Scottish Combinatorics Meeting 2025

Abstracts

Thomas Erlebach (Durham University) - *On reachability graphs of temporal traphs*

Temporal graphs are graphs that have a fixed vertex set but whose edge set can change in every time step. They can model time-varying or dynamic networks arising in application areas such as public transport networks, social networks, and mobile or reconfigurable communication networks. A temporal graph can be represented by a graph $G = (V, E)$ together with a labelling function that maps each edge to the set of time steps during which it is present. An edge e present at time t is referred to as a time edge. A temporal path from u to v is then a sequence of time edges that form a path from u to v and have increasing time labels. The reachability graph of a temporal graph is the directed graph with an arc (u, v) if and only if the temporal graph contains a temporal path from u to v. In this talk, we will discuss the directed graphs that can arise as reachability graphs of temporal graphs, the computational complexity of recognizing such graphs, and efficient algorithms for special cases of that problem.

Based on joint work with Othon Michail and Nils Morawietz.

Jane Tan (University of Oxford) - *Colouring t-perfect graphs*

The class of t-perfect graphs consists of graphs whose stable set polytopes are defined by their non-negativity, edge inequalities, and odd circuit inequalities. These were first studied by Chv/'atal in 1975, motivated by the related and well-studied class of perfect graphs. While perfect graphs are easy to colour, the same is not true for t-perfect graphs; numerous questions and conjectures have been posed, and even the most basic, on whether there exists some k such that every t-perfect graph is k-colourable, has remained open since 1994.

I will talk about joint work with Maria Chudnovsky, Linda Cook, James Davies, and Sang-il Oum in which we establish the first finite bound and show that a little less than 200 000 colours suffice.

Natalie Behague (University of Warwick) - *Colour-biased Hamilton cycles in subgraphs of the random graph*

Dirac's Theorem says that every n-vertex graph with minimum degree at least $n/2$ contains a Hamilton cycle. Lee and Sudakov extended Dirac's theorem to the setting of random graphs, showing that a random graph $G(n, p)$ (with p above the threshold to contain a Hamilton cycle) typically has the property that every spanning subgraph of it with minimum degree at least $(1/2 + o(1))np$ contains a Hamilton cycle.

In a different direction, a 'discrepancy' version of Dirac's theorem states that every r-colouring of the edge set of an n-vertex graph with minimum degree at least $(1/2+1/2r+o(1))n$ contains a Hamilton cycle where one of the colour appears (a lot) more than n/r times. Such Hamilton cycles are called colour-biased.

This talk concerns a combination of the above lines of research. Specifically, I will discuss a discrepancy result proving that the random graph $G(n, p)$ (where p is above the Hamiltonicity threshold) typically has the property that every r-colouring of the edge set of every spanning subgraph with minimum degree at least $(1/2 + 1/2r + o(1))np$ contains a colour-biased Hamilton cycle. This is asymptotically optimal. I will finish with a brief discussion of some potential generalisations.

This is joint work with Debsoumya Chakraborti and Jared Leon.

Colva Roney-Dougal (University of St. Andrews) - *Counting groups*

The symmetries of any object are described by a group, so it is natural to ask: What does a random group look like? This talk will start with a brief survey of how we might go about counting various algebraic structures. We'll then go on to see what a random group might be, in various different contexts.

A symmetric group on some set Ω is the group of all permutations of Ω, under composition of functions. Every group arises as a subgroup of some symmetric group, so fully understanding the symmetric group means understanding all groups. An elementary argument shows that there are at least $2^{n^2/16}$ subgroups of a symmetric group on n points, and it was conjectured by Pyber in 1993 that up to lower order error terms this is also an upper bound. The same year, Kantor conjectured that a random subgroup of the symmetric group is nilpotent. This talk will present a proof of one of these conjectures, and a disproof of the other.

The new results in this talk are joint work with Gareth Tracey (Warwick).

He Sun (University of Edinburgh) - *Dynamic spectral clustering with provable approximation guarantee*

Spectral clustering is one of the most fundamental clustering algorithms in machine learning and has comprehensive applications in other fields of computer science. In this talk I will introduce the basics of spectral clustering, starting with its roots in spectral graph theory and its connection to eigenvalues and eigenvectors of graph Laplacians. I will further present a spectral clustering algorithm in dynamic settings and the techniques for analysing its performance. Several open problems will be discussed at the end of the talk.

This is based on joint work with Steinar Laenen from the University of Edinburgh, and the work appeared at ICML 2024.

Francesca Arrigo (University of Strathclyde) - *Walk this way: non-k-cycling walks in complex networks*

Powers of the adjacency matrix of a graph allow for an extremely simple and cost efficient way to count walks around it. Walks can in turn be used to define centrality measures, thus allowing one to identify the most relevant entities in a network. However, a lot of the information accounted for in walks is somehow redundant. For instance, backtracking (i.e., behaviour of the type $i \to j \to i$, for two nodes i and j) is best avoided in certain message passing settings, yet backtracking walks carry the same weight in centrality measures than non-backtracking ones.

In this talk, after a general introduction to the topic of networks and walk-based centralities, we will describe how to count walks that avoid backtracking. Further, we will show how the presented strategy can be extended to count walks that avoid cycles, i.e., closed paths, of up to a given length k. The derived expressions will then be used to construct families of walk-based centralities that only consider the desired types of walks.

This talk collates a series of results from a few papers published in collaboration with Prof. Peter Grindrod (Oxford), Prof. Desmond Higham (Edinburgh), and Prof. Vanni Noferini (Aalto).

Yiannis Giannakopoulos (University of Glasgow) - *On the Smoothed Complexity of Combinatorial Local Search, and Congestion Games*

In this talk I will present a unifying framework for smoothed analysis of combinatorial local optimization problems and discuss how a diverse selection of problems within the complexity class PLS can be cast within this model. Our framework includes a black-box tool for bounding the expected maximum number of steps needed until local search reaches an *exact* local optimum. We instantiate this tool to rederive (and in some cases improve) existing positive results from the literature for various local search problem.

As a prominent application, we provide the first polynomial *smoothed* running time bounds, for PLS-hard instances of the problem of computing (pure Nash) equilibria in congestion games. For the more relaxed notion of $(1+\varepsilon)$-approximate equilibria (under which the problem is known to remain PLS-hard, for **any** constant ε) we provide a totally different analysis that guarantees a smoothed FPTAS for general instances, i.e. running time polynomial on $\frac{1}{\varepsilon}$, ϕ, and the game's description, where ϕ is an upper bound on the density of the random noise imposed on the resource costs under smoothed analysis.

The results presented are based on the following two papers:
https://arxiv.org/abs/2211.07547 [ICALP 2024]; with Alexander Grosz (TU Munich) and Themistoklis Melissourgos (Essex)
https://arxiv.org/abs/2306.10600 [EC 2024]

Jan van den Heuvel (LSE) - *Independent sets in triangle-free graphs*

Independent sets in triangle-free graphs has been the subject of research for more than a century. In this talk I will give some idea where this interest comes from, and then report on some new results and in particular the surprising proofs of those new results.

This is joint work with Pjotr Buys and Ross Kang (University of Amsterdam).

Short Postdoc Talks

Abhiram Natarajan (University of Warwick) - *Partitioning theorems for sets of semi-Pfaffian sets, with applications*

We generalize the seminal polynomial partitioning theorems of Guth and Katz [1, 2] to a set of semi-Pfaffian sets. Specifically, given a set $\Gamma \subseteq \mathbb{R}^n$ of k-dimensional semi-Pfaffian sets, where each $\gamma \in \Gamma$ is defined by a fixed number of Pfaffian functions, and each Pfaffian function is in turn defined with respect to a Pfaffian chain $\vec{q}$ of length r, for any $D \geq 1$, we prove the existence of a polynomial $P \in \mathbb{R}[X_1, \ldots, X_n]$ of degree at most D such that each connected component of $\mathbb{R}^n \setminus Z(P)$ intersects at most $\sim \frac{|\Gamma|}{D^{n-k-r}}$ elements of Γ. Also, under some mild conditions on $\vec{q}$, for any $D \geq 1$, we prove the existence of a Pfaffian function P' of degree at most D defined with respect to $\vec{q}$, such that each connected component of $\mathbb{R}^n \setminus Z(P')$ intersects at most $\sim \frac{|\Gamma|}{D^{n-k}}$ elements of Γ. To do so, given a k-dimensional semi-Pfaffian set $\gamma \subseteq \mathbb{R}^n$, and a polynomial $P \in \mathbb{R}[X_1, \ldots, X_n]$ of degree at most D, we establish a uniform bound on the number of connected components of $\mathbb{R}^n \setminus Z(P)$ that γ intersects; that is, we prove that the number of connected components of $(\mathbb{R}^n \setminus Z(P)) \cap \gamma$ is at most $\sim D^{k+r}$. Finally, as applications, we derive Pfaffian versions of Szemerédi-Trotter-type theorems and also prove bounds on the number of joints between Pfaffian curves.

[1] Larry Guth, Polynomial partitioning for a set of varieties, Mathematical Proceedings of the Cambridge Philosophical Society, vol. 159, Cambridge University Press, 2015, pp. 459-–469.

[2] Larry Guth and Nets Hawk Katz, On the Erdös distinct distances problem in the plane, Annals of mathematics (2015), 155—190.

David Kutner (University of Glasgow) - *MIS, begotten: Generalizing the maximal independent set algorithm with Boolean Networks*

A simple greedy algorithm begets a maximal independent set (MIS) in a graph by starting with the empty set and visiting every vertex, adding it to the set if and only if none of its neighbours are already in the set.

In this talk, we generalize this MIS algorithm so that any starting set is allowed. We view the MIS algorithm as a sequential update of a Boolean network according to a permutation of the vertex set.

We establish that deciding whether all maximal independent sets can be reached from some configuration is coNP-complete; that fixing words (which reach a MIS from any starting configuration) and fixing permutations (briefly, permises) are coNP-complete to recognize; and that permissible graphs (graphs with a permis) are coNP-hard to recognize. We also exhibit large classes of permissible and non-permissible graphs.