will overview the interpretation of [5], based on intuitionistic linear logic, and its extensions accounting for (session) polymorphism [4], data dependencies [10] and higher-order process passing [11] (forming the basis for a concurrent functional programming language **SILL**). I will also briefly give an account of asynchronous communication within the framework [6] and more advanced reasoning techniques based on logical relations [4, 9].

The work of [5] also sparked a renewed interest in the connections of logic and concurrency. In the second part of the talk I will discuss the work of Wadler [12], which develops a similar interpretation for the classical setting, and of Lindley and Morris [8] who study recursion within the classical framework, and compare it with the analogous works carried out in the intuitionistic setting.

The final part of the talk will discuss existing challenges, ongoing research efforts and potentially interesting avenues of future work on this topic.

## References

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