ROADSTRIX

RADIX SORT

Radix Sort

- Unlike other sorting methods, radix sort considers the structure of the keys

- Assume keys are represented in a base M number system (M = radix), i.e., if M = 2, the keys are represented in binary

	8	4	2	1	weight
9 =	1	0	0	1	(b = 4)
	3	2	1	0	bit #

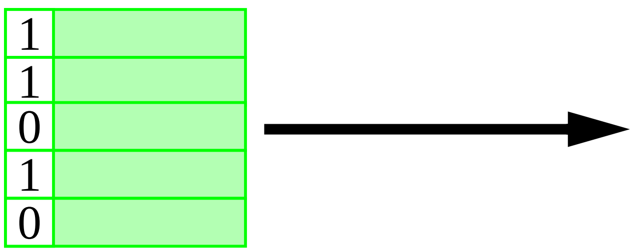
- Sorting is done by comparing bits in the same position

- Extension to keys that are alphanumeric strings

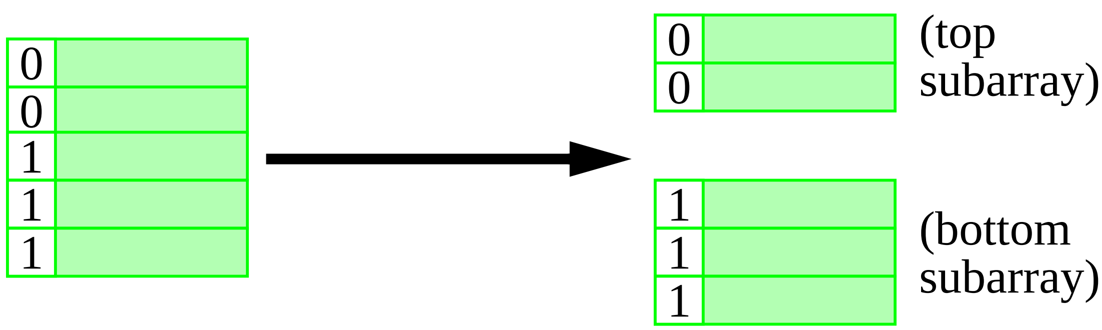
Radix Exchange Sort

Examine bits from ***left*** to ***right***:

1. **Sort array with respect to leftmost bit**



2. **Partition array**



3. **<u>Recursion</u>**
   - recursively sort top subarray, ignoring leftmost bit
   - recursively sort bottom subarray, ignoring leftmost bit

Time: O(b N)

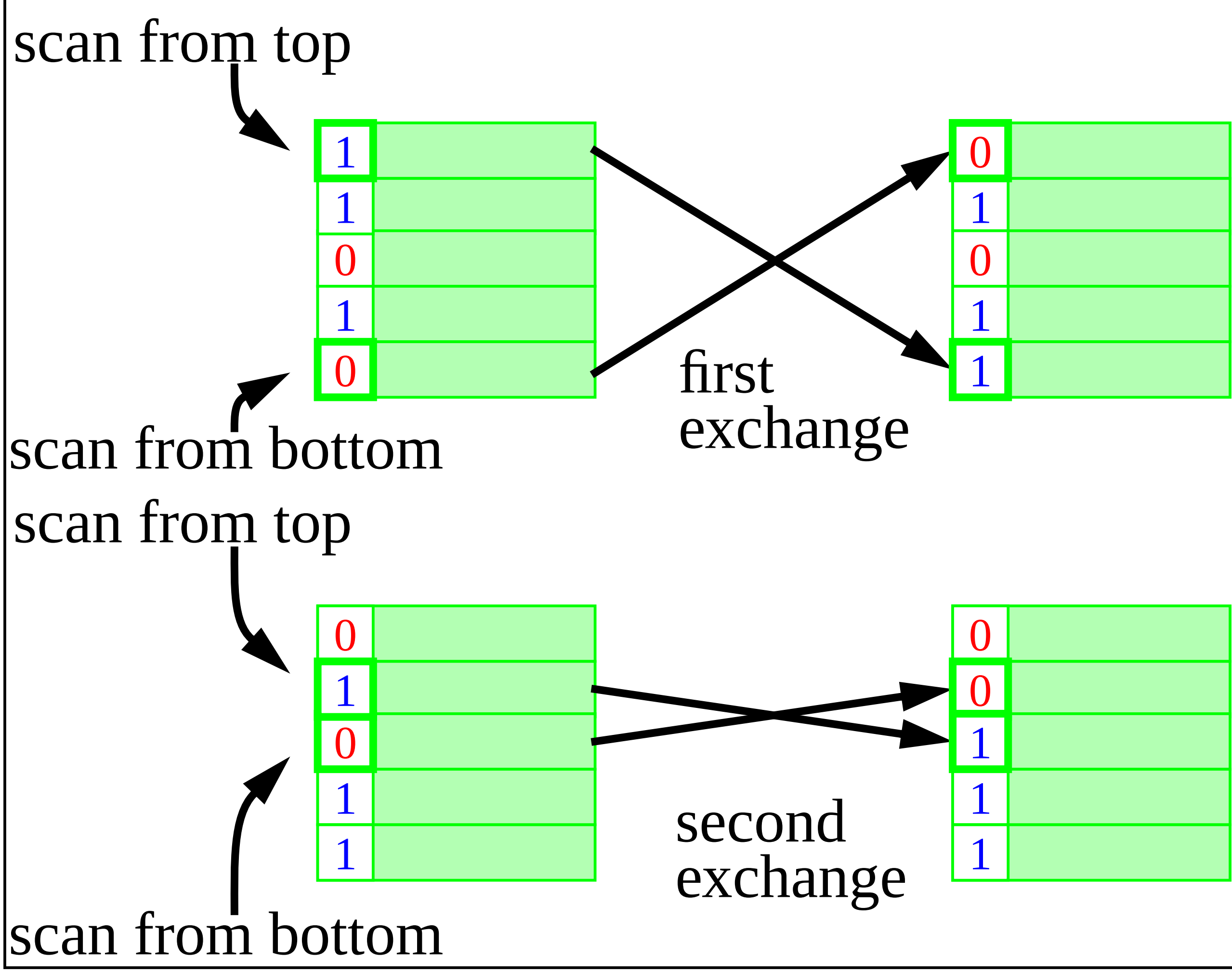
Radix Exchange Sort

How do we do the sort from the previous page?
Same idea as partition in Quicksort.

repeat

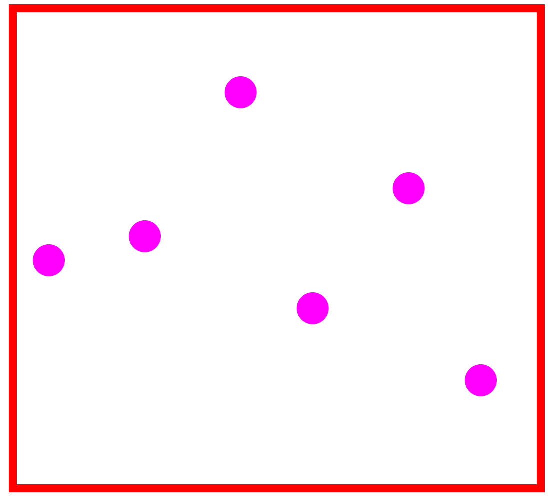
scan top-down to find key starting with 1;
scan bottom-up to find key starting with 0;
exchange keys;

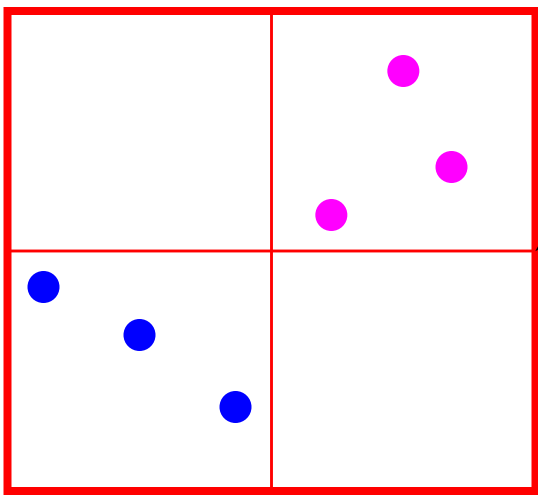
until scan indices cross;



Radix Exchange Sort

array before sort

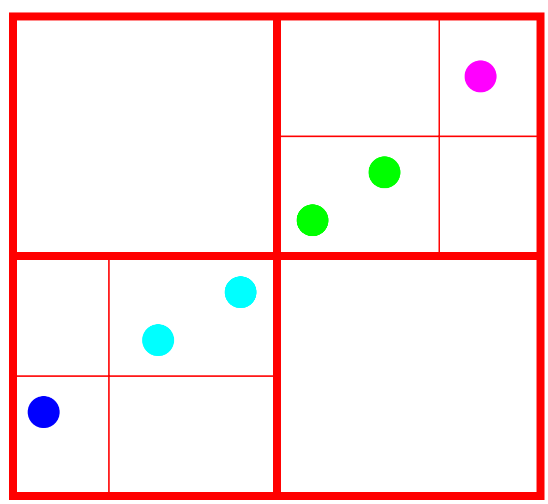




2^{b-1}

array after sort
on leftmost bit

array after recursive
sort on second from
leftmost bit



Radix Exchange Sort vs. Quicksort

Similarities

- both partition array
- both recursively sort sub-arrays

Differences

- ***Method of partitioning***
  - radix exchange divides array based on greater than or less than 2^{b-1}
  - quicksort partitions based on greater than or less than some element of the array
- ***Time complexity***
  - Radix exchange O (bN)
  - Quicksort average case O (N log N)
  - Quicksort worst case O (N^2)

Straight Radix Sort

Examines bits from ***right*** to ***left***

for k := 0 **to** b–1
sort the array in a *stable* way,
looking only at bit k

First, sort these

Next, sort these digits

Last, sort these.

0	1	0
0	0	0
1	0	1
0	0	1
1	1	1
0	1	1
1	0	0
1	1	0

0	1	$\boxed{0}$
0	0	$\boxed{0}$
1	0	$\boxed{0}$
1	1	$\boxed{0}$
1	0	$\boxed{1}$
0	0	$\boxed{1}$
1	1	$\boxed{1}$
0	1	$\boxed{1}$

0	$\boxed{0}$	0
1	$\boxed{0}$	0
1	$\boxed{0}$	1
0	$\boxed{0}$	1
0	$\boxed{1}$	0
1	$\boxed{1}$	0
1	$\boxed{1}$	1
0	$\boxed{1}$	1

$\boxed{0}$	0	0
$\boxed{0}$	0	1
$\boxed{0}$	1	0
$\boxed{0}$	1	1
$\boxed{1}$	0	0
$\boxed{1}$	0	1
$\boxed{1}$	1	0
$\boxed{1}$	1	1

Note order of these bits after sort.

I forgot what it means to "sort in a stable way"!!!

In a stable sort, the initial relative order of equal keys is unchanged.

For example, observe the first step of the sort from the previous page:

0	1	0
0	0	0
1	0	1
0	0	1
1	1	1
0	1	1
1	0	0
1	1	0

0	1	0
0	0	0
1	0	0
1	1	0
1	0	1
0	0	1
1	1	1
0	1	1

Note that the relative order of those keys ending with 0 is unchanged, and the same is true for elements ending in 1

The Algorithm is Correct (right?)

- We show that any two keys are in the correct relative order at the end of the algorithm

- Given two keys, let ***k*** be the leftmost bit-position where they differ

0	1	0	1	1

0	1	1	0	1
		k		

- At step ***k*** the two keys are put in the correct relative order

- Because of <u>stability</u>, the successive steps do not change the relative order of the two keys

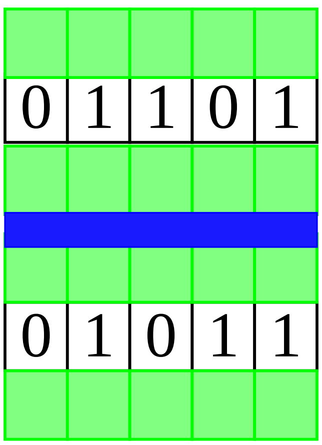
For Instance,

Consider a sort on an array with these two keys:

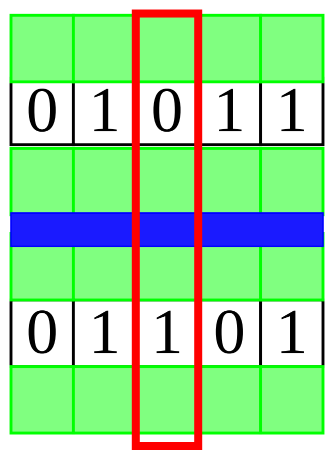
0	1	0	1	1

0	1	1	0	1

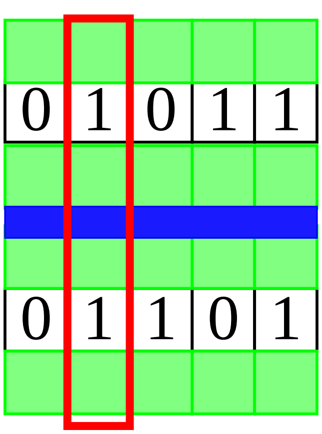
k



It makes no difference what order they are in when the sort begins.



When the sort visits bit ***k***, the keys are put in the correct relative order.



Because the sort is stable, the order of the two keys will not be changed when bits > ***k*** are compared.

Radix sorting can be applied to decimal numbers

First, sort these digits

Next, sort these digits

Last, sort these.

0	3	2
2	2	4
0	1	6
0	1	5
0	3	1
1	6	9
1	2	3
2	5	2

0	3	1
0	3	2
2	5	2
1	2	3
2	2	4
0	1	5
0	1	6
1	6	9

0	1	5
0	1	6
1	2	3
2	2	4
0	3	1
0	3	2
2	5	2
1	6	9

0	1	5
0	1	6
0	3	1
0	3	2
1	2	3
1	6	9
2	2	4
2	5	2

Note order of these bits after sort.

Voila!

Straight Radix Sort

Time Complexity

for k := 0 **to** b-1

sort the array in a *stable* way,

looking only at bit k

Suppose we can perform the stable sort above in O(N) time. The total time complexity would be

O(bN).

As you might have guessed, we can perform a stable sort based on the keys' $\boldsymbol{k}^{\text{th}}$ digit in O(N) time.

The method, you ask? Why it's Bucket Sort, of course.

Bucket Sort

- N numbers
- Each number $\in$ {1, 2, 3, ... M}
- Stable
- Time: O (N + M)

For example, M = 3 and our array is:

2	1	3	1	2

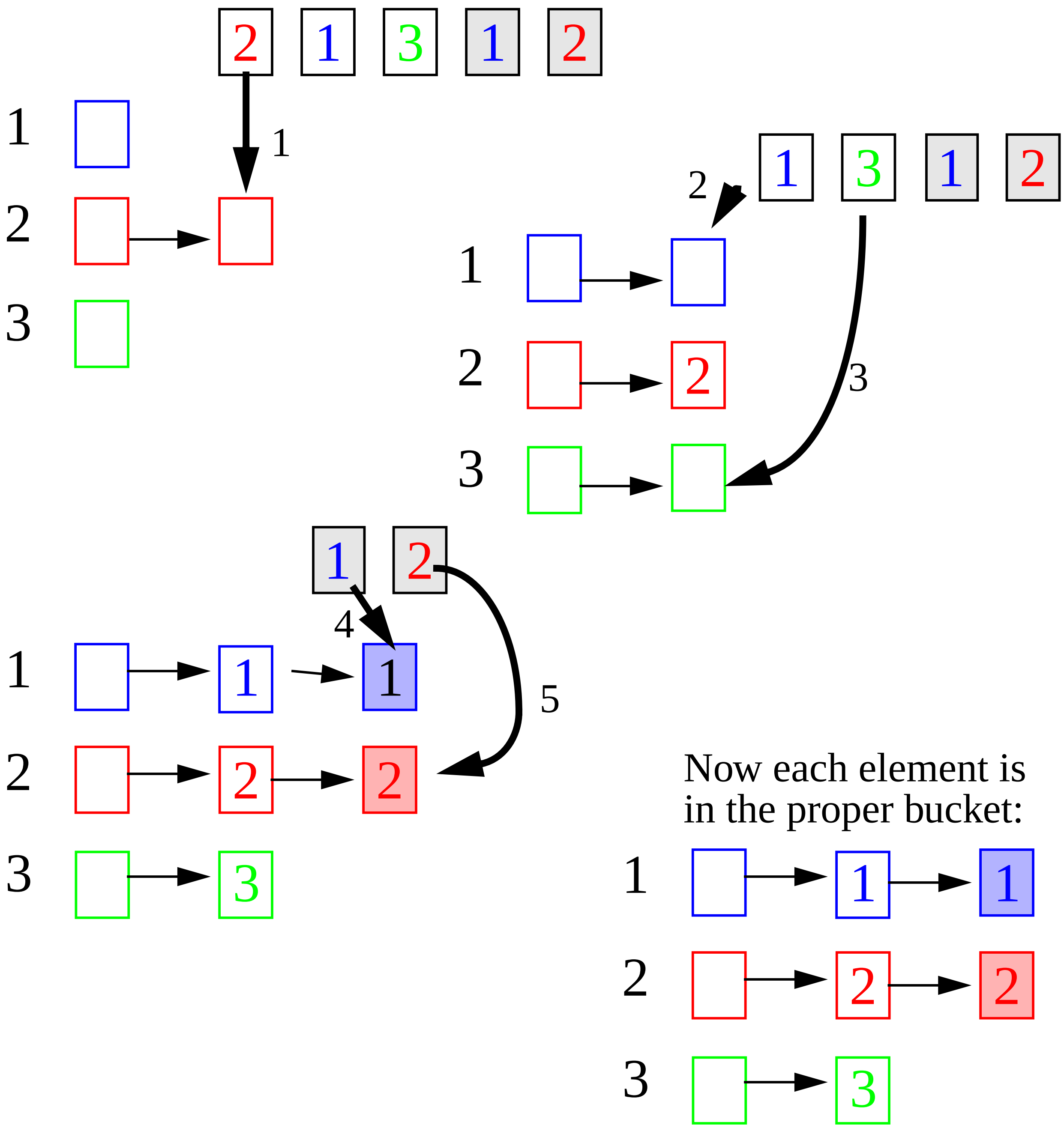
(note that there are two “2”s and two “1”s)

First, we create M “buckets”

1	
2	
M = 3	

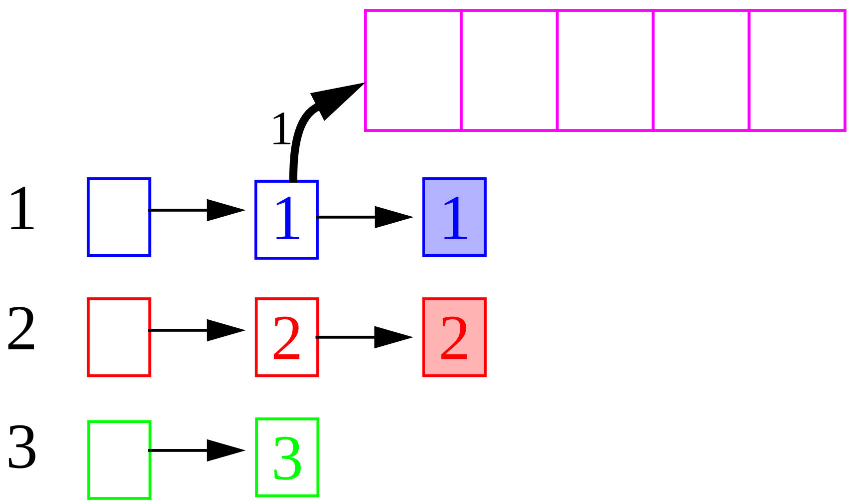
Bucket Sort

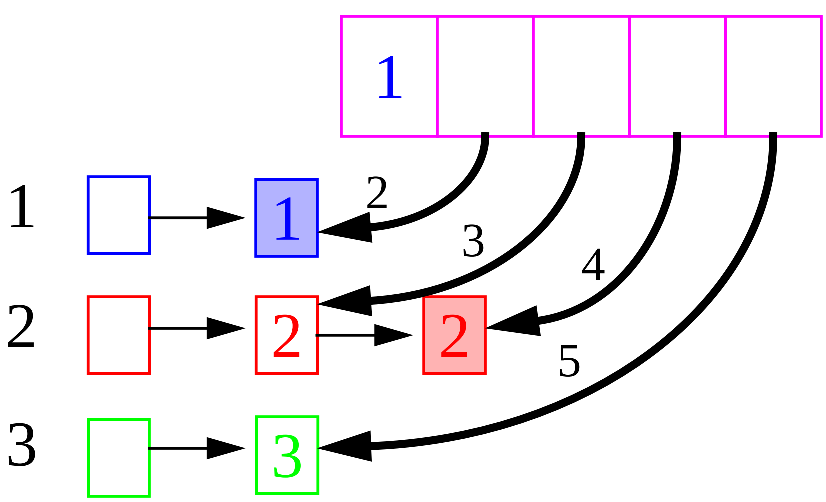
Each element of the array is put in one of the M "buckets"



Bucket Sort

Now, pull the elements from the buckets into the array





At last, the sorted array (sorted in a stable way):

1	1	2	2	3